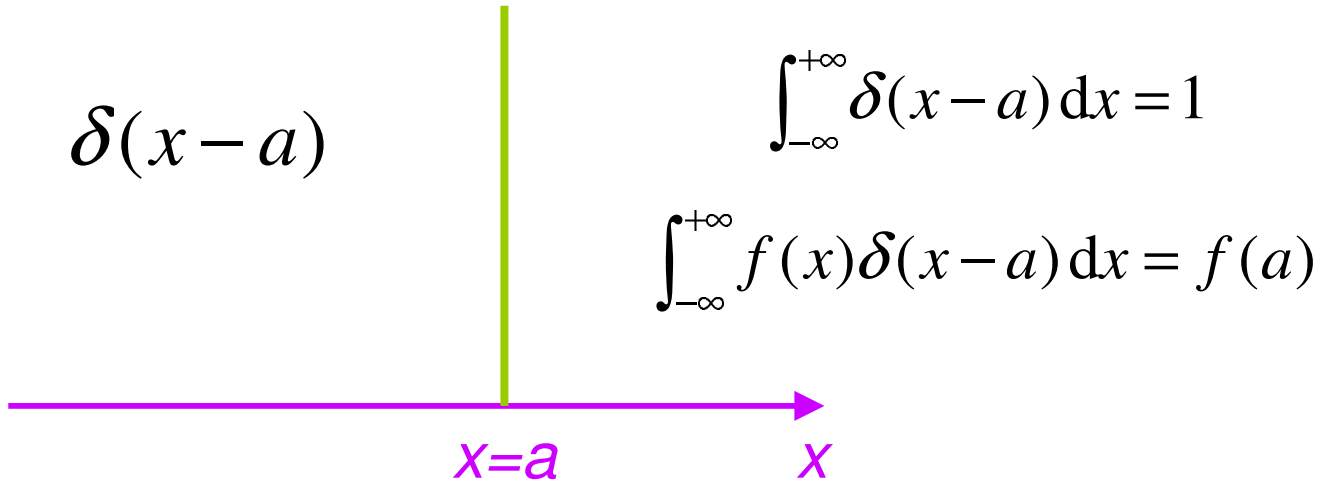


Dirac δ function

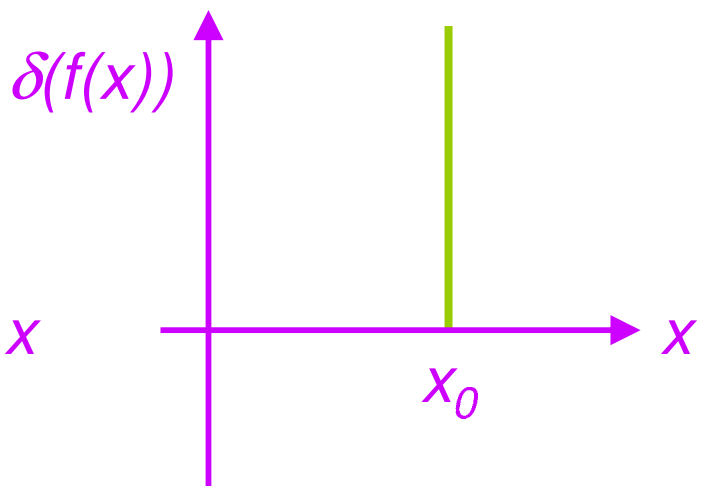
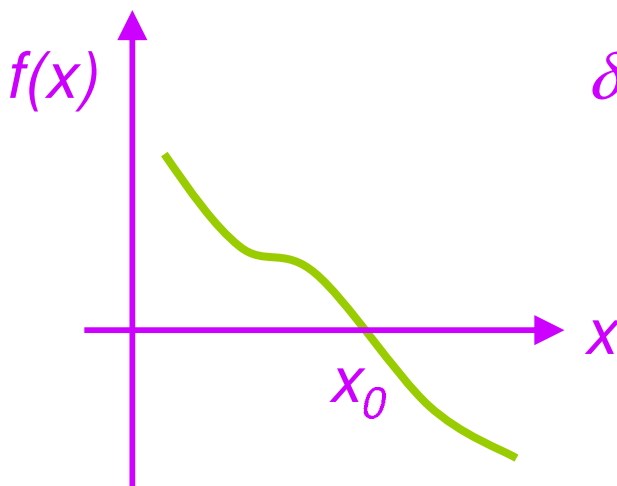
- ◆ Infinitely narrow spike with unit area:



