Parity Violation

- 1957: C.S.Wu et al studied beta decay of polarised cobalt-60 nuclei:

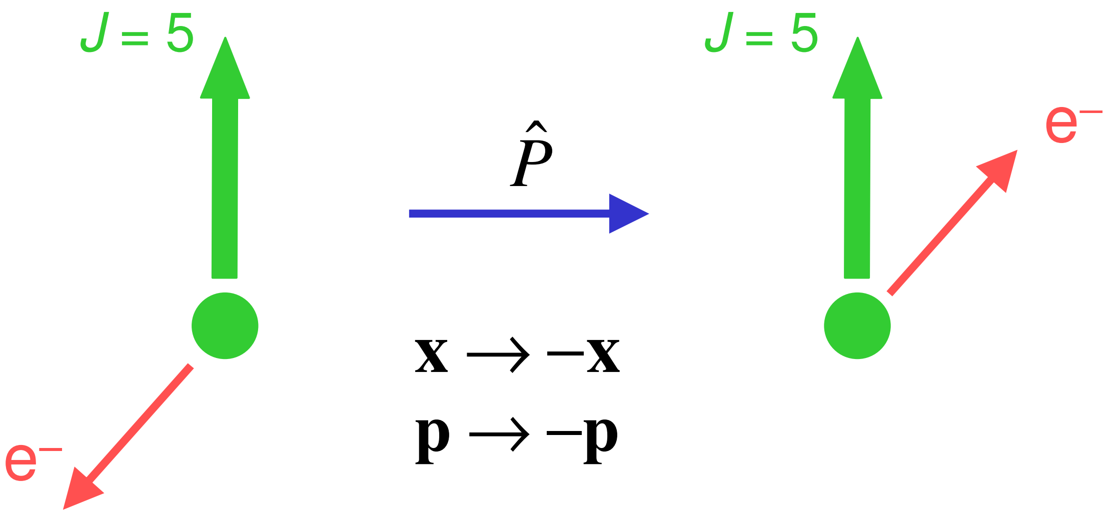
$$^{60}\text{Co}\ (J=5) \rightarrow {}^{60}\text{Ni}^*\ (J=4) + e^- + \bar{\nu}_e$$

^{60}Co cooled to 0.01K inside solenoid

$\Longrightarrow$ high proportion of ^{60}Co have spins aligned along **B**

- Found that e^- were emitted preferentially in direction opposite to that of nuclear spin

But: under parity transformation:



$$\mathbf{x} \rightarrow -\mathbf{x}$$

$$\mathbf{p} \rightarrow -\mathbf{p}$$

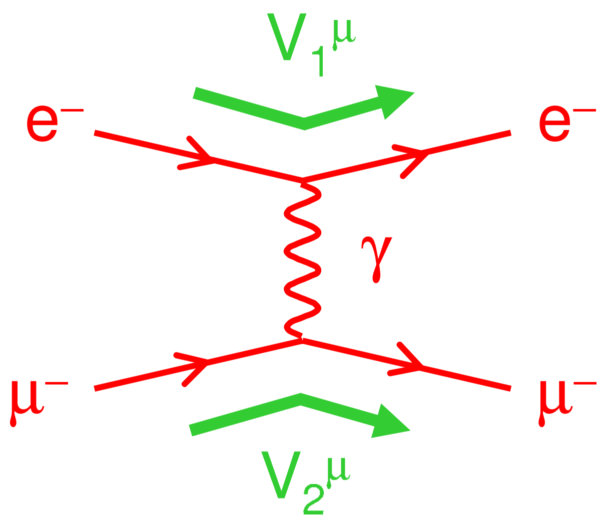
$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \rightarrow (-\mathbf{r}) \times (-\mathbf{p}) = \mathbf{L}$$

$\Longrightarrow$ if parity conserved: would observe equal # of e^- along and opposite to nuclear spin

➡ Direct observation of parity violation

How can P violation be accounted for ?

- Recall QED: structure of matrix element is:



$$M_{\mathrm{fi}} \sim g_{\mu\nu}.\overline{\psi}\gamma^\mu\psi.\overline{\psi}\gamma^\nu\psi$$

$$\equiv g_{\mu\nu}V_1^\mu V_2^\nu$$

"current-current" interaction

i.e. scalar product of two 4-vector currents

Under parity: $\psi \xrightarrow{\hat{P}} \gamma^0\psi$

$$\Rightarrow \quad \overline{\psi} \equiv \psi^+\gamma^0 \xrightarrow{\hat{P}} (\gamma^0\psi)^+\gamma^0 = \psi^+(\gamma^0)^+\gamma^0 = \overline{\psi}\gamma^0$$

$$\Rightarrow \quad \overline{\psi}\gamma^0\psi \xrightarrow{\hat{P}} \overline{\psi}\gamma^0\gamma^0\gamma^0\psi = +\overline{\psi}\gamma^0\psi$$

$$\overline{\psi}\gamma^k\psi \xrightarrow{\hat{P}} \overline{\psi}\gamma^0\gamma^k\gamma^0\psi = -\overline{\psi}\gamma^k\psi$$

$(k = 1,2,3)$

i.e. spatial components change sign:

$$(V^0, \mathbf{V}) \xrightarrow{\hat{P}} (V^0, -\mathbf{V})$$

just like

$$(t, \mathbf{x}) \xrightarrow{\hat{P}} (t, -\mathbf{x})$$

$$(E, \mathbf{p}) \xrightarrow{\hat{P}} (E, -\mathbf{p})$$

4-vectors V^μ transforming in this way under P are known as vector quantities

- Hence, under parity:

$$M_{fi} \sim (V_1^0, \mathbf{V}_1).(V_2^0, \mathbf{V}_2) = V_1^0 V_2^0 - \mathbf{V}_1.\mathbf{V}_2$$

$$\xrightarrow{\hat{P}} V_1^0 V_2^0 - (-\mathbf{V}_1).(-\mathbf{V}_2) = V_1^0 V_2^0 - \mathbf{V}_1.\mathbf{V}_2$$

i.e. M_{fi} is left unchanged

$\Longrightarrow$ QED conserves parity

- Now consider $\overline{\psi}\gamma^\mu\gamma^5\psi$:

$$(\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3)$$
$$(\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5)$$

$$\overline{\psi}\gamma^0\gamma^5\psi \xrightarrow{\hat{P}} \overline{\psi}\gamma^0\gamma^0\gamma^5\gamma^0\psi = -\overline{\psi}\gamma^0\gamma^5\psi$$
$$\overline{\psi}\gamma^k\gamma^5\psi \xrightarrow{\hat{P}} \overline{\psi}\gamma^0\gamma^k\gamma^5\gamma^0\psi = +\overline{\psi}\gamma^k\gamma^5\psi$$

i.e. time component changes sign:

$$(A^0, \mathbf{A}) \xrightarrow{\hat{P}} (-A^0, \mathbf{A})$$

and such quantities are known as axial vectors

- A scalar product of two axial vectors is still invariant under parity, but a mixture of V and A is not :

e.g. $$M_{fi} \sim g_{\mu\nu}(V_1^\mu - A_1^\mu)(V_2^\nu - A_2^\nu)$$

$$= (V_1^0 - A_1^0)(V_2^0 - A_2^0) - (\mathbf{V}_1 - \mathbf{A}_1).(\mathbf{V}_2 - \mathbf{A}_2)$$

$$\xrightarrow{\hat{P}} (V_1^0 + A_1^0)(V_2^0 + A_2^0) - (-\mathbf{V}_1 - \mathbf{A}_1).(-\mathbf{V}_2 - \mathbf{A}_2)$$

which are **not** equal to each other

- In fact, it can be shown that the most general parity violating weak current must be a linear combination of 5 terms:

$\overline{\psi}\psi$	Scalar	S	(1)
$\overline{\psi}\gamma^5\psi$	Pseudoscalar	P	(1)
$\overline{\psi}\gamma^\mu\psi$	Vector	V	(4)
$\overline{\psi}\gamma^\mu\gamma^5\psi$	Axial-vector	A	(4)
$\overline{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\psi$	Tensor	T	(6)
			(4x4=16)

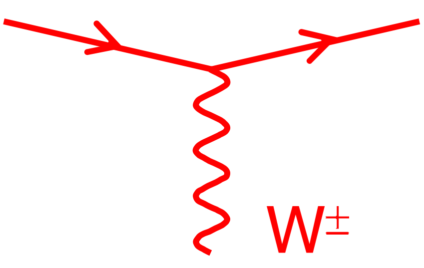
- The correct combination can only be determined by experiment

Answer: **V – A**

i.e. the weak charged current is proportional to

$$\overline{\psi}(\gamma^\mu - \gamma^\mu\gamma^5)\psi$$

- The Feynman rule for the $W^\pm$ vertex is



$$-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)$$

g_W = weak coupling constant

(Z^0 vertex similar but more complicated: see later)

- A weak charged current of the V – A form

$$\bar{\phi}\gamma^{\mu}\,\tfrac{1}{2}(1-\gamma^{5})\psi$$

is non-zero only for the left-handed chiral components of both spinors :

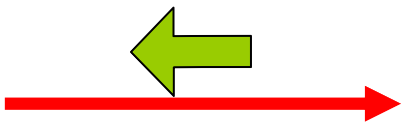
$$\bar{\phi}\gamma^{\mu}\,\tfrac{1}{2}(1-\gamma^{5})\psi=\bar{\phi}_{L}\gamma^{\mu}\,\tfrac{1}{2}(1-\gamma^{5})\psi_{L}$$

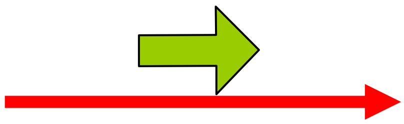
(examples sheet)

where
$$\begin{cases}\psi_{L}\equiv\tfrac{1}{2}(1-\gamma^{5})\psi\\ \phi_{L}\equiv\tfrac{1}{2}(1-\gamma^{5})\phi\end{cases}$$

> only the left-handed chiral components of particle or antiparticle spinors participate in charged current weak interactions

- At very high energy $(E \gg m)$, the left-handed chiral components are helicity eigenstates :

particle:  –ve helicity

antiparticle:  +ve helicity

> only left-handed particles, or right-handed antiparticles, participate in charged current weak interactions (in the relativistic limit)

- In the Standard Model: ($\beta = 1$ always)

  neutrinos are exactly massless
  and interact only via the weak interactions

$\Rightarrow$ neutrinos are always left-handed
antineutrinos are always right-handed

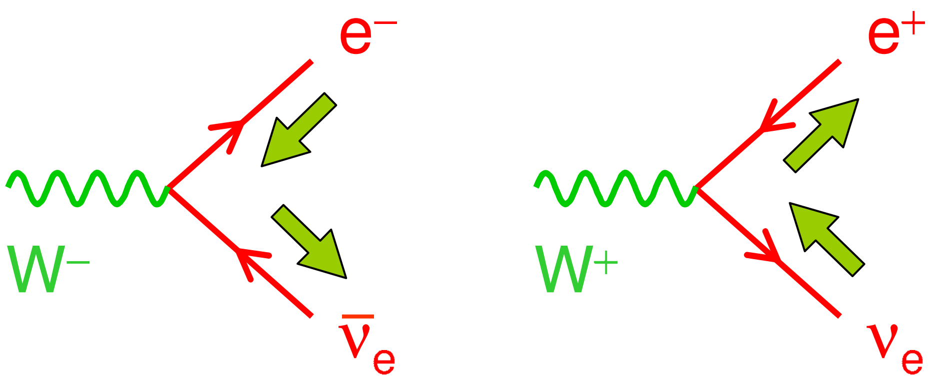
or equivalently:

ν always –ve helicity $\bar{\nu}$ always +ve helicity

(will return later to recent evidence for a small but finite neutrino mass)

- Note that helicity conservation applies ….

