Electric Potential, Electric Potential Energy and Capacitance

Chapter 18

Electric Potential Energy

Conservation of Energy

Potential of Point Charges

Equipotential Surfaces

Capacitance & Capacitors

Part 1

Electric Potential Energy

Energy: Definitions

Webster's dictionary:

Energy – the capacity to do work

Work – the transfer of energy

Richard Feynman – Nobel Prize in physics (1965)

The Feynman Lectures on Physics. "...**in physics today, we have no knowledge of what energy is**."

We know how to calculate its value for a great variety of situations, but beyond that it's just an abstract thing which has only one really important property - conservation

Physics 111: Potential Energy

Potential energy U is energy that can be associated with the configuration of a system of objects that exert forces on one another. If the configuration of the system changes, then the potential energy of the system can also change

Potential energy can be defined for conservative forces only

Examples:

- gravitational potential energy
- spring elastic potential energy

Connection between energy and force

$$\Delta K = \frac{mv^2}{2} - \frac{mv_0^2}{2} \qquad v^2 - v_0^2 = 2a(x - x_0)$$

$$\Delta K = \frac{1}{2}m(v^2 - v_0^2) = ma(x - x_0)$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F(x - x_0)$$

Left side – the kinetic energy has been changed

Right side – the change is equal to $F_x \cdot d$

$$W = F(x - x_0)$$

Similarities between Coulomb's law and Newton's gravitational law

Equations are similar

$$F_g = G\frac{m_1 m_2}{r^2} \qquad F_c = k\frac{q_1 q_2}{r^2}$$

Both forces are conservative ones.

Conservative force:

Definition 1. A conservative force does zero total work on any closed path

Definition 2. The work done by a conservative force in going from an arbitrary point A to an arbitrary point B is *independent of the path* from A to B

Connection between a <u>conservative</u> force and potential energy

Calculus based physics

$$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx$$

Algebra based physics (gravitational and spring forces)

$$\Delta U = mgh \qquad F = mg$$

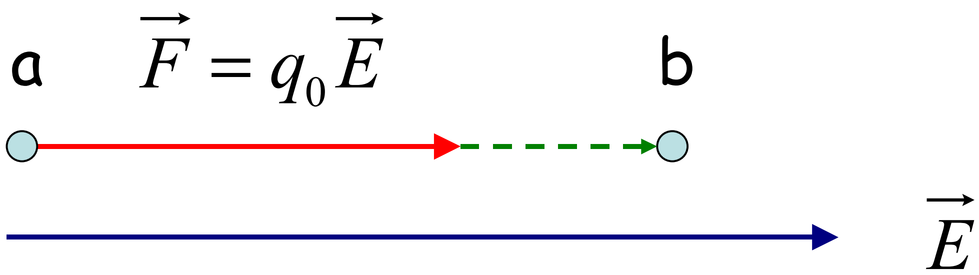
$$\Delta U = -\frac{1}{2}kx^2 \qquad F = -kx$$

Definition for electric potential energy

Electric potential energy can be defined similar to definition of the gravitational potential energy

Potential energy change

$$U_a = U_b + Fd = U_b + q_0 E d$$



d is the distance between points a and b

Definition is true for a constant electric force working along the path of the motion.

Definition for electric potential

Electric potential is **NOT** the same as Electric potential energy but the connection is very simple

For applications it is very useful to introduce the potential energy per unit charge

$$\Delta V = \frac{\Delta U}{q_0}$$

Units: J/C (the joule per coulomb),

1 volt = 1 joule per coulomb

with this definition E = N/C = V/m

Electric potential is a scalar, not a vector

Confusion with definitions for “change in electric potential”, and “change in electric potential energy”

There are two definitions for a “change of something”

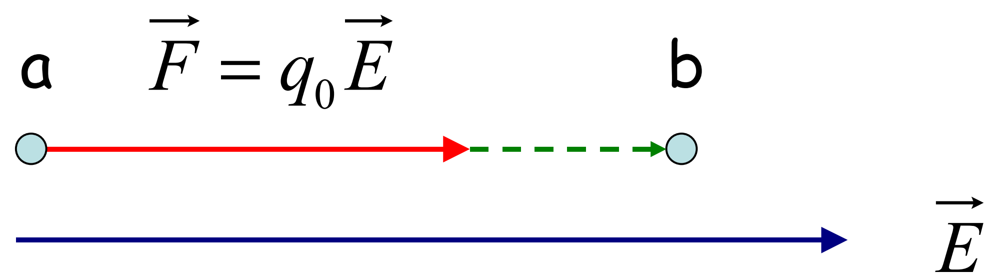
$$\Delta x = x_i - x_f \quad and \quad \Delta x = x_f - x_i$$

In many textbooks the change is defined as

Therefore

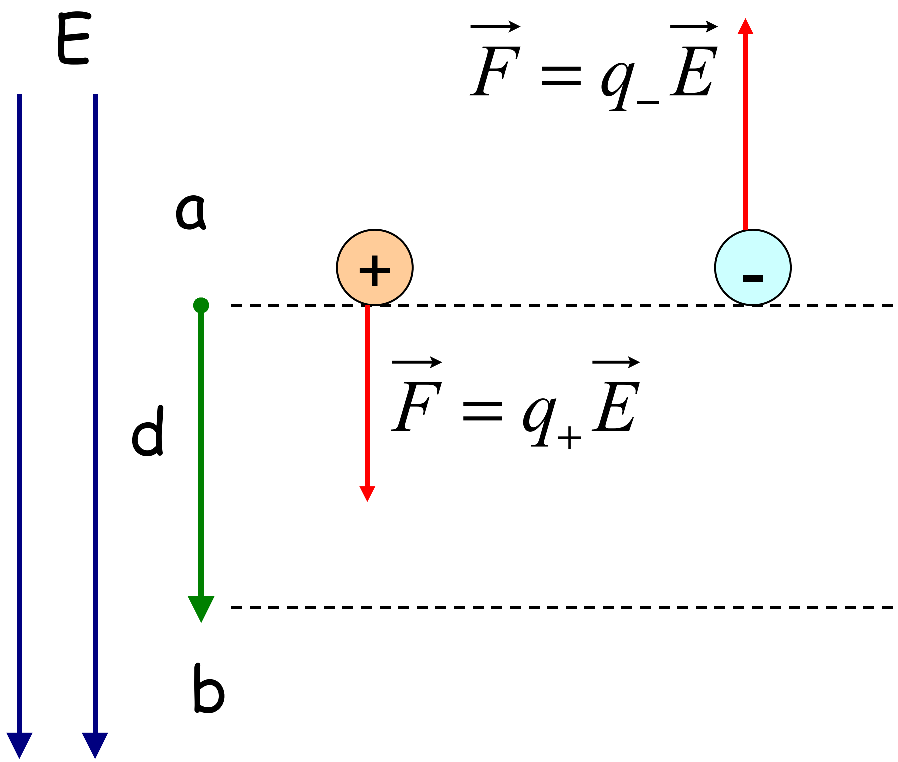
$$\Delta U = U_b - U_a = -q_0 E d$$

$$\Delta V = V_b - V_a = -Ed$$



Helpful approach (less confusion)

1. Always draw a diagram with fields and forces
2. Mark initial and final points
3. Watch the field direction and the sign of the charge



$$U_a = U_b + q_+ Ed$$

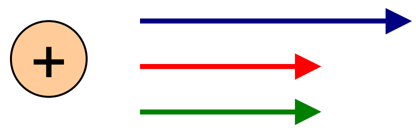
$$U_a = U_b - q_- Ed$$

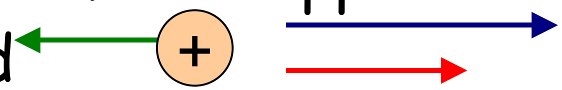
in most textbooks

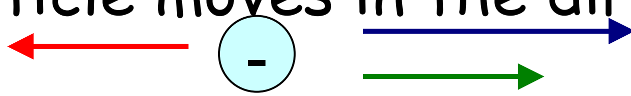
$$\Delta U = U_b - U_a = -q_+ Ed$$

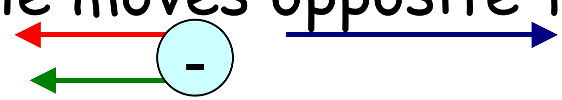
blue field
red force
green displacement

Four possible combinations

1. A positive particle moves in the direction of the electric field