(Handout 3.11)

- ◆ Will need two results:

$$1) \quad \delta(f(x)) = \left| \frac{1}{df/dx} \right|_{x=x_0} \delta(x-x_0)$$



$$2) \quad \delta(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ikx} dk$$

Transition Rates

- ◆ Fermi's Golden Rule: (Handout 3.1)

$$d\Gamma = 2\pi |T_{fi}|^2 \delta(E_i - E_f) \times \left(\begin{array}{l} \text{density of} \\ \text{final states} \end{array} \right)$$

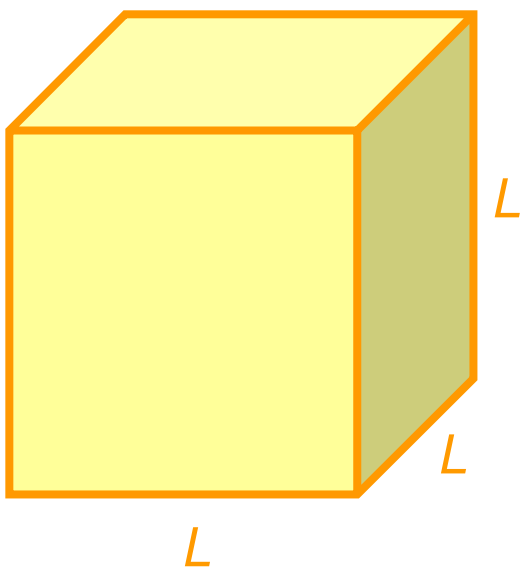
↑
energy conservation

1st order perturbation theory: $T_{fi} = \langle f | H' | i \rangle$

- ◆ Density of final states: (Handout 3.2)

normalise wavefunctions inside cube of volume $V = L^3$

impose periodic boundary conditions



$$k_x = \frac{2\pi n_x}{L}$$

$$k_y = \frac{2\pi n_y}{L}$$

$$k_z = \frac{2\pi n_z}{L}$$

each state takes up volume
(in **k** space)

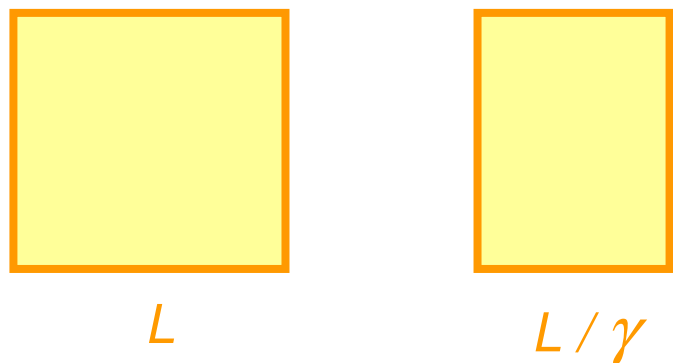
$$\left(\frac{2\pi}{L} \right)^3 = \frac{(2\pi)^3}{V}$$

- ◆ Normalising to one particle per unit volume, and using $p = k$

$$\Rightarrow \text{factor } \frac{d^3 p}{(2\pi)^3} \text{ for each particle}$$

- ◆ In relativity, cube is Lorentz contracted by a factor

$$\gamma = \frac{E}{m}$$



\Rightarrow convenient to normalise to $2E$ particles per unit volume

- ◆ Matrix elements are then Lorentz invariant

$$M_{fi} = \langle f' | H' | i' \rangle \quad \begin{aligned} |i'\rangle &= \sqrt{2E} |i\rangle \\ |f'\rangle &= \sqrt{2E} |f\rangle \end{aligned}$$

and phase space factor becomes

$$\frac{d^3 p}{(2\pi)^3 2E} \text{ per particle}$$

“Lorentz Invariant Phase Space” (LIPS)

Particle Decays

◆ For decays $a \rightarrow 1 + 2 + \dots + n$

$$\Gamma = \frac{1}{2E_a} \times \int |M_{fi}|^2 \times \delta(\dots) \times d(\text{LIPS})$$

Time dilation

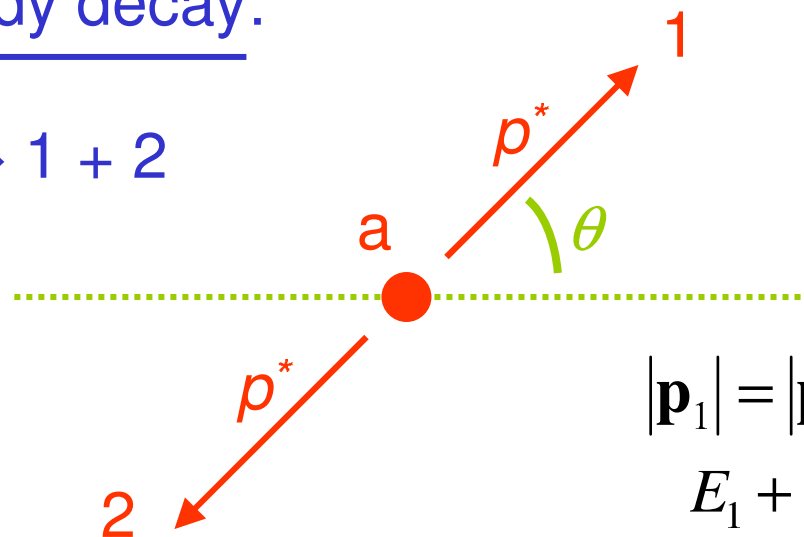
Lorentz invariant
matrix element

E, p
cons.

Lorentz Invariant
Phase Space

◆ Two-body decay:

$a \rightarrow 1 + 2$



$$|\mathbf{p}_1| = |\mathbf{p}_2| = p^*$$

$$E_1 + E_2 = m_a$$

Total decay rate is

(Handout 3.3)

$$\Gamma = \frac{p^*}{32\pi^2 m_a^2} \times \int |M_{fi}|^2 \times d\cos\theta d\phi$$

For isotropic decay :
(e.g. spin 0)

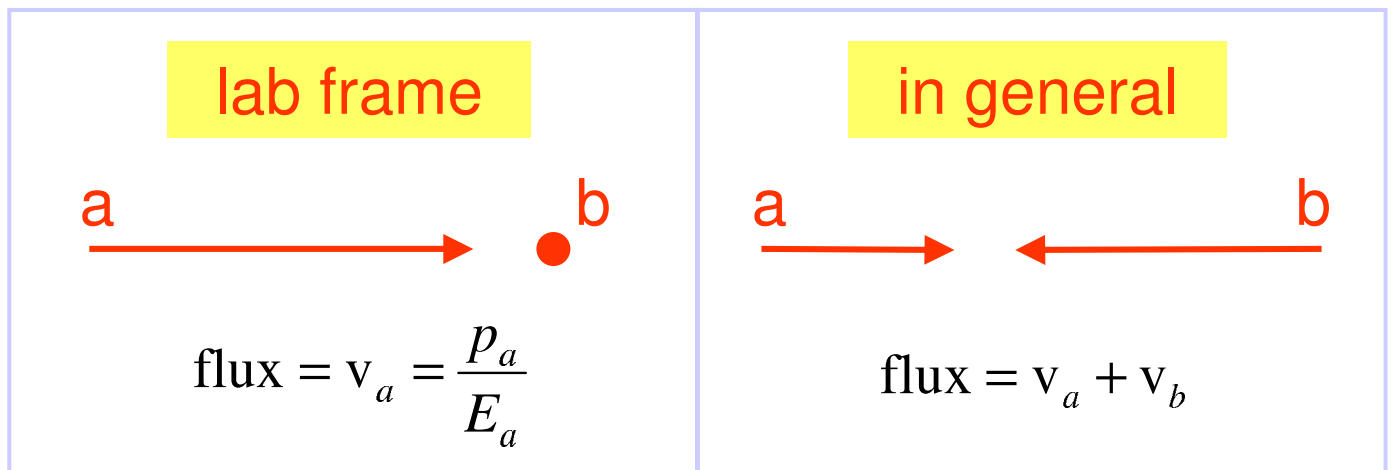
$$\Gamma = \frac{p^*}{8\pi m_a^2} |M_{fi}|^2$$

Particle Scattering

◆ For $a + b \rightarrow 1 + 2 + \dots + n$

$$\Gamma = \frac{1}{2E_a 2E_b} \times \int |M_{fi}|^2 \times \delta(\dots) \times d(\text{LIPS})$$

◆ Convert to a cross section by dividing by the incident flux: (Handout 3.4)



$$\sigma = \frac{1}{F} \times \int |M_{fi}|^2 \times \delta(\dots) \times d(\text{LIPS})$$

where F is the Lorentz invariant flux:

$$\begin{aligned}
 F &= (v_a + v_b) \cdot 2E_a \cdot 2E_b \\
 &= 4 p_a m_b = 4 p_i^* \sqrt{s} \\
 &\quad \text{(lab)} \qquad \qquad \text{(cms)}
 \end{aligned}$$

◆ Total cross section:

$$\sigma = \frac{\text{no of scatters per unit time}}{\text{incident flux}}$$

— Lorentz invariant

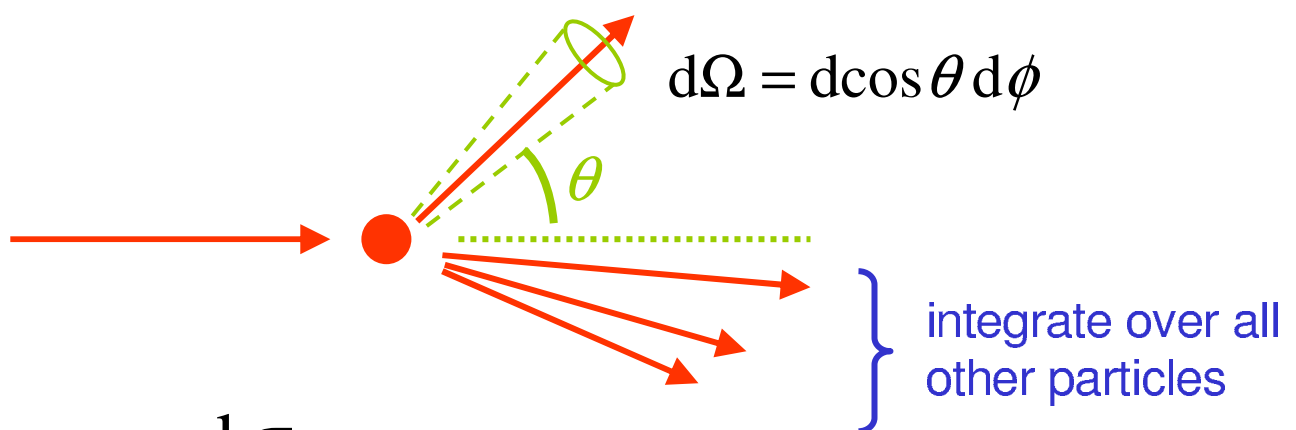
— independent of experimental conditions (flux, target density, ...)

➡ depends only on physics of scattering process itself

— effective area presented by each scattering centre

◆ Differential cross section:

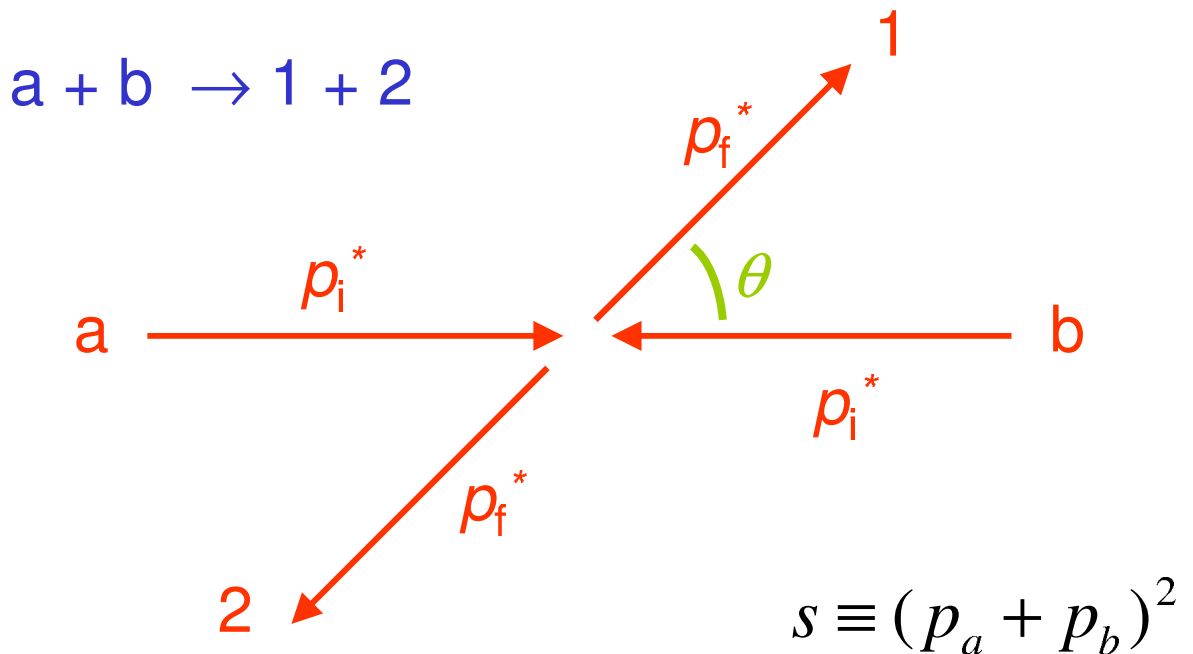
$$\frac{d\sigma}{d\Omega} = \frac{\text{no of particles per sec into } d\Omega}{\text{incident flux} \cdot d\Omega}$$



$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

Two Body Scattering

◆ In centre of mass frame : (Handout 3.5)



$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s} \cdot \frac{p_f^*}{p_i^*} |M_{fi}|^2$$

\sqrt{s} is the total centre of mass energy:

$$\begin{aligned} p_a + p_b &= (E_a^*, p_i^*, 0, 0) + (E_b^*, -p_i^*, 0, 0) \\ &= (E_a^* + E_b^*, 0, 0, 0) \end{aligned}$$

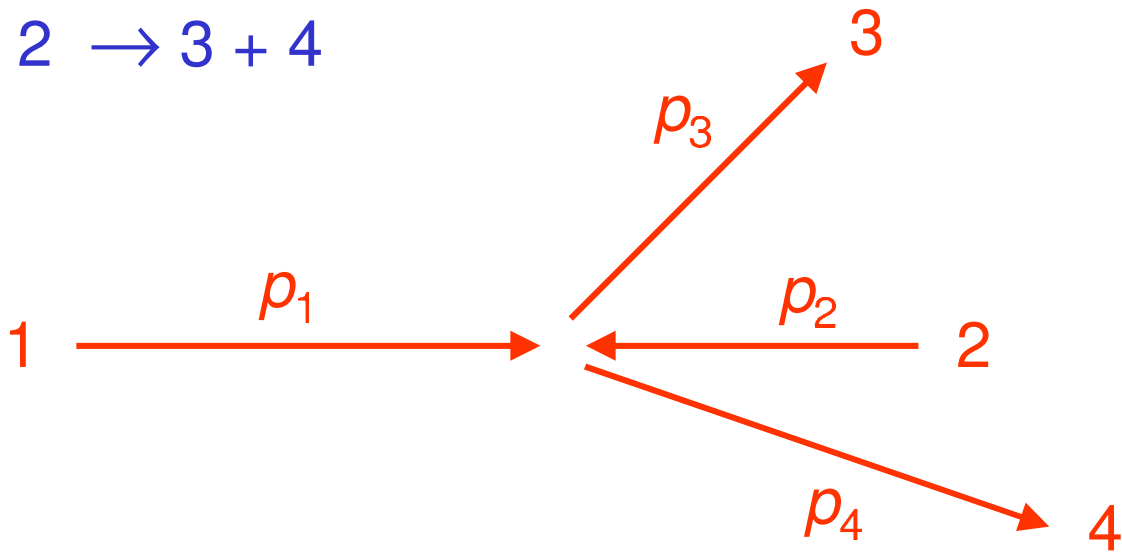
➡ $s = (p_a + p_b)^2 = (E_a^* + E_b^*)^2$

➡ $\sqrt{s} = E_a^* + E_b^*$

◆ In any frame :

(Handout 3.6)

$$1 + 2 \rightarrow 3 + 4$$



Lorentz-invariant cross section :

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s (p_i^*)^2} |M_{fi}^2|$$

where $s \equiv (p_1 + p_2)^2$

$$t \equiv (p_1 - p_3)^2$$

At high energy (neglecting m_1) :

$$\frac{d\sigma}{dt} = \frac{1}{16\pi (s - m_2^2)^2} |M_{fi}^2|$$

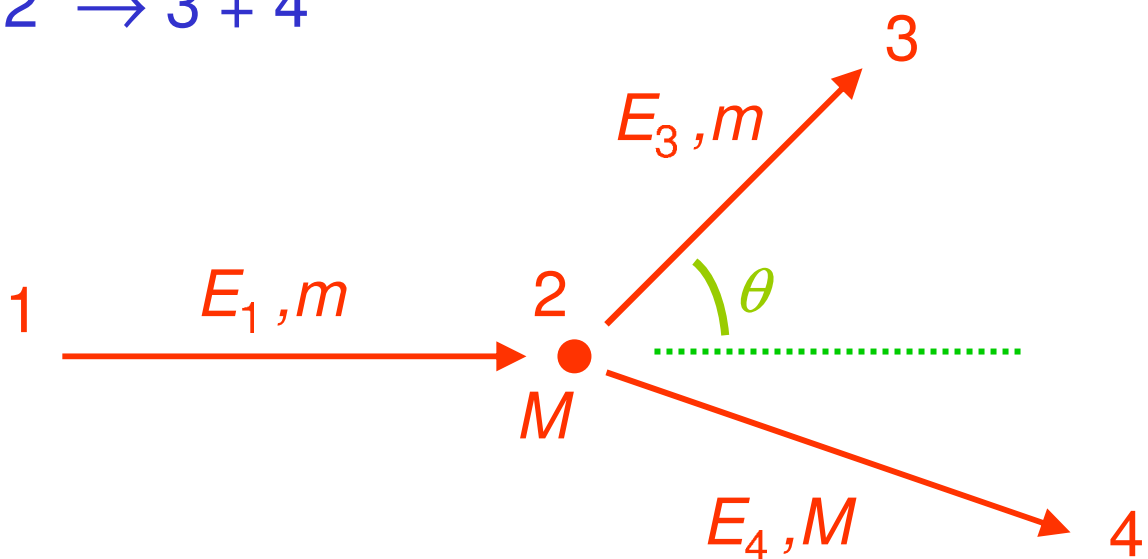
◆ In lab frame :

(Handout 3.7)

For elastic scattering at high energies:

$$\begin{aligned} m_1 &= m_3 = m & E_1 \gg m \\ m_2 &= m_4 = M \end{aligned}$$

$$1 + 2 \rightarrow 3 + 4$$



$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left(\frac{E_3}{ME_1} \right)^2 |M_{fi}^2|$$

where

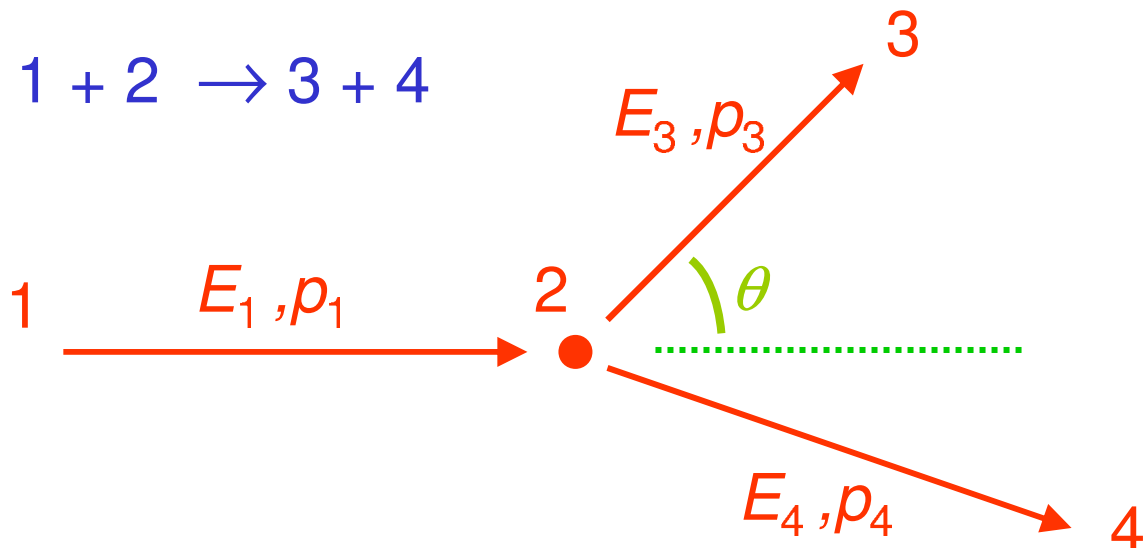
$$E_3 = \frac{E_1 M}{M + E_1 - E_1 \cos \theta}$$

$$d\Omega = 2\pi d \cos \theta$$

◆ In lab frame:

(Handout 3.8, 3.9)

General case (valid for all energies, masses):



$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{p_1 m_2} \cdot \frac{p_3^2 |M_{\text{fi}}^2|}{p_3 (E_1 + m_2) - E_3 p_1 \cos \theta}$$

where $E_3 = \sqrt{p_3^2 + m_3^2}$ and p_3 is given by

$$E_1 + m_2 = \sqrt{p_3^2 + m_3^2} + \sqrt{p_1^2 + p_3^2 - 2p_1 p_3 \cos \theta + m_4^2}$$

(for a given scattering angle θ)

➡ only one independent variable (θ)