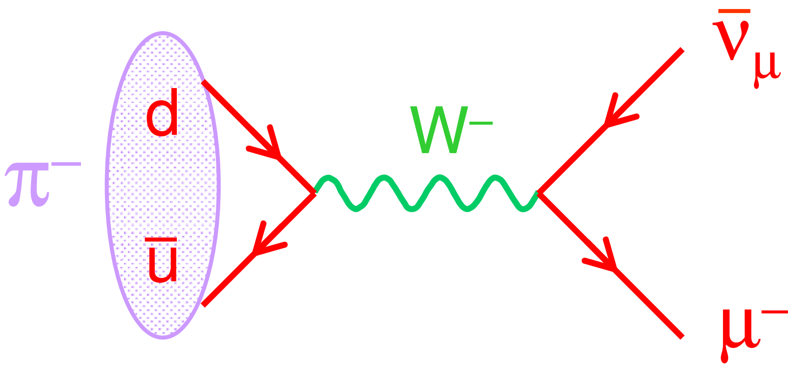
In the relativistic limit, W vertices preserve particle helicity, just as in QED and QCD



- For massive particles, at finite energies, the other helicity states also participate:

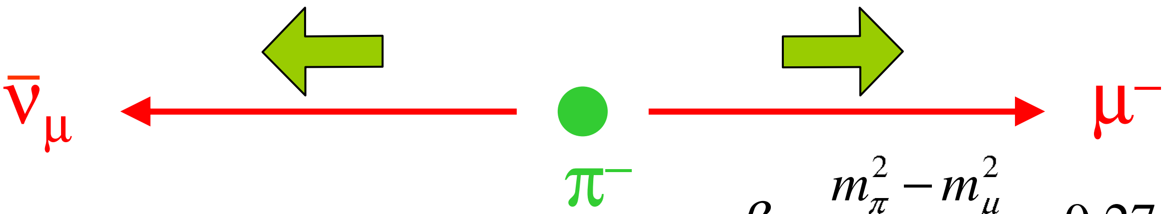
Typically contributes a factor $1-\beta = 1-\mathrm{v}/c$ to the non-invariant matrix element squared

e.g. $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$



massless antineutrino is always right-handed

Muon must also be right-handed to conserve overall angular momentum (pion is spin-zero)



$$\beta = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2} \approx 0.27$$

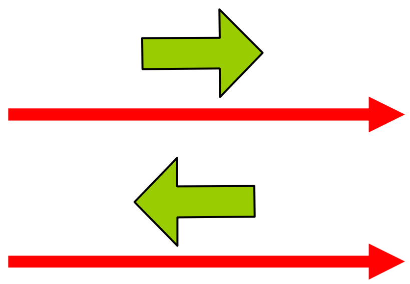
$$|T_{\mathrm{fi}}|^2 = \frac{1}{4}\left(\frac{g_W}{2m_W}\right)^4 f_\pi^2 m_\pi (1-\beta)$$

(examples sheet)

Note that matrix element vanishes as $m_\mu \rightarrow 0$

(cf $\pi^- \rightarrow e^- \bar{\nu}_e$)

- For particles, the typical relative interaction strengths are:

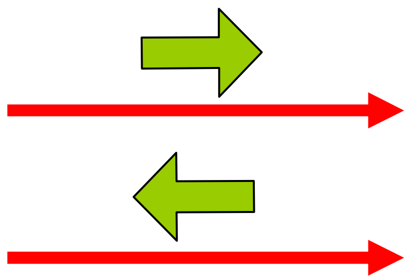


$$\tfrac{1}{2}(1-\beta)$$

$$\tfrac{1}{2}(1+\beta)$$

while for antiparticles we find:

(usually dominant)



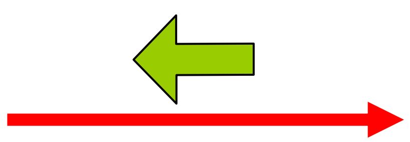
$$\tfrac{1}{2}(1+\beta)$$

$$\tfrac{1}{2}(1-\beta)$$

- Hence, other things being equal, when produced in CC weak interactions:

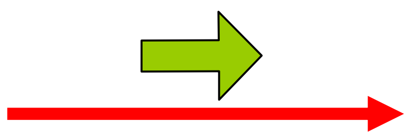
"Charged Current", i.e. $W^{\pm}$

Particles have –ve helicity on average:



$$\langle\lambda\rangle = -\beta = -v/c$$

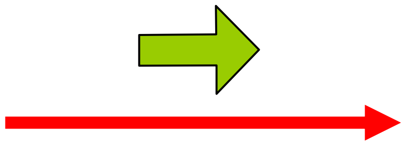
while antiparticles prefer +ve helicity:



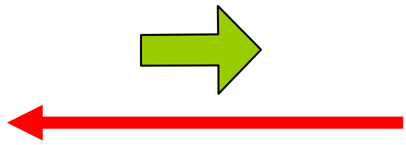
$$\langle\lambda\rangle = +\beta = +v/c$$

- The different behaviour of the +ve and –ve helicity states can easily be seen to violate parity:

Under parity transformation, ordinary momenta change sign but angular momenta don’t :



$$\xrightarrow{\hat{P}}$$



Hence helicity changes sign :

+ve helicity $\xrightarrow{\hat{P}}$ –ve helicity

- Therefore P invariance demands equal interaction strengths for both helicities

(as in QED and QCD)

Evidence for V – A

- An important test of V – A theory was the small branching ratio for $\pi^+ \to e^+ \nu_e$

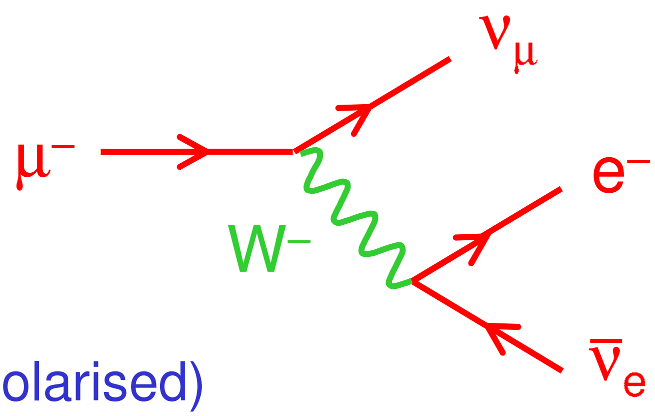
V ± A :
$$\frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = 1.3 \times 10^{-4}$$
(examples sheet)

S, P :
$$\frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = 5.5$$
(Tripos 2000)

Experiment = 1.23×10^{-4}

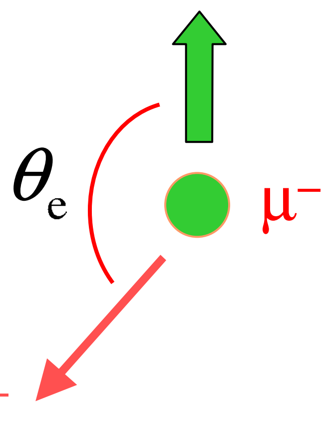
- Muon decay :

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$



(polarised or unpolarised)

Measure electron energy and angular distributions relative to muon spin direction :
$$\frac{d\Gamma}{dE_e \, d\theta_e}$$



- In general, depends on ten free parameters :

P and S $\longrightarrow$ g^{S}_{RR} g^{S}_{RL} g^{S}_{LR} g^{S}_{LL}

V and A $\longrightarrow$ g^{V}_{RR} g^{V}_{RL} g^{V}_{LR} g^{V}_{LL}

T $\longrightarrow$ g^{T}_{RL} g^{T}_{LR}

e.g. V–A predicts $g^{V}_{LL} = 1$, rest = 0

V+A predicts $g^{V}_{RR} = 1$, rest = 0 etc.

Data:

$\lvert g^{S}_{RR} \rvert < 0.066$	$\lvert g^{V}_{RR} \rvert < 0.033$	
$\lvert g^{S}_{LR} \rvert < 0.125$	$\lvert g^{V}_{LR} \rvert < 0.060$	$\lvert g^{T}_{LR} \rvert < 0.036$
$\lvert g^{S}_{RL} \rvert < 0.424$	$\lvert g^{V}_{RL} \rvert < 0.110$	$\lvert g^{T}_{RL} \rvert < 0.122$
$\lvert g^{S}_{LL} \rvert < 0.550$	$\lvert g^{V}_{LL} \rvert > 0.960$	

Some linear combinations of these parameters can be measured very accurately :

e.g. the "Michel rho parameter"

$$\rho = 0.75080 \pm 0.00105$$

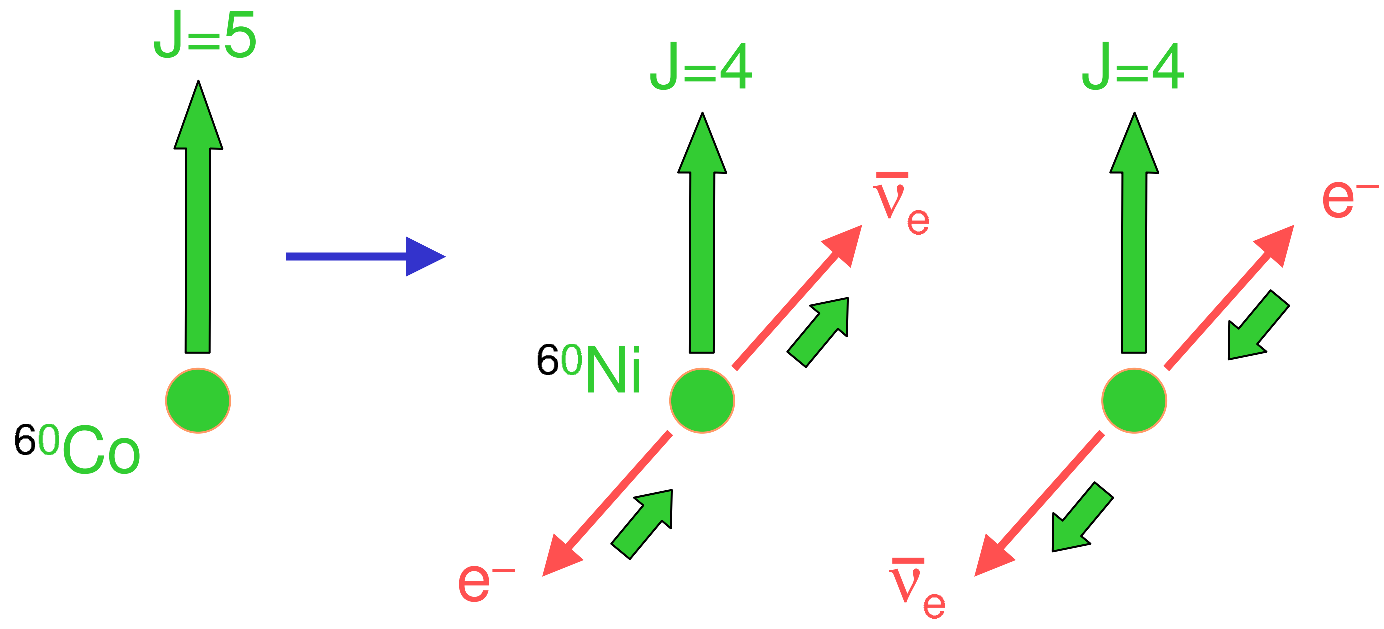
(V–A) predicts 0.75

TWIST expt. (2004)
(hep-ex/0409063)
$6\mathrm{x}10^{9}$ μ decays

- Also neutron decay, kaon decays, etc….

$\Rightarrow$ no evidence of any S, P, T contributions to the weak charged current

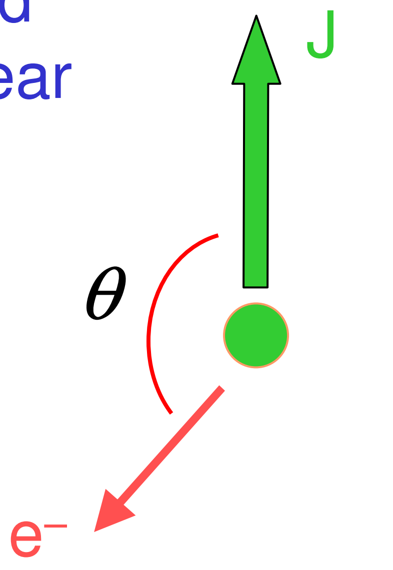
Return to β–decay of ^{60}Co :



favoured disfavoured

by overlap of spin wavefunctions

$\Longrightarrow$ electron prefers to be emitted <u>opposite</u> to direction of nuclear spin (as observed)



Electron intensity was found to be consistent with V–A prediction : $I(\theta) = 1 - \beta\cos\theta$

Note that V+A would give <u>left</u>-handed antineutrinos and electron would be emitted <u>along</u> the nuclear spin : $I(\theta) = 1 + \beta\cos\theta$

Determination of ν Helicity

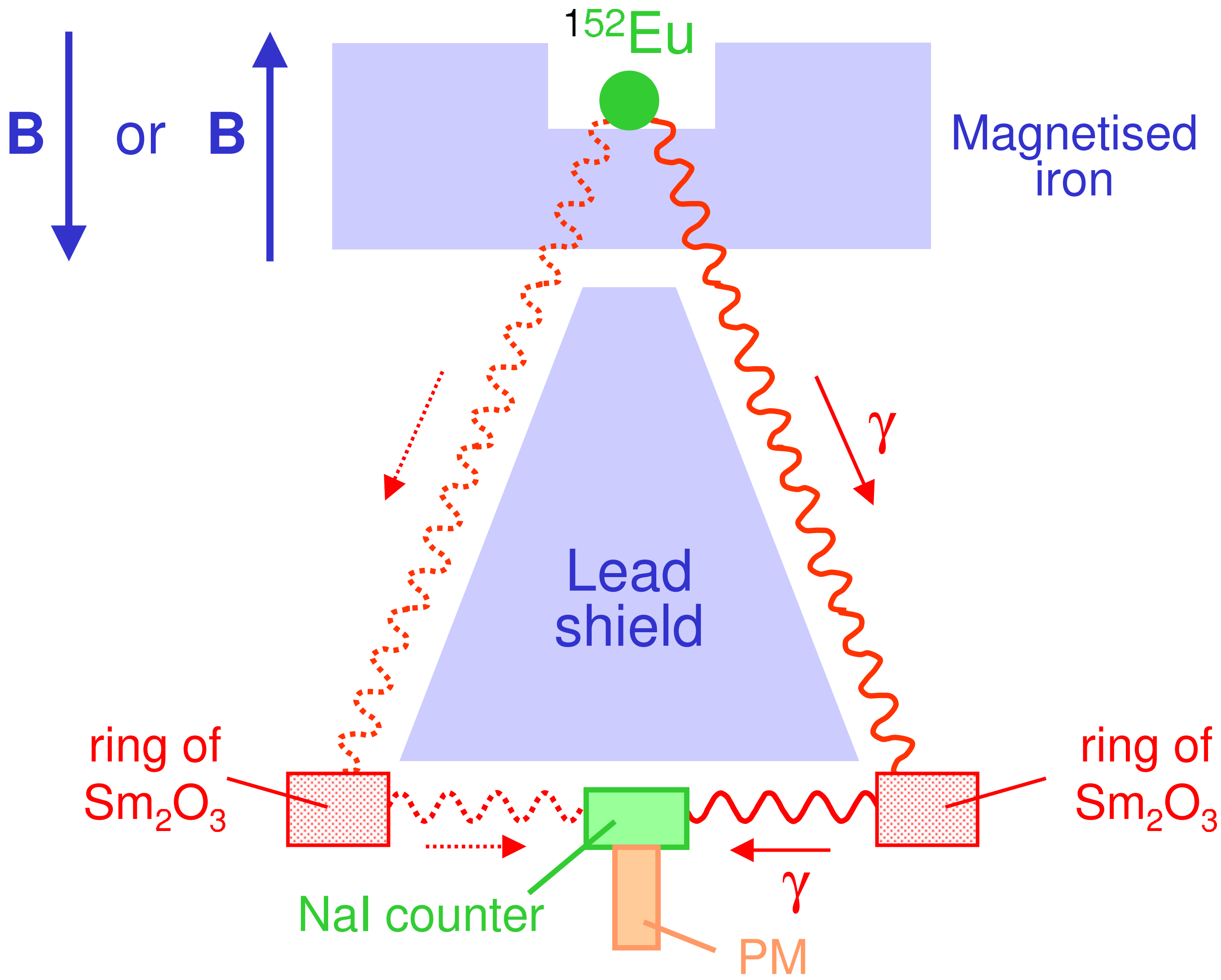
(non-examinable)

- Ingenious experiment by Goldhaber et al : (1958)

Studied electron capture in ^{152}Eu ...

$$e^- + {}^{152}Eu(J=0) \rightarrow {}^{152}Sm^*(J=1) + \nu_e$$

$$\hookrightarrow {}^{152}Sm\ (J=0) + \gamma$$

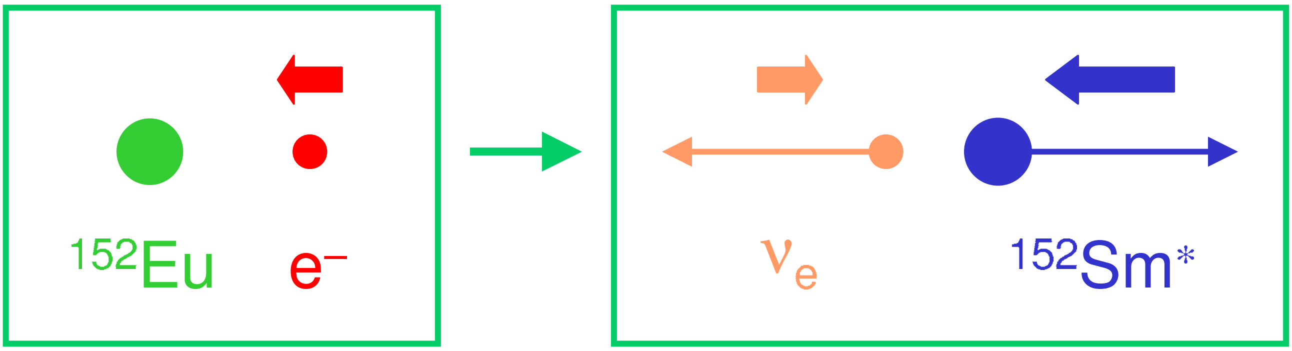


… and detected photons after resonant scattering on ring of ^{152}Sm ...

$$\gamma + {}^{152}Sm \rightarrow {}^{152}Sm^* \rightarrow {}^{152}Sm + \gamma$$

Step 1:

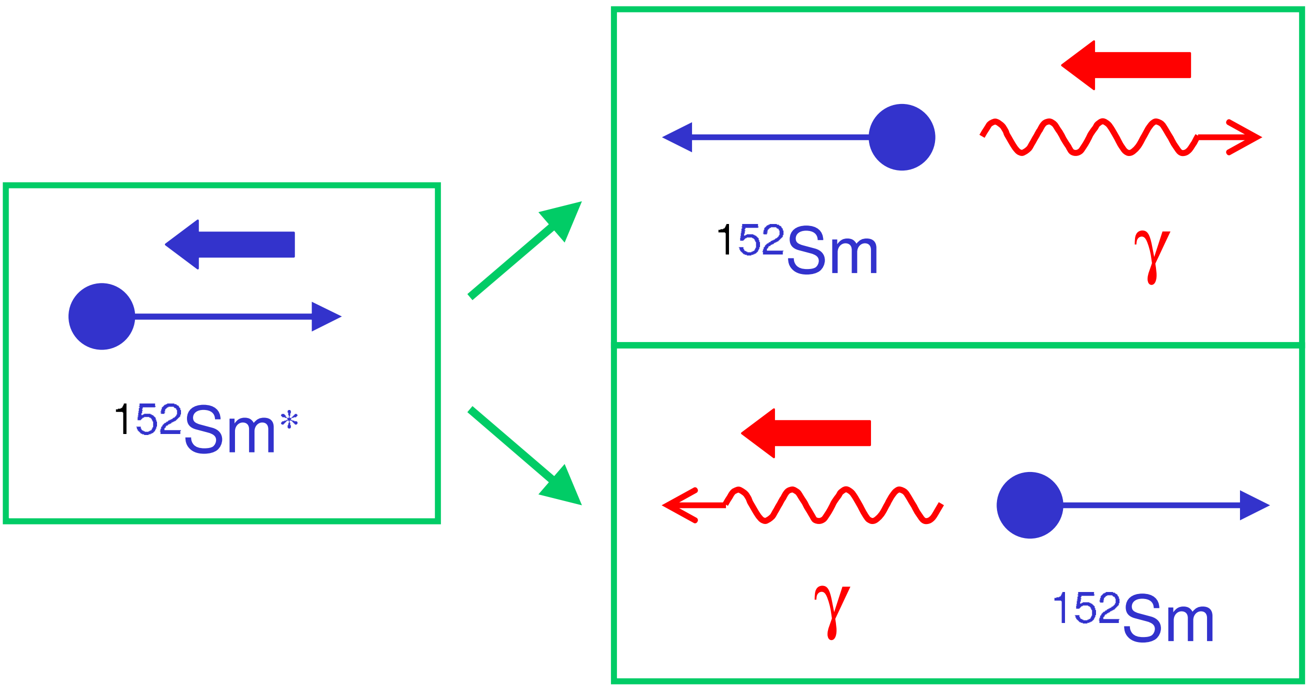
e^- capture from K-shell of ^{152}Eu emits neutrino:



⟹ ^{152}Sm* must recoil with <u>same</u> <u>helicity</u> as neutrino

Step 2:

^{152}Sm* decays to ground state ^{152}Sm + γ :



⟹ photons which happen to be emitted along the same line of flight as the ^{152}Sm* will have the <u>same</u> <u>helicity</u> as the neutrino

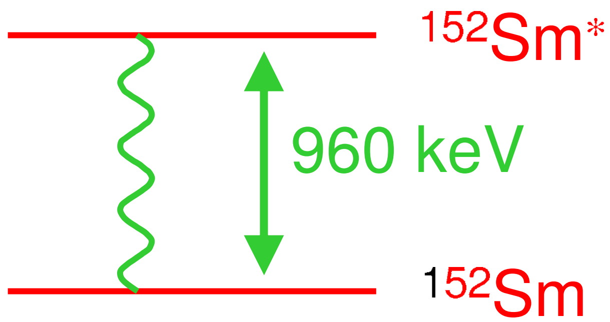
Step 3:

Resonant scattering of γ by more ^{152}Sm:

$$\gamma + {}^{152}Sm \rightarrow {}^{152}Sm^{*} \rightarrow {}^{152}Sm + \gamma$$

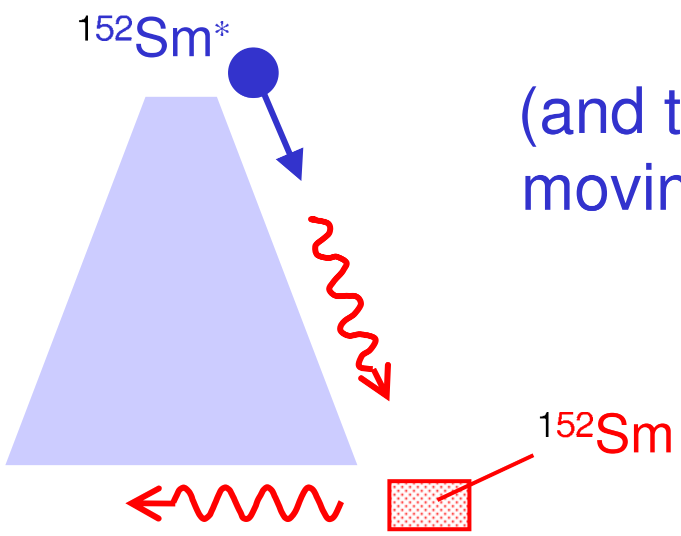
Only those γ with just the right energy to "hit" the resonance can scatter in this way

i.e. only those γ with a bit of extra energy



γ energy must be slightly greater than 960 keV to allow for recoil of $^{152}Sm^{*}$

i.e. only those γ which happened to be emitted along the line of flight of the $^{152}Sm^{*}$



(and the $^{152}Sm^{*}$ was moving downwards)

i.e. only those γ with the <u>same</u> <u>helicity</u> as the original neutrino

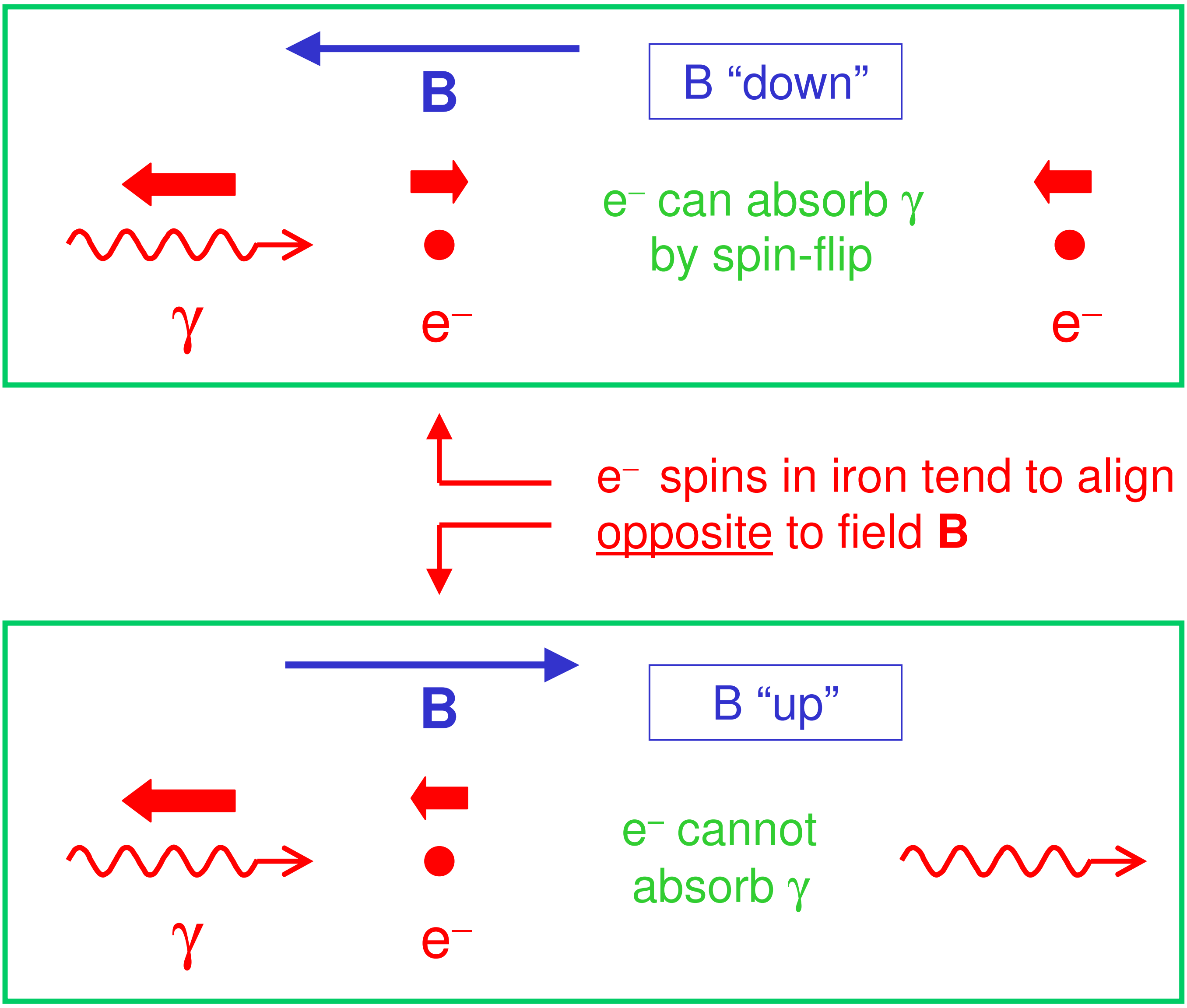
Step 4:

Measure the helicity of the emitted photons

Do this by comparing the photon counting rate for both directions of the magnetic field **B**

Photon absorption in iron depends on photon spin direction relative to magnetic field **B**

e.g. for left-handed photons:



$\Longrightarrow$ if the photons are left-handed, will see greater absorption with **B** down than **B** up

$\Longrightarrow$ will see lower counting rate with **B** down than **B** up

Measure counting rate with both field directions

→ gives photon helicity

→ infer neutrino helicity

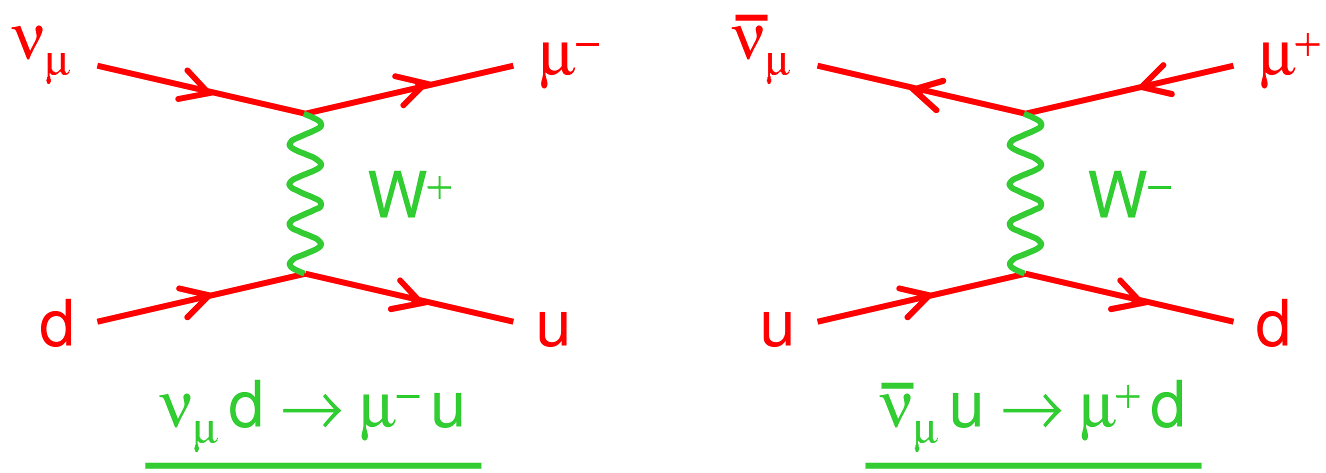
Result:

neutrinos have negative helicity

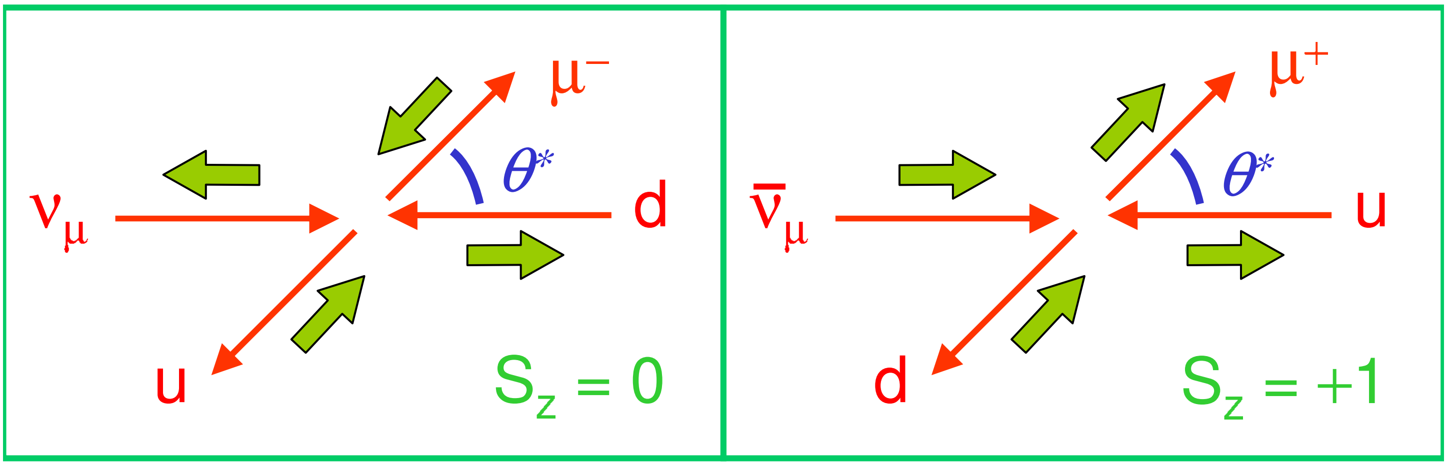
(as expected for V–A weak charged current)

Neutrino Scattering

- For ν, $\bar{\nu}$ scattering on u,d quarks in nucleon, leading order Feynman diagrams are:



- In centre of mass frame: (Handout 7.1, 7.2)



$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{4\pi^2}\hat{s}$$

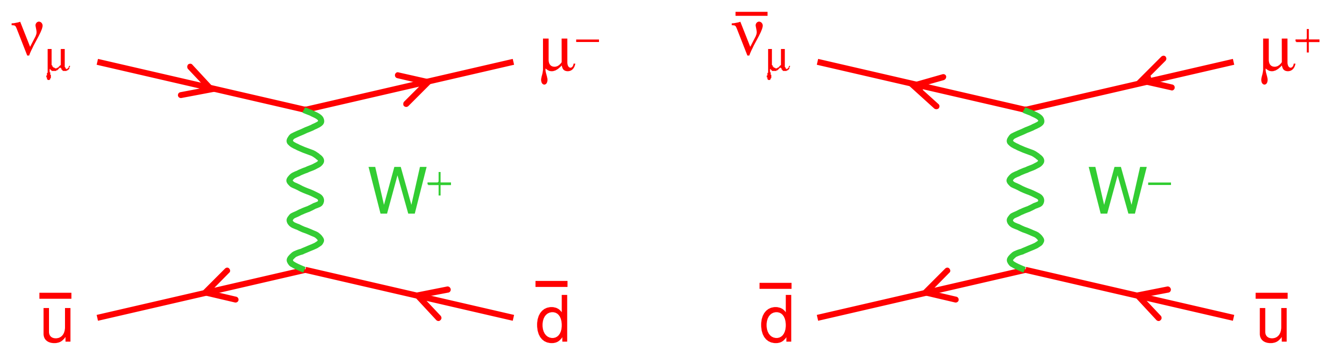
$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{16\pi^2}\hat{s}(1+\cos\theta^*)^2$$

isotropic

extra factor $\frac{1}{4}(1+\cos\theta^*)^2$

$$\hat{s} = (p_1 + p_2)^2 = (\nu q \text{ cms energy})^2$$

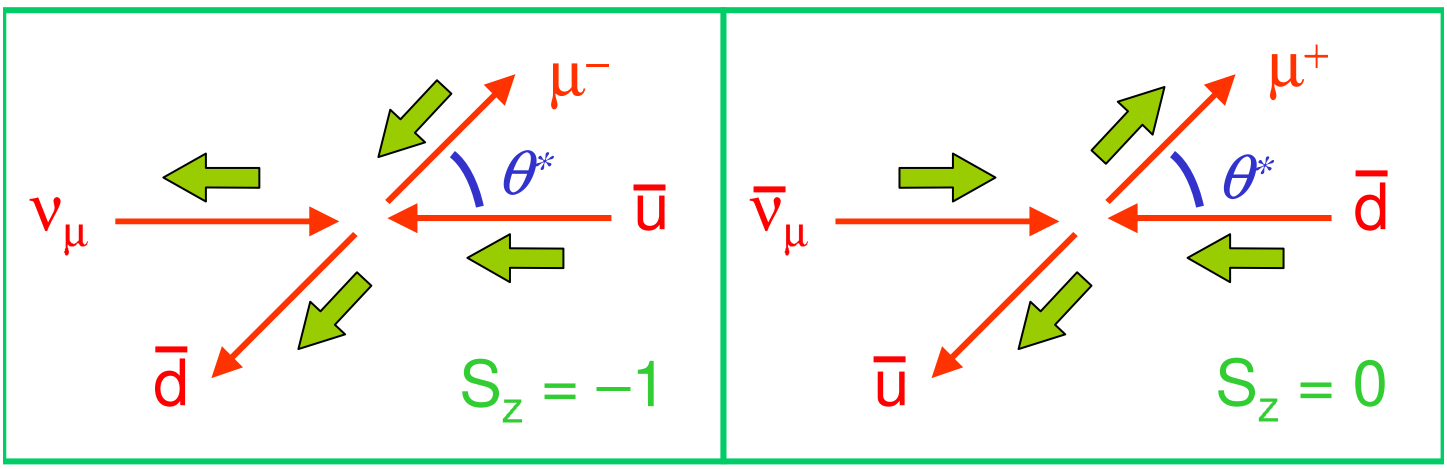
For scattering from antiquark in nucleon:



$\nu_\mu \bar{u} \rightarrow \mu^- \bar{d}$ $\quad\quad$ $\bar{\nu}_\mu \bar{d} \rightarrow \mu^+ \bar{u}$

In centre of mass frame:

(Handout 7.3)



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{G_F^2}{16\pi^2}\hat{s}\,(1+\cos\theta^*)^2$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{G_F^2}{4\pi^2}\hat{s}$$

In terms of scaling variable *y* :

$$y = \frac{1}{2}(1-\cos\theta^*)$$

$$\frac{\mathrm{d}\sigma^{\nu\bar{q}}}{\mathrm{d}y} = \frac{\mathrm{d}\sigma^{\bar{\nu}q}}{\mathrm{d}y} = \frac{G_F^2}{\pi}\hat{s}\,(1-y)^2$$

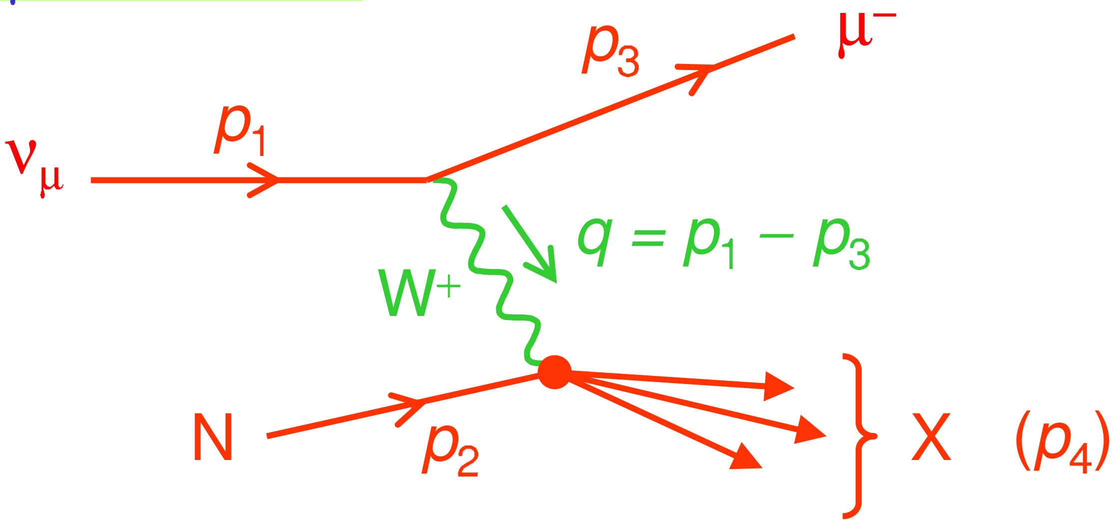
$$\frac{\mathrm{d}\sigma^{\nu q}}{\mathrm{d}y} = \frac{\mathrm{d}\sigma^{\bar{\nu}\bar{q}}}{\mathrm{d}y} = \frac{G_F^2}{\pi}\hat{s}$$

(Lorentz invariant)

(Handout 7.4)

Deep Inelastic ν Scattering

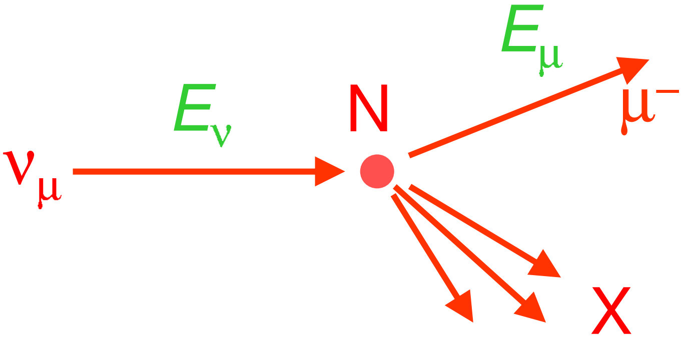
e.g. $\nu_\mu N \rightarrow \mu^- X$



Take the two (Lorentz invariant) independent variables describing the scattering to be

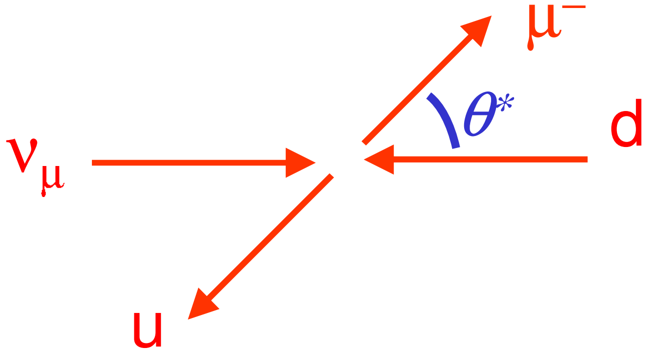
$$x \equiv \frac{Q^2}{2p_2.q} \equiv \frac{Q^2}{2M\nu} \qquad y \equiv \frac{p_2.q}{p_2.p_1} \qquad \begin{array}{l} 0 < x < 1 \\ 0 < y < 1 \end{array}$$

lab frame



$$y = \frac{E_\nu - E_\mu}{E_\nu} = \frac{\nu}{E_\nu}$$

νq cms frame



$$y = \frac{1}{2}(1 - \cos\theta^*)$$

For electromagnetic DIS , ep $\rightarrow$ eX:

- The most general Lorentz-invariant form for the cross section was

$$\frac{\mathrm{d}^2\sigma^{\mathrm{em}}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2x^2y^2}{Q^2}\right)\frac{F_2(x)}{x} + \frac{y^2}{2}\frac{2xF_1(x)}{x}\right]$$

(assumes single virtual photon exchange and parity conservation)

- Can be converted from (x,Q^2) to (x,y) :

$$Q^2 = \left(s - M^2\right)x\,y$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}y} = \left|\frac{\mathrm{d}Q^2}{\mathrm{d}y}\right|\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \left(s - M^2\right)x\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2}$$

$$\frac{\mathrm{d}^2\sigma^{\mathrm{em}}}{\mathrm{d}x\mathrm{d}y} = \frac{4\pi\alpha^2(s - M^2)}{Q^4}\left[\left(1 - y - \frac{M^2xy}{s - M^2}\right)F_2(x) + \frac{y^2}{2}2xF_1(x)\right]$$

At high energy $(s >> M^2)$:

$$\boxed{\frac{\mathrm{d}^2\sigma^{\mathrm{em}}}{\mathrm{d}x\mathrm{d}y} = \frac{4\pi\alpha^2 s}{Q^4}\left[(1 - y)F_2(x) + \frac{y^2}{2}2xF_1(x)\right]}$$

(Handout 7.5.1)

For neutrino DIS , νp → eX, μX, τX :

- Same form for cross section, except:

  1) $4\pi\alpha^2/Q^4 \rightarrow G_F^2/2\pi$

  2) need extra structure function (F_3) to allow for parity violation

$$\frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu p}(x) + \frac{y^2}{2}2xF_1^{\nu p}(x) + y\left(1-\frac{y}{2}\right)xF_3^{\nu p}(x)\right]$$

$$\frac{d^2\sigma^{\bar{\nu} p}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\bar{\nu} p}(x) + \frac{y^2}{2}2xF_1^{\bar{\nu} p}(x) - y\left(1-\frac{y}{2}\right)xF_3^{\bar{\nu} p}(x)\right]$$

plus similar expressions for νn, ν̄n :

$$\frac{d^2\sigma^{\nu n}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu n}(x) + \frac{y^2}{2}2xF_1^{\nu n}(x) + y\left(1-\frac{y}{2}\right)xF_3^{\nu n}(x)\right]$$

$$\frac{d^2\sigma^{\bar{\nu} n}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\bar{\nu} n}(x) + \frac{y^2}{2}2xF_1^{\bar{\nu} n}(x) - y\left(1-\frac{y}{2}\right)xF_3^{\bar{\nu} n}(x)\right]$$

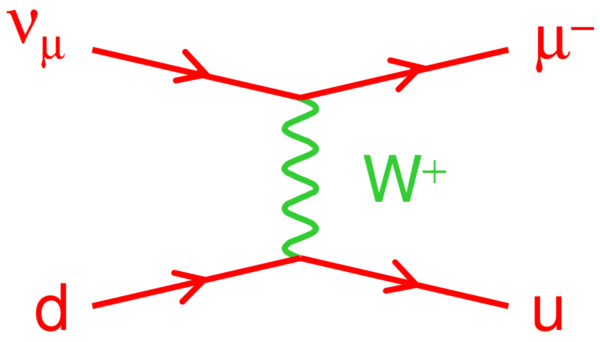
(Handout 7.5.2)

- In general:

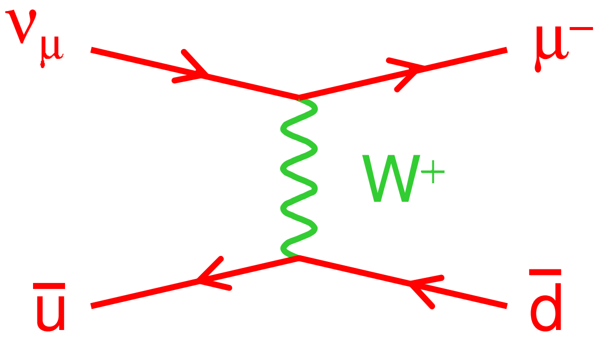
  F_1 , F_2 , F_3 depend on both x and y

  Bjorken scaling $\Longrightarrow$ functions of x only

Parton model for νp scattering:



$\nu_\mu d \rightarrow \mu^- u$



$\nu_\mu \bar{u} \rightarrow \mu^- \bar{d}$

$d(x)\mathrm{d}x$ = # of d quarks with momentum fraction x

$\bar{u}(x)\mathrm{d}x$ = # of ū with momentum fraction x

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y} = \frac{G_F^2}{\pi}\hat{s}\cdot d(x)\,\mathrm{d}x \qquad \frac{\mathrm{d}\sigma}{\mathrm{d}y} = \frac{G_F^2}{\pi}\hat{s}(1-y)^2\cdot \bar{u}(x)\,\mathrm{d}x$$

$$\Rightarrow \frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi} sx\, d(x) \qquad \frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi} sx(1-y)^2 \bar{u}(x)$$

where: $\hat{s}$ = νq (cms energy)^2, s = νp (cms energy)^2 $\Rightarrow$ $\hat{s} = sx$

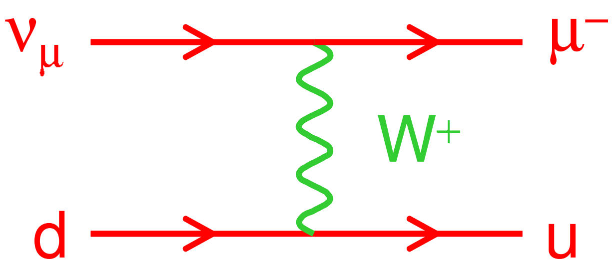
In total:

$$\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi} sx\left[d(x) + (1-y)^2\bar{u}(x)\right]$$

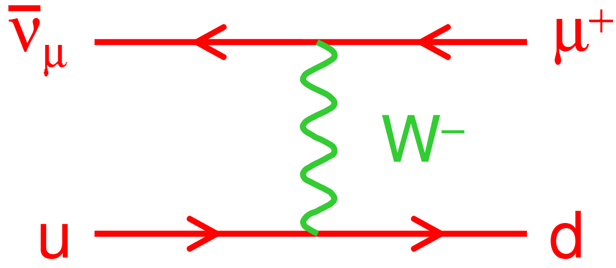
Similarly, for antineutrinos:

$$\frac{\mathrm{d}^2\sigma^{\bar{\nu} p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi} sx\left[(1-y)^2 u(x) + \bar{d}(x)\right]$$

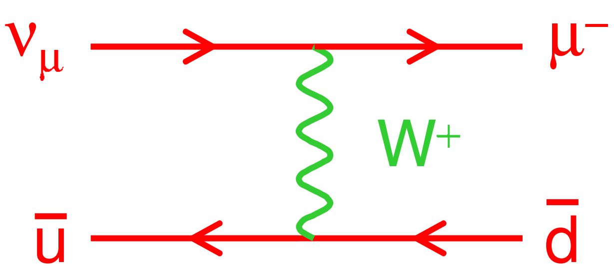
ν	$\bar{\nu}$



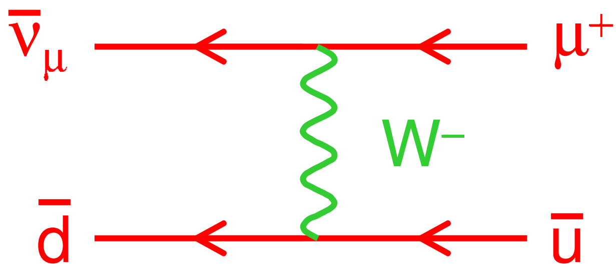
$$\frac{d\sigma}{dy} = \frac{G_F^2}{\pi}\hat{s}$$



$$\frac{d\sigma}{dy} = \frac{G_F^2}{\pi}\hat{s}(1-y)^2$$



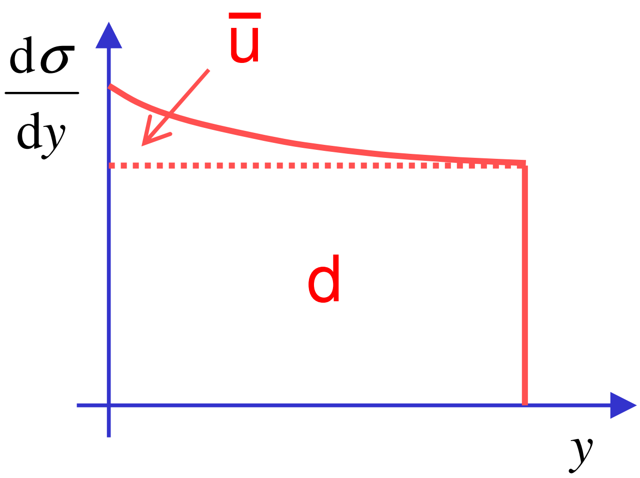
$$\frac{d\sigma}{dy} = \frac{G_F^2}{\pi}\hat{s}(1-y)^2$$

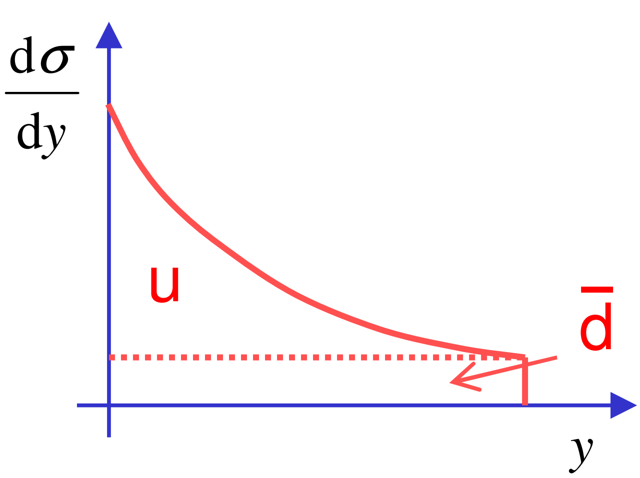


$$\frac{d\sigma}{dy} = \frac{G_F^2}{\pi}\hat{s}$$

giving sum:

νp	$\bar{\nu} p$





- Parton model cross sections on protons :

$$\frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2}{\pi} sx\left[d(x) + (1-y)^2 \bar{u}(x)\right]$$
$$\frac{d^2\sigma^{\bar{\nu} p}}{dxdy} = \frac{G_F^2}{\pi} sx\left[(1-y)^2 u(x) + \bar{d}(x)\right]$$

(p)

- Similarly for neutrons :

$$\frac{d^2\sigma^{\nu n}}{dxdy} = \frac{G_F^2}{\pi} sx\left[d^n(x) + (1-y)^2 \bar{u}^n(x)\right]$$

$$\frac{d^2\sigma^{\bar{\nu} n}}{dxdy} = \frac{G_F^2}{\pi} sx\left[(1-y)^2 u^n(x) + \bar{d}^n(x)\right]$$

But p=(uud) n=(udd) so

$$u^n(x) = d^p(x) \equiv d(x) \qquad \bar{u}^n(x) = \bar{d}^p(x) \equiv \bar{d}(x)$$
$$d^n(x) = u^p(x) \equiv u(x) \qquad \bar{d}^n(x) = \bar{u}^p(x) \equiv \bar{u}(x)$$

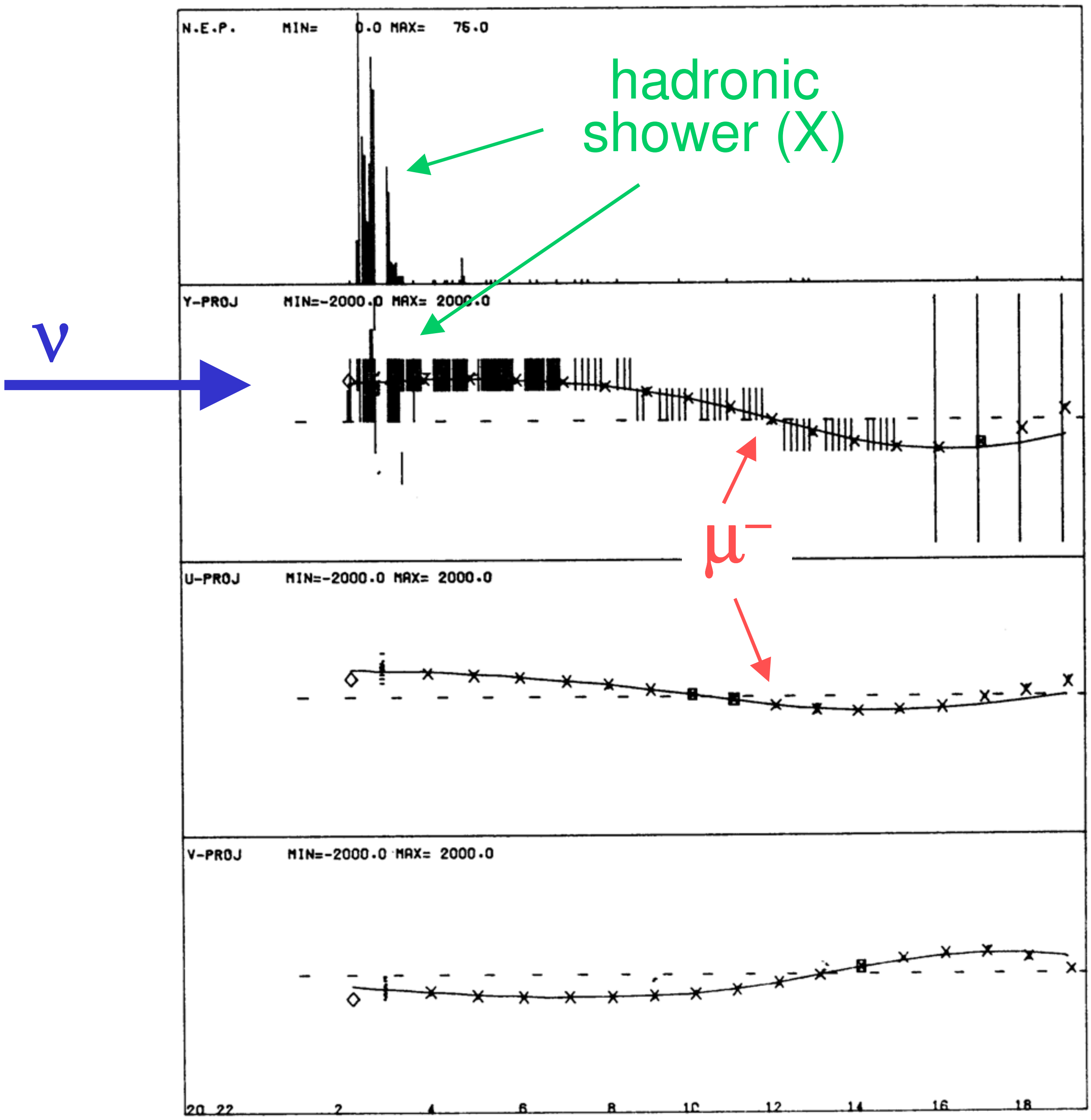
and neutron cross sections become

$$\frac{d^2\sigma^{\nu n}}{dxdy} = \frac{G_F^2}{\pi} sx\left[u(x) + (1-y)^2 \bar{d}(x)\right]$$
$$\frac{d^2\sigma^{\bar{\nu} n}}{dxdy} = \frac{G_F^2}{\pi} sx\left[(1-y)^2 d(x) + \bar{u}(x)\right]$$

(n)

CDHS Neutrino Experiment (CERN)

Example of DIS $\nu + Fe \rightarrow \mu^- + X$ event



A highly penetrating charged particle is the characteristic signature of a <u>muon</u>

(muons don't feel the strong interactions)

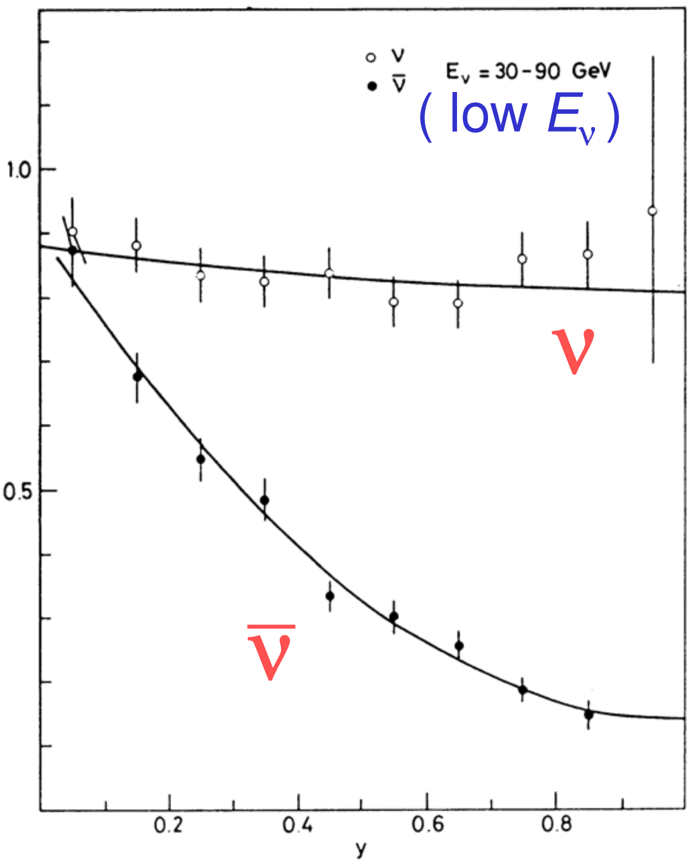
(electrons give electromagnetic showers)

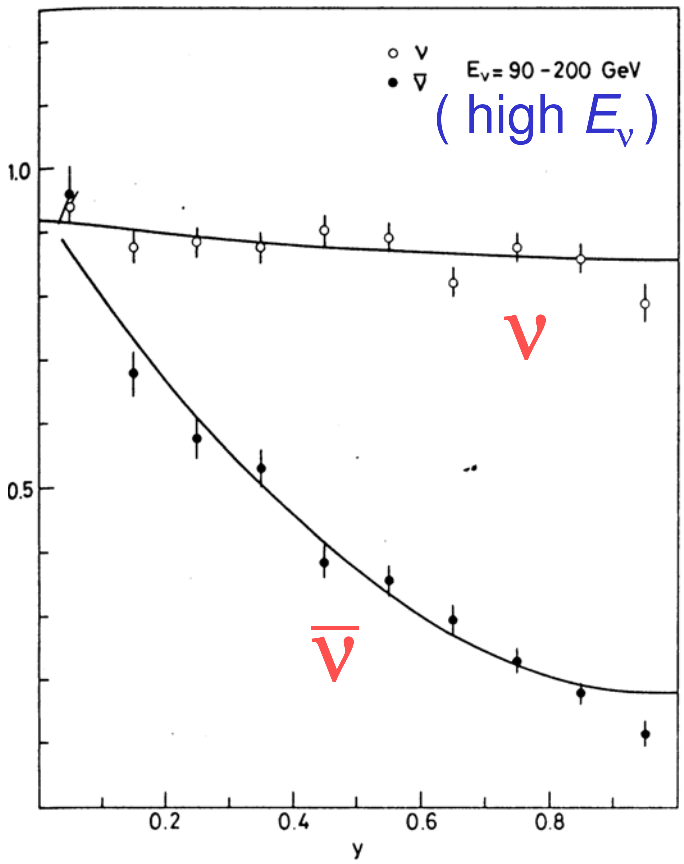
(tau leptons decay very rapidly)

Measured y distributions

e.g. from CDHS experiment at CERN :

J. de Groot et al., Z.Phys. **C1** (1979) 143





Scattering from iron target :

$\Rightarrow$ approximately an <u>isoscalar</u> target

(equal numbers of protons and neutrons)

$$\frac{d^2\sigma^{\nu N}}{dxdy} = \frac{1}{2}\left(\frac{d^2\sigma^{\nu p}}{dxdy} + \frac{d^2\sigma^{\nu n}}{dxdy}\right) = \frac{G_F^2}{2\pi} sx\left[(u+d) + (1-y)^2(\bar{u}+\bar{d})\right]$$

$$\frac{d^2\sigma^{\bar{\nu} N}}{dxdy} = \frac{1}{2}\left(\frac{d^2\sigma^{\bar{\nu} p}}{dxdy} + \frac{d^2\sigma^{\bar{\nu} n}}{dxdy}\right) = \frac{G_F^2}{2\pi} sx\left[(1-y)^2(u+d) + (\bar{u}+\bar{d})\right]$$

- Total ν, ν̄ cross sections from parton model :

$$\sigma^{\nu N} = \frac{G_F^2 s}{2\pi}\left[f_q + \frac{1}{3} f_{\bar{q}}\right] \qquad \sigma^{\bar{\nu} N} = \frac{G_F^2 s}{2\pi}\left[\frac{1}{3} f_q + f_{\bar{q}}\right]$$

$f_q = f_u + f_d =$ fraction of proton's momentum carried by quarks

$f_{\bar{q}} = f_{\bar{u}} + f_{\bar{d}} =$ fraction of proton's momentum carried by antiquarks

(examples sheet)

$\Rightarrow$ can measure quark, antiquark, gluon momentum fractions

Find: $f_q \approx 0.41 \qquad f_{\bar{q}} \approx 0.08$

$\Rightarrow$ ~ 50% of momentum is carried by gluons

(which cannot interact with the virtual W boson)

- If no antiquarks in nucleon, expect

$$\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} = 3$$

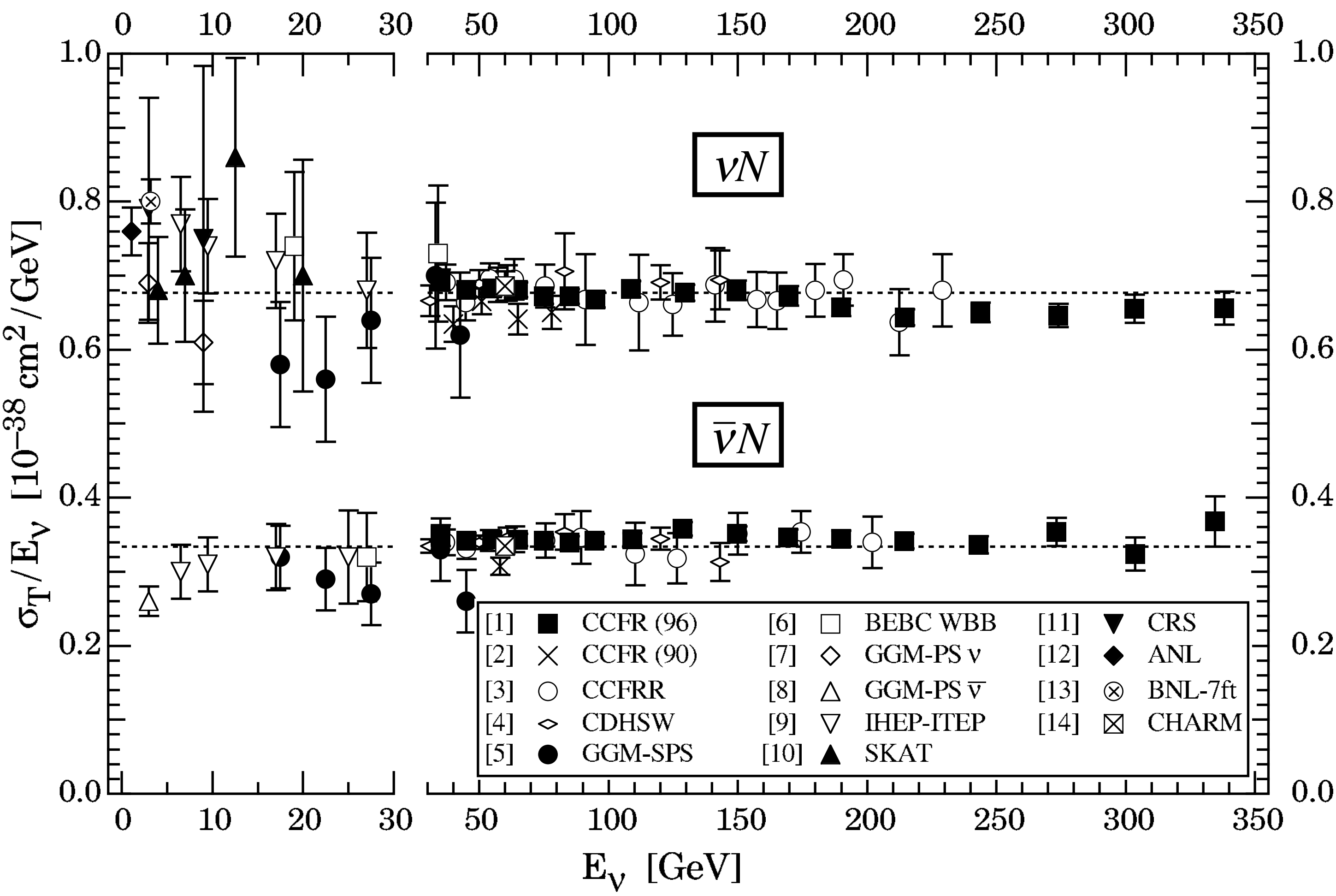
Taking antiquarks into account modifies this to

$$\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} = \frac{f_q + \frac{1}{3} f_{\bar{q}}}{\frac{1}{3} f_q + f_{\bar{q}}} \approx 2$$

νN, $\bar{\nu}$N total cross sections

- In lab frame:

$$s \approx 2ME_\nu^{\rm lab} \quad \Longrightarrow \quad \begin{cases} \sigma^{\nu N} \propto E_\nu^{\rm lab} \\ \sigma^{\bar{\nu} N} \propto E_{\bar{\nu}}^{\rm lab} \end{cases}$$



These cross sections are very small !

e.g. $\sigma \sim 10^{-38}$ cm^2 $\Longrightarrow$

mean free path in water $\sim 10^{13}$ m

~ 0.01 light years

Structure Function Predictions

- For νp: comparing coefficients of 1, y, y^2 in

$$\frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2}{\pi} sx\left[d(x) + (1-y)^2 \bar{u}(x)\right]$$

$$\frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu p}(x) + \frac{y^2}{2} 2xF_1^{\nu p}(x) + y\left(1-\frac{y}{2}\right)xF_3^{\nu p}(x)\right]$$

gives

$$F_2^{\nu p} = 2xF_1^{\nu p} = 2x[d(x) + \bar{u}(x)]$$
$$xF_3^{\nu p} = 2x[d(x) - \bar{u}(x)]$$

Callan-Gross relation again (quarks are pointlike spin 1/2 particles)

- Measure both F_2 and F_3

$\Longrightarrow$ can determine $d(x)$, $\bar{u}(x)$ separately

(possible because scattering from quarks or antiquarks gives different angular distributions)

- νn scattering same with d ↔ u :

$$F_2^{\nu n} = 2xF_1^{\nu n} = 2x[u(x) + \bar{d}(x)]$$
$$xF_3^{\nu n} = 2x[u(x) - \bar{d}(x)]$$

$\Longrightarrow$ can determine $u(x)$, $\bar{d}(x)$ separately

- For an isoscalar target :

$$\left.\begin{aligned} F_2^{\nu N} &= \frac{1}{2}\left(F_2^{\nu p} + F_2^{\nu n}\right) = x\left[u + d + \bar{u} + \bar{d}\right] \\ xF_3^{\nu N} &= \frac{1}{2}\left(xF_3^{\nu p} + xF_3^{\nu n}\right) = x\left[u + d - \bar{u} - \bar{d}\right] \end{aligned}\right\}$$

- But for eN scattering, we had:

$$\left.\begin{aligned} F_2^{ep} &= x\left[\frac{4}{9}u + \frac{1}{9}d + \frac{4}{9}\bar{u} + \frac{1}{9}\bar{d}\right] \\ F_2^{en} &= x\left[\frac{4}{9}d + \frac{1}{9}u + \frac{4}{9}\bar{d} + \frac{1}{9}\bar{u}\right] \end{aligned}\right\}$$

$$\Rightarrow \quad F_2^{eN} = \frac{1}{2}\left(F_2^{ep} + F_2^{en}\right) = \frac{5}{18}x\left[u + d + \bar{u} + \bar{d}\right]$$

$$\Rightarrow \quad \boxed{F_2^{eN} = \frac{5}{18}F_2^{\nu N}} \qquad \left(= 0.2\dot{7}\, F_2^{\nu N}\right)$$

comes from mean squared quark charge $= \frac{1}{2}\left[\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right]$

Experiment $\rightarrow$ 0.29 ± 0.02

eN scattering: depends on z_q^2

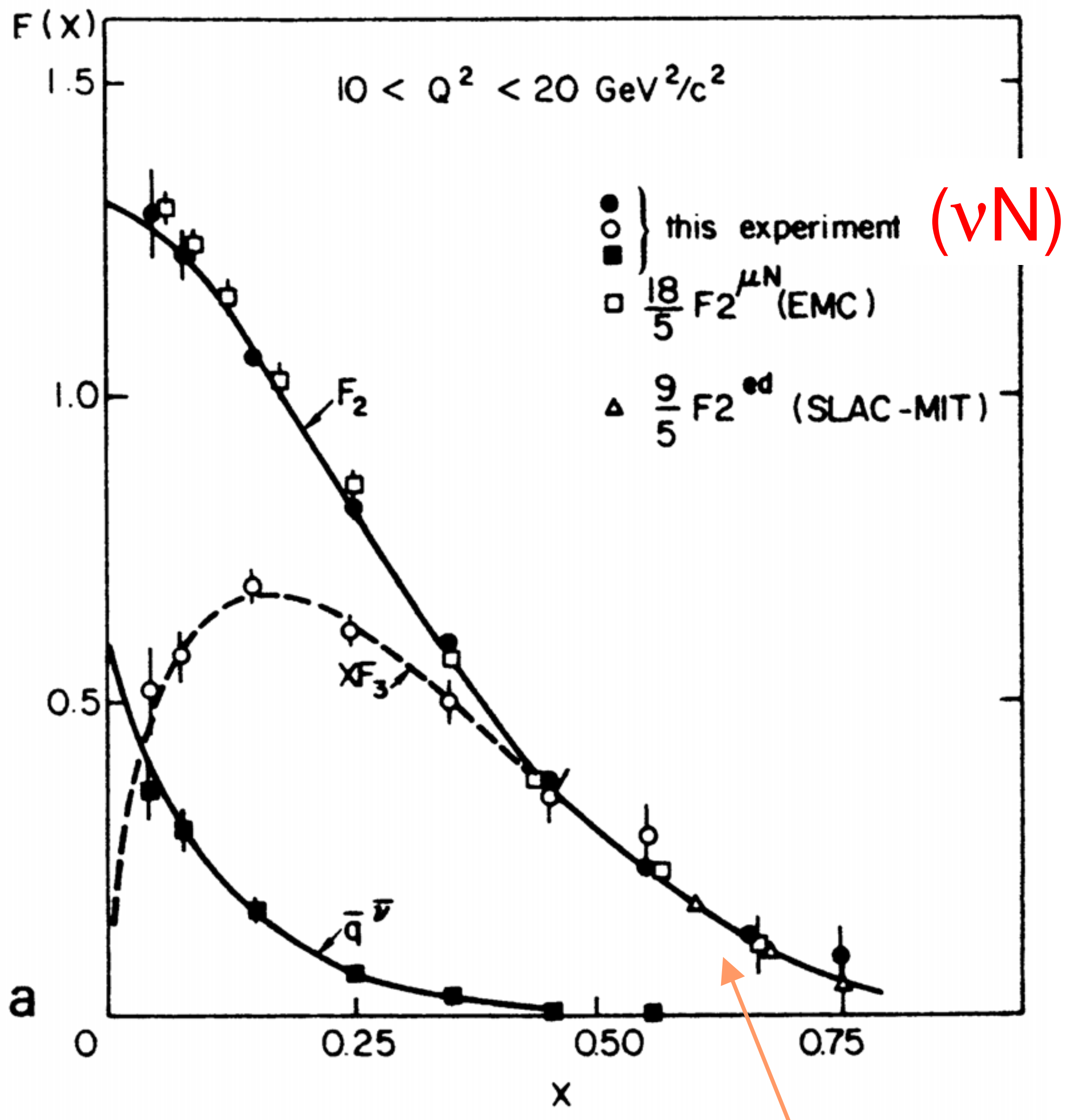
νN scattering: independent of quark charge

Measurements of F_2, F_3

e.g. CDHS experiment again :

$$\nu + \mathrm{Fe} \rightarrow \mu^- + X$$

H. Abramowicz et al., Z.Phys. **C17** (1983) 283



$$F_2^{\nu N} = \frac{18}{5} F_2^{\mu N} = x[u + d + \bar{u} + \bar{d}]$$

$$xF_3^{\nu N} = x\left[u + d - \bar{u} - \bar{d}\right]$$

sea contribution small at large x, so $F_2 \approx xF_3$

- Bring in "valence" and "sea" components:

$$u(x) = u_V(x) + u_S(x) = u_V(x) + S(x)$$
$$d(x) = d_V(x) + d_S(x) = d_V(x) + S(x)$$
$$\bar{u}(x) = \bar{u}_S(x) = S(x)$$
$$\bar{d}(x) = \bar{d}_S(x) = S(x)$$

with normalisation $\int_0^1 u_V(x)\,\mathrm{d}x = 2 \qquad \int_0^1 d_V(x)\,\mathrm{d}x = 1$

$$\Rightarrow \qquad F_3^{\nu N} = u_V(x) + d_V(x)$$

$$\Rightarrow \qquad \boxed{\int_0^1 F_3^{\nu N}(x)\,\mathrm{d}x = \int_0^1 [u_V(x) + d_V(x)]\,\mathrm{d}x = 3}$$

"Gross-Llewellyn-Smith sum rule"

only 2 people!

⟶ measures # of valence quarks in nucleon

experiment = 3.0 ± 0.2

- For antineutrinos: parton model predicts

$$F_2^{\bar{\nu}p} = F_2^{\nu n} \qquad F_3^{\bar{\nu}p} = F_3^{\nu n} \qquad F_2^{\bar{\nu}n} = F_2^{\nu p} \qquad F_3^{\bar{\nu}n} = F_3^{\nu p}$$

Summary of ν, $\bar{\nu}$ Scattering

1) ν couples to d and $\bar{u}$; $\bar{\nu}$ couples to u and $\bar{d}$

   $\Longrightarrow$ can investigate flavour content of nucleon

2) $\nu\bar{q}$ suppressed by factor $(1-y)^2$ wrt νq
   $\bar{\nu}q$ suppressed by factor $(1-y)^2$ wrt $\bar{\nu}\bar{q}$

   $\Longrightarrow$ can measure antiquark content of nucleon

3) νq, $\bar{\nu}q$ scattering depends on same quark momentum distributions as eq scattering, but not on quark charge

   $\Longrightarrow$ can measure quark charges

(mean square)

$$F_2^{eN} = \frac{5}{18} F_2^{\nu N}$$

4) New structure function F_3 due to parity violation

$$\int_0^1 F_3^{\nu N}(x)\,\mathrm{d}x = 3$$

   $\Longrightarrow$ can count number of valence quarks in nucleon