X. Charge conjugation and parity in weak interactions

REMINDER:

Parity

❖ The parity transformation is the transformation by reflection:

→ $\vec{x}_i \rightarrow \vec{x}'_i = -\vec{x}_i$

A parity operator $\hat{P}$ is defined as

→ $\hat{P}\psi((\vec{x}, t) = p\psi(-\vec{x}, t))$ where p = $\pm$1

Charge conjugation

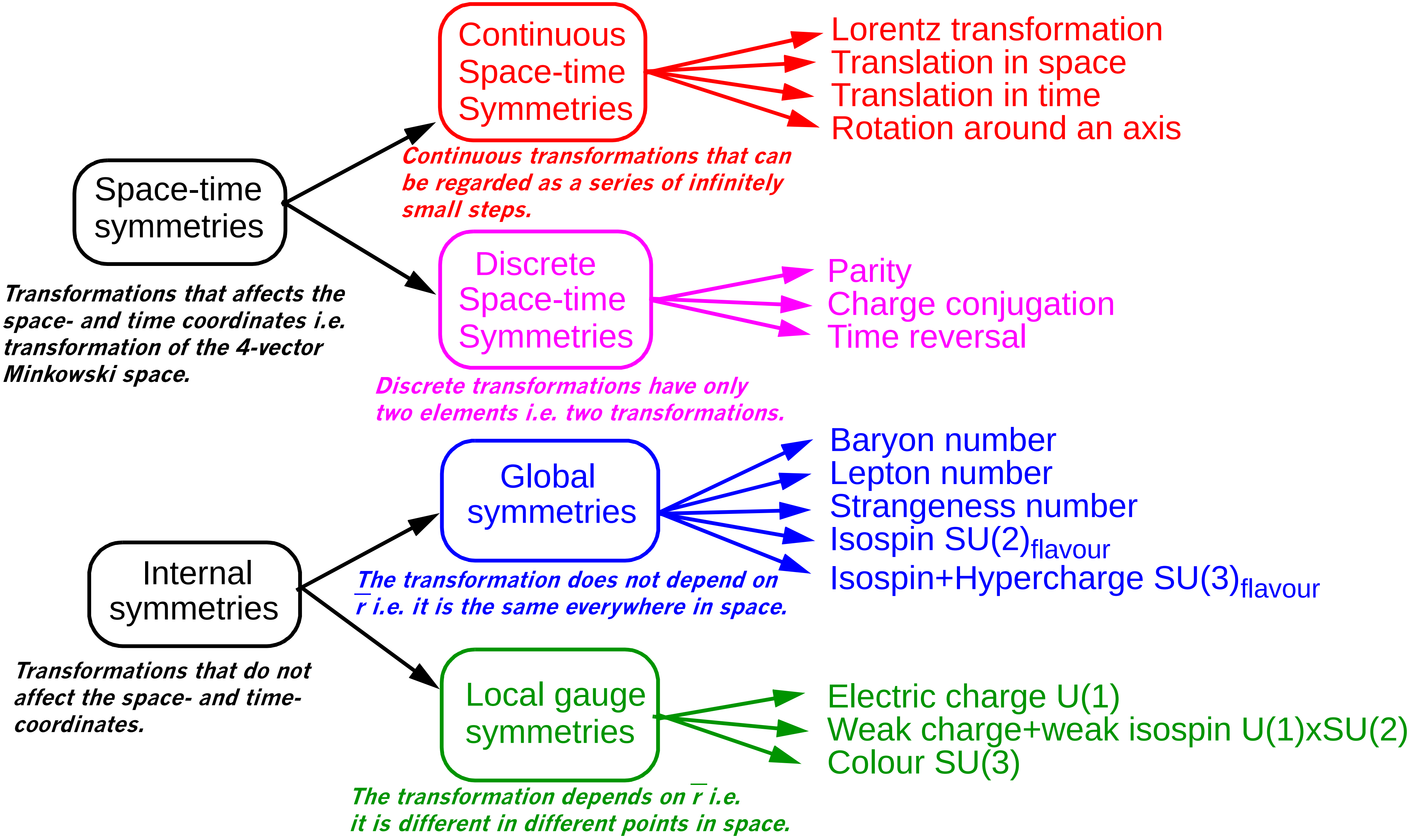
❖ The charge conjugation replaces particles by their antiparticles, reversing charges and magnetic moments

→ $\hat{C}\Psi_a = c\Psi_{\bar{a}}$ where c = $\pm$1

meaning that from the particle in the initial state we go to the antiparticle in the final state.

Reminder

Symmetries



Space-time symmetries

Transformations that affects the space- and time coordinates i.e. transformation of the 4-vector Minkowski space.

Continuous Space-time Symmetries

Continuous transformations that can be regarded as a series of infinitely small steps.

- Lorentz transformation
- Translation in space
- Translation in time
- Rotation around an axis

Discrete Space-time Symmetries

Discrete transformations have only two elements i.e. two transformations.

- Parity
- Charge conjugation
- Time reversal

Internal symmetries

Transformations that do not affect the space- and time-coordinates.

Global symmetries

The transformation does not depend on $\overline{r}$ i.e. it is the same everywhere in space.

- Baryon number
- Lepton number
- Strangeness number
- Isospin $SU(2)_{flavour}$
- Isospin+Hypercharge $SU(3)_{flavour}$

Local gauge symmetries

The transformation depends on $\overline{r}$ i.e. it is different in different points in space.

- Electric charge U(1)
- Weak charge+weak isospin U(1)xSU(2)
- Colour SU(3)

- While parity is conserved in strong and electromagnetic interactions, it is violated in weak processes:

– 1956: Based on the measurements of Kaon decays, Lee & Yang propose that parity is violated in weak processes:

Two known decays of the K^+ were:

$K^+ \to \pi^0 + \pi^+$ and $K^+ \to \pi^+ + \pi^+ + \pi^-$

The intrinsic parity of a pion $P_\pi = -1$, and for the $\pi^0\pi^+$ and $\pi^+\pi^+\pi^-$ states the parities are

$$P_{\pi\pi} = P_\pi^2(-1)^L = 1 \quad \text{since } L = L_{12} = 0$$

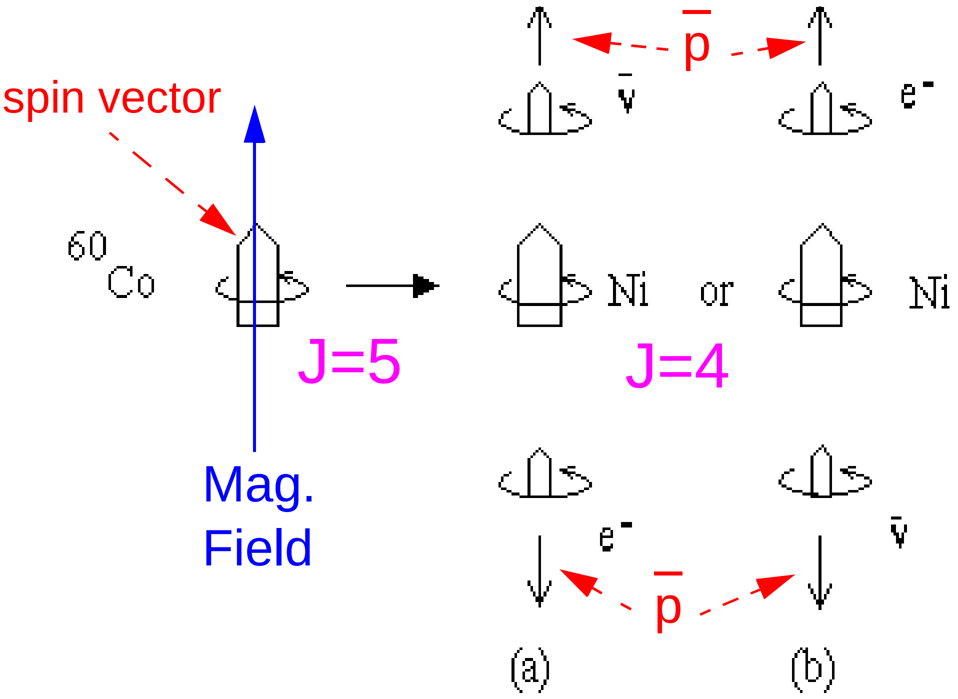
$$P_{\pi\pi\pi} = P_\pi^3(-1)^L = -1 \quad \text{since } L = L_{12} + L_3 = 0$$

- Since the two final states have opposite parities, one of the K^+ decays must **violate** parity!

– 1957: Wu carries out studies of parity violation in β-decay. The ^{60}Co β-decay into $^{60}\mathrm{Ni}^{*}+e^{-}+\bar{\nu}_e$ was studied.

– The ^{60}Co sample was cooled to 0.01 K to prevent thermal disorder.

– The sample was placed in a magnetic field ⇒ the nuclear spins were aligned along the field direction



Possible β-decays of ^{60}Co: case (a) is preferred.

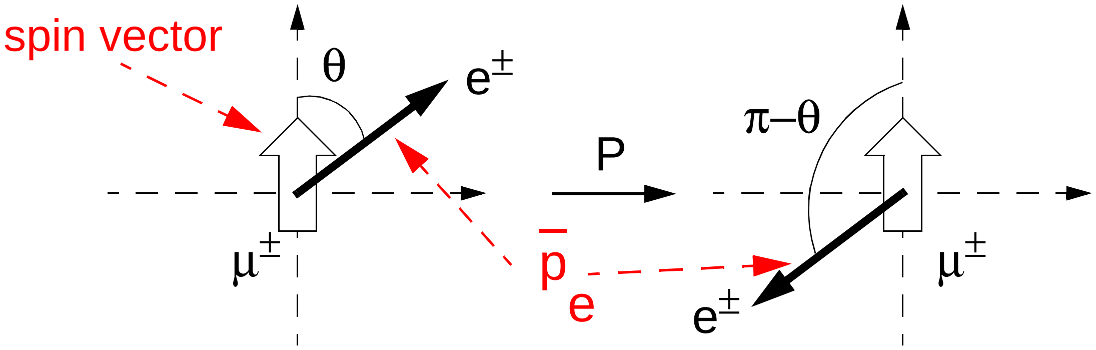
– If parity is conserved, processes (a) and (b) must have equal rates.

❖ *Electrons were emitted predominantly in the direction opposite to the* ^{60}*Co spin*

→ Another case of both parity and C-parity violation was observed in muon decays:

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$



Effect of a parity transformation on the muon decays above

The angular distribution of the electrons (positrons) emitted in μ^- (μ^+) decay are given by

$$\Gamma_{\mu^\pm}(\cos\theta) = \frac{1}{2}\Gamma_\pm\left(1 - \frac{\xi_\pm}{3}\cos\theta\right) \qquad (136)$$

here $\xi_\pm$ are constants – "*asymmetry parameters*", and $\Gamma_\pm$ are total decay rates ⇒ inverse lifetimes

$$\Gamma_\pm = \int_{-1}^{1} \Gamma_\pm(\cos\theta)\, d\cos\theta \equiv \frac{1}{\tau_\pm} \qquad (137)$$

→ If the process is invariant under charge conjugation (C-invariance) ⇒

$$\Gamma_+ = \Gamma_- \qquad \xi_+ = \xi_- \qquad (138)$$

(rates and angular distributions are the same for e^- and e^+)

→ If the process is P-invariant, then angular distributions in forward and backward directions are the same:

$$\Gamma_{\mu^\pm}(\cos\theta) = \Gamma_{\mu^\pm}(-\cos\theta) \qquad \xi_+ = \xi_- = 0 \quad (139)$$

→ Experimental results:

$$\Gamma_+ = \Gamma_- \qquad \xi_+ = -\xi_- = 1{,}00 \pm 0{,}04 \qquad (140)$$

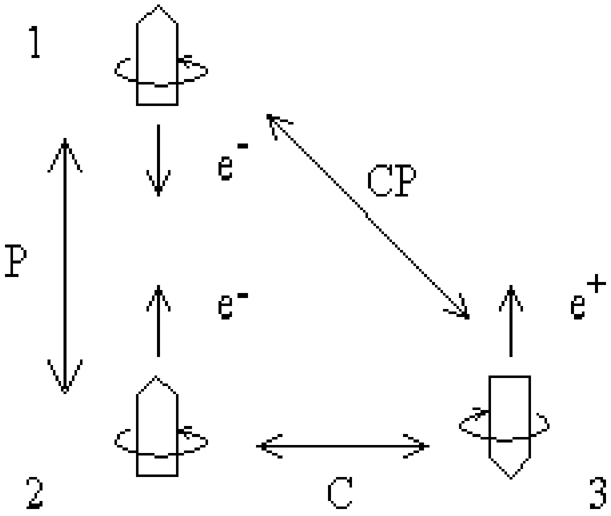
❖ Both C- and P-invariance are violated!

→ However, the combined operation CP is conserved since that requires

$$\Gamma_{\mu^+}(\cos\theta) = \Gamma_{\mu^-}(-\cos\theta) \tag{141}$$

$$\Gamma_+ = \Gamma_- \qquad \xi_+ = -\xi_- \tag{142}$$

which is in agreement with the experiments.



P-, C- and CP-transformation of an electron

❖ The combined transformation CP is a weaker requirement than the individual transformations P and C and it is conserved.

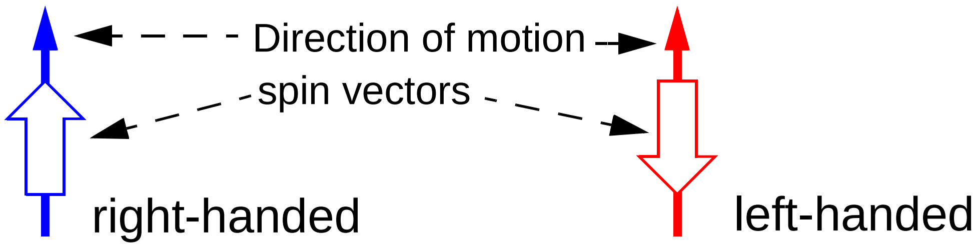
Helicity

helicity – the spin is quantized along the particle´s direction of motion instead of along an arbitrary z-direction

$$\hat{\Lambda} = \frac{\vec{s} \bullet \vec{p}}{|\vec{p}|} \qquad (143)$$

$$\hat{\Lambda}\psi = \lambda\psi$$

The eigenvalues of the helicity operator are $\lambda = -s, -s+1, \ldots, +s$, ⇒ for spin-1/2 particle it can be either -1/2 or 1/2



Helicity states of spin-1/2 particle

A particle with $\lambda = +1/2$ is called right handed.

A particle with $\lambda = -1/2$ is called left handed.

A subscript R or L is used to denote if a state is right or left handed e.g. e^-_R and ν_L

- 1958: Goldhaber et al. measured the helicity of the neutrino by studying electron capture in europium:

$$e^- + {}^{152}\mathrm{Eu} \rightarrow {}^{152}\mathrm{Sm}^* + \nu_e \qquad (144)$$

$$\hookrightarrow {}^{152}\mathrm{Sm} + \gamma \qquad (145)$$

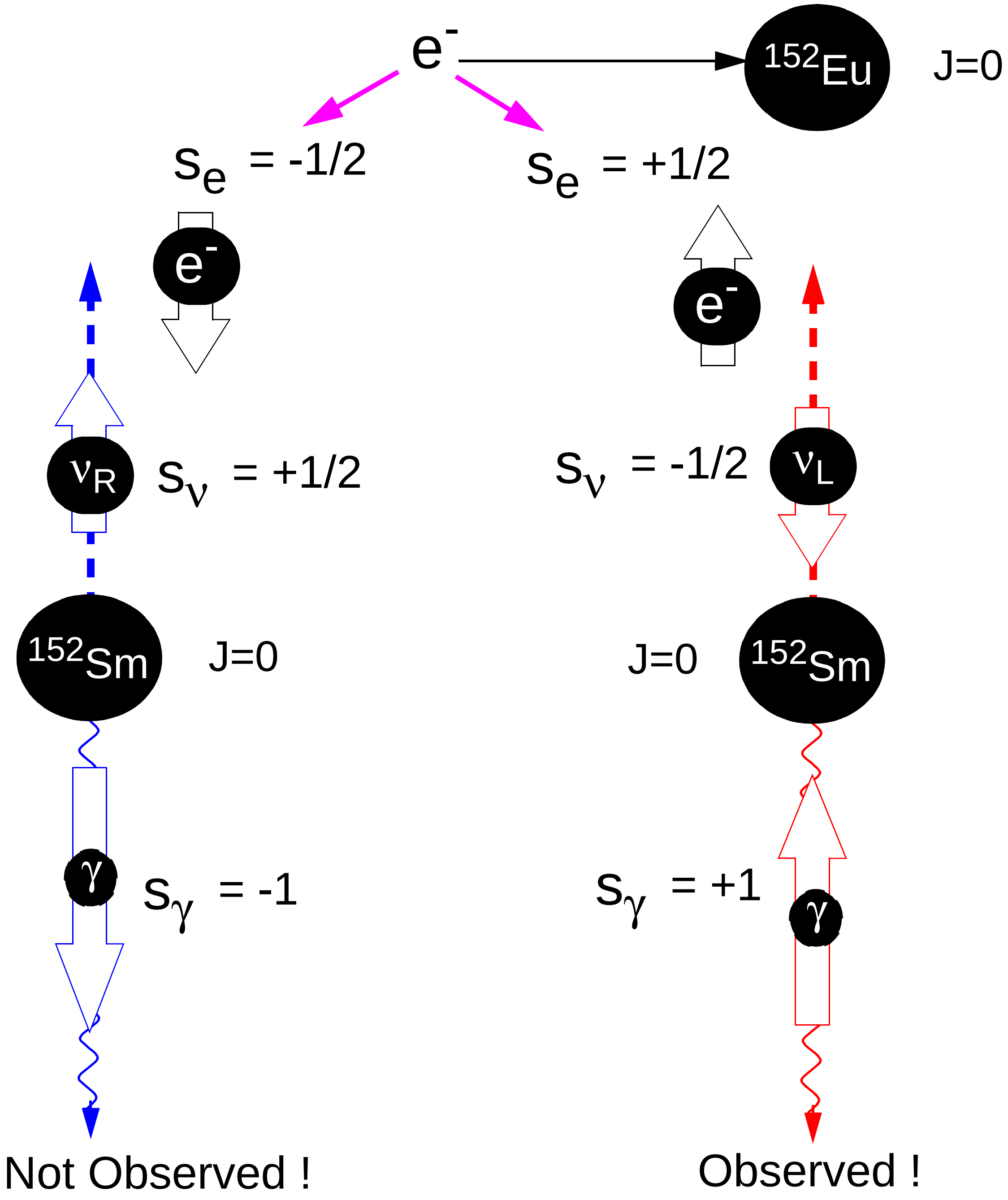
- In this reaction the initial state has zero momentum and ${}^{152}\mathrm{Sm}^*$ and ν_e recoil in opposite directions.

- Events with the γ emitted in the direction of motion of the ${}^{152}\mathrm{Sm}^*$ were selected so that the overall observed reaction was:

$$e^- + {}^{152}\mathrm{Eu}\,(J=0) \rightarrow {}^{152}\mathrm{Sm}(J=0) + \nu_e + \gamma \qquad (146)$$

- The spin of the neutrino (+1/2 or -1/2) and the photon (+1 or -1) must add to give the spin of the electron (+1/2 or -1/2).

- The helicity (polarization) of the photons was determined by studying their absorption in magnetized iron.



From the helicity of the photons it is possible to determine the helicity of the neutrinos.

❖ From the measured photon helicity it was concluded that neutrinos must be left-handed.

V-A interaction

- V-A interaction theory was introduced by Fermi as an analytic description of spin dependence of charged current interactions.

- It denotes “polar Vector - Axial vector” interaction

– *A Polar vector* is one which direction is reversed by parity transformation e.g. momentum $\vec{p}$

– *An Axial vector* is one which direction is not changed by parity transformation e.g. spin $\vec{s}$ or orbital angular momentum $\vec{L} = \vec{r} \times \vec{p}$

– The weak current has both vector and axial components, hence parity is not conserved in weak interactions

→ Main conclusion: if $v \approx c$, only left-handed fermions ν_L, e^-_L etc. are emitted, and right-handed antifermions.

→ *The very existence of preferred states violates both C- and P- invariance*

- Neutrinos (antineutrinos) are always relativistic and hence always left(right)-handed

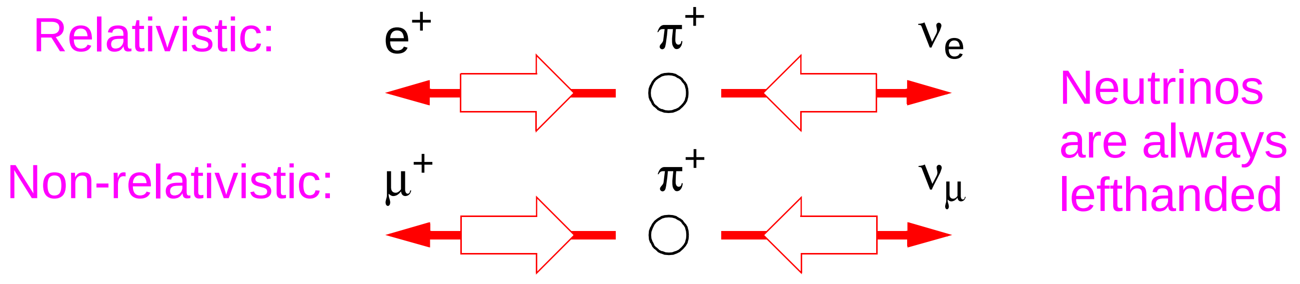
- For other fermions, the preferred states are left-handed. Right-handed states are not completely forbidden but suppressed by the factor

$$\left(1 - \frac{v}{c}\right) \approx \frac{m^2}{2E^2} \qquad (147)$$

Consider the two pion decay modes:

$$\pi^+ \rightarrow e^+ + \nu_e \qquad (148)$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \qquad (149)$$



Neutrinos are always lefthanded

Helicities of leptons emitted in a pion decay

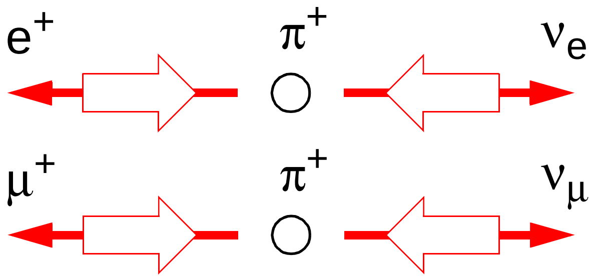
– The π^+ has spin-0 and it is at rest ⇒ the spins of the charged lepton and the neutrino must be opposite.

– The neutrinos are always left-handed ⇒ the charged leptons have to be left-handed as well.

– BUT: the e^+ and the μ^+ should be right-handed since they are anti-fermions.

- In these decays the electron will be relativistic but not the muon (due to its large mass).

- It follows that the pion to muon decay should be allowed but the pion to positron decay should be suppressed.



Left-handed

Should be right-handed
since it is relativistic

Can be left-handed
since it is non-relativistic

– The suppression factor for positrons is expected to be of the order 10^{-5}.

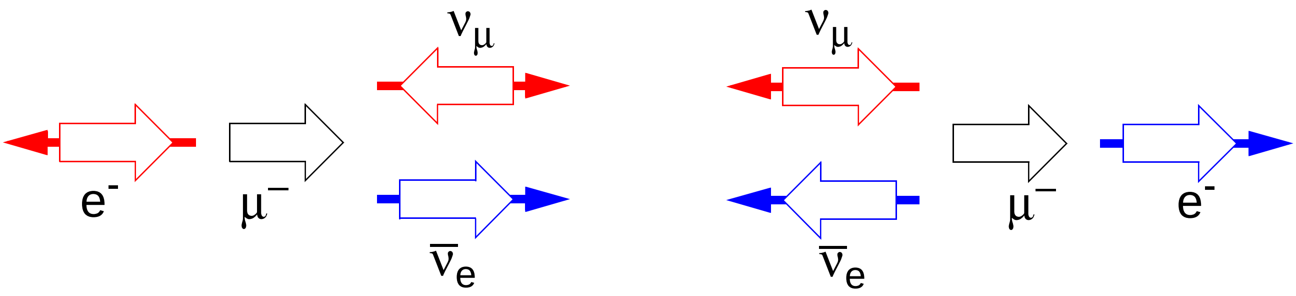
The measured ratio:

$$\frac{\Gamma(\pi^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = (1{,}230 \pm 0{,}004)\times 10^{-4} \qquad (150)$$

❖ Muons emitted in pion decays are always polarized and this can be used to measure muon decay symmetries by detecting the relativistic electrons in the following decays:

$$\begin{tikzcd}[column sep=small] \pi^- \arrow[r] & \mu^- \arrow[dr] & + \bar{\nu}_\mu \\ & & e^- + \bar{\nu}_e + \nu_\mu \end{tikzcd} \qquad (151)$$

The electrons are emitted in decays when both the ν_μ and the $\bar{\nu}_e$ are emitted in the direction opposite to the e^-:



(a) Left-handed electrons Favoured

(b) Right-handed electrons Surpressed

Muon decays with high energy electron emission.

The electron must have a spin parallel to the muon spin ⇒ configuration (a) with left-handed electrons is strongly preferred ⇒ this is observed experimentally as a forward-backward asymmetry.

Neutral kaons

- It is possible to produce the neutral kaons $K^0 = d\bar{s}$ and $\bar{K}^0 = s\bar{d}$ in πp-collisions. This is a strong interaction process and strangeness has to be conserved:

$$\pi^- + p \rightarrow K^0 + \Lambda$$

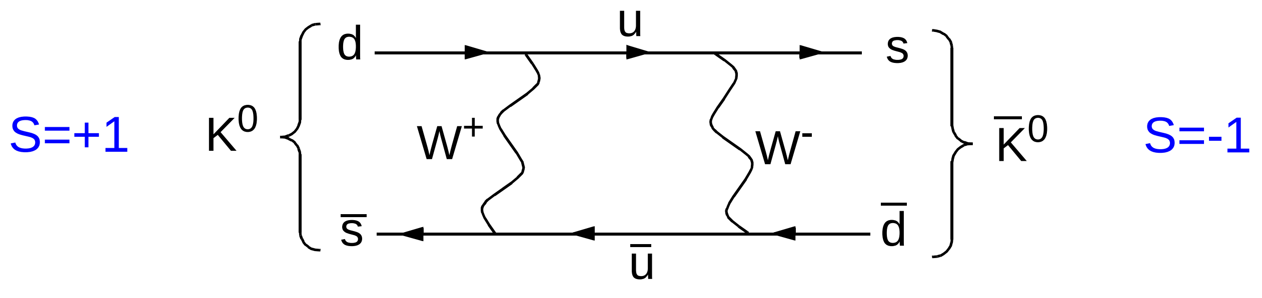
s: 0 0 +1 -1

$$\pi^+ + p \rightarrow \bar{K}^0 + p + K^+$$

s: 0 0 -1 0 +1

- The kaons that are produced in this way are pure $K^0 = d\bar{s}$ and $\bar{K}^0 = s\bar{d}$ states.

- However, K^0 and $\bar{K}^0$ can be converted into each other since strangeness is not conserved in weak interactions:



Example of a process converting K^0 to $\bar{K}^0$.

- The observed physical particles are linear combinations of K^0 and $\bar{K}^0$, since there is no conserved quantum number to distinguish them. The phenomenon is called *K^0–$\bar{K}^0$ mixing* .

- We know that neither parity nor charge conjugation are conserved in weak decays. The combined operation CP is, however, almost conserved.

- In this case the CP operators eigenstates can be written as a mixture of K^0 and $\bar{K}^0$:

$$K_1^0 = \frac{1}{\sqrt{2}}\{K^0 + \bar{K}^0\} \tag{152}$$

$$K_2^0 = \frac{1}{\sqrt{2}}\{K^0 - \bar{K}^0\} \tag{153}$$

so that

$$\hat{C}PK_1^0 = K_1^0 \text{ and } CPK_2^0 = -K_2^0 \tag{154}$$

i.e. the CP eigenvalues are cp=+1 for K_1^0 and cp=-1 for K_2^0 and the K_1^0 can therefore only decay to cp-even states while the K_2^0 only to cp-odd states.

Experimentally observed are two types of neutral kaons: K_S^0 ("S" for "short", lifetime $\tau = 0{,}9 \times 10^{-10} s$) and K_L^0 ("long", $\tau = 500 \times 10^{-10} s$).

◆ Can the K_S^0 be identified with the K_1^0 CP-eigenstate, and the K_L^0 with the K_2^0 ?

→ If CP-invariance holds for neutral kaons, K_S^0 should decay only into states with cp=1 such as 2π-states, and K_L^0 into states with cp=-1 such as 3π-states:

$$K_S^0 \to \pi^+\pi^-, \qquad K_S^0 \to \pi^0\pi^0 \tag{155}$$

– The parity of a two-pion state is $P = P_\pi^2(-1)^L = 1$

– The C-parity of a $\pi^0\pi^0$ state is $C = (C_{\pi^0})^2 = 1$, and of a $\pi^+\pi^-$ state: $C = (-1)^L = 1$

– i.e. cp= 1 for the $\pi^+\pi^-$ and $\pi^0\pi^0$ states

– i.e. the assumption that $K_S^0 = K_1^0$ seem to be correct.

$$K_L^0 \to \pi^+\pi^-\pi^0, \qquad K_L^0 \to \pi^0\pi^0\pi^0 \qquad (156)$$

– The parity of the 3-π states are -1

– The C-parity of $\pi^0\pi^0\pi^0$ is $C = (C_{\pi^0})^3 = 1$

– The C-parity of $\pi^+\pi^-\pi^0$ is $C = C_{\pi^0}(-1)^{L_{\pi\pi}} = 1$

– i.e. the 3-π final states above have cp=-1

– i.e. the assumption that $K_L^0 = K_2^0$ seem to be correct.

❖ Summary:

The neutral Kaon eigenstates in strong interactions are:

$$\boxed{\begin{aligned} K^0 &= d\bar{s} \\ \bar{K}^0 &= s\bar{d} \end{aligned}}$$

The neutral Kaon eigenstates in weak interactions (if CP is conserved) are:

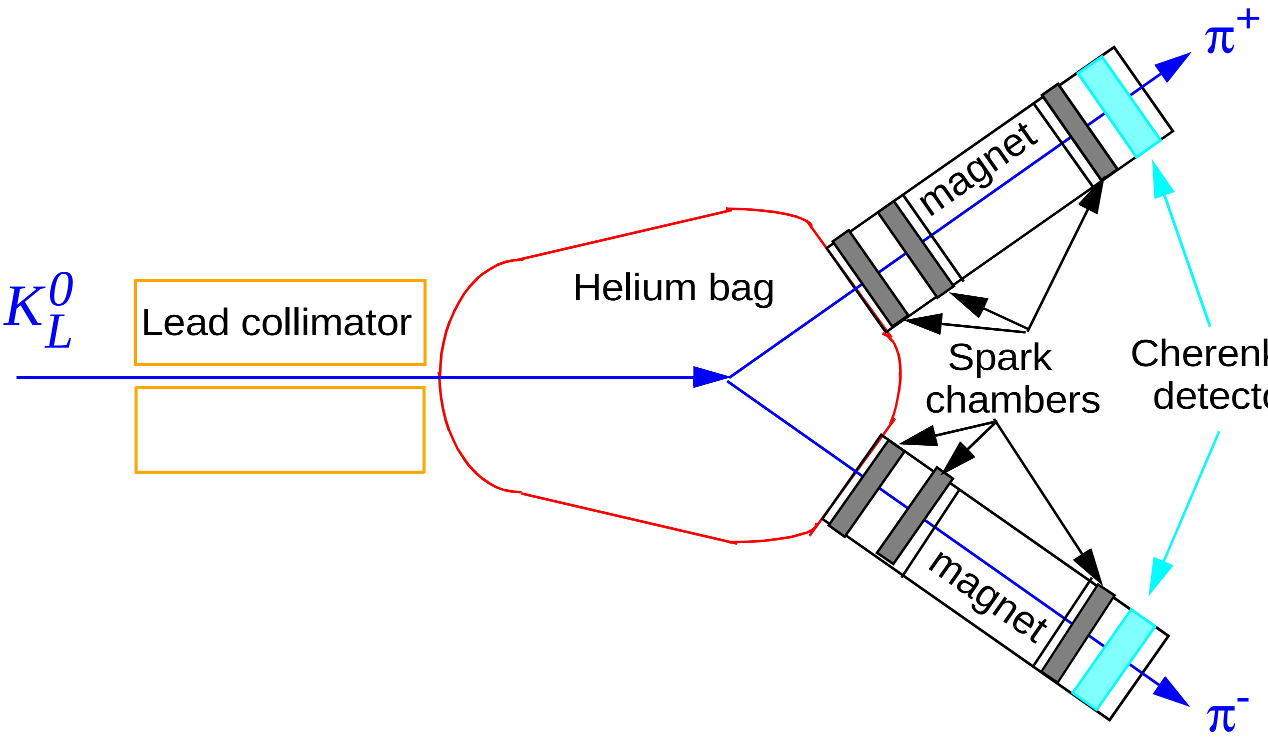
$$\boxed{\begin{aligned} K_s^0 = K_1^0 &= \frac{1}{\sqrt{2}}\{K^0 + \bar{K}^0\} \\ K_L^0 = K_2^0 &= \frac{1}{\sqrt{2}}\{K^0 - \bar{K}^0\} \end{aligned}}$$

CP-violation

The *CP-violating* decay

$$K_L^0 \rightarrow \pi^+ \pi^- \qquad (157)$$

was first observed in 1964, with a branching ratio of B≈10^{-3}.



Sketch of the experiment that discovered CP-violation in weak decays.

❖ In general, the physical states K_S^0 and K_L^0 don’t have to correspond to pure CP-eigenstates K_1^0 and K_2^0. Instead

$$\boxed{\begin{aligned} K_S^0 &= \frac{1}{\sqrt{1+|\varepsilon|^2}}\{K_1^0 + \varepsilon K_2^0\} \\ K_L^0 &= \frac{1}{\sqrt{1+|\varepsilon|^2}}\{\varepsilon K_1^0 + K_2^0\} \end{aligned}}$$

where ε is a small complex parameter: $|\varepsilon| = 2\times 10^{-3}$

❖ K_S^0 contains mostly K_1^0 but has also a small K_2^0 component while K_L^0 consists mostly of K_2^0 with a small component of K_1^0.

→ Mixing occur also for neutral B-mesons ($B^0 = d\bar{b}$, $\bar{B}^0 = b\bar{d}$, $B_s = s\bar{b}$ and $\bar{B}_s = b\bar{s}$) and for neutral D-mesons ($D^0 = c\bar{u}$ and $\bar{D}^0 = u\bar{c}$).

→ There can be different mechanisms for CP-violation, especially in the B^0-$\bar{B}^0$ systems. Several dedicated experiments have been built to study this system.

Summary

- **Parity and charge conjugation**

  a) Parity is violated in weak processes.

  b) Parity violation was first observed in ^{60}Co-decays.

  c) Muon decays can be used to show that both parity and charged conjugation is violated while the combined CP operation is conserved.

- **Helicity**

  d) Helicity is the spin quantized along the direction of motion.

  e) Neutrinos are left-handed and antineutrinos right-handed.

  f) This was first observed in reactions between electrons and ^{152}Eu atoms.

• V-A interactions

g) While neutrinos are always left-handed other fermions are exclusively left-handed only when they are relativistic.

• Neutral kaons

h) The neutral kaons that are observed experimentally (K^0_S and K^0_L) are due to K^0-$\overline{K}^0$ mixing

i) CP-violating decays of neutral kaons have been observed with a small branching ratios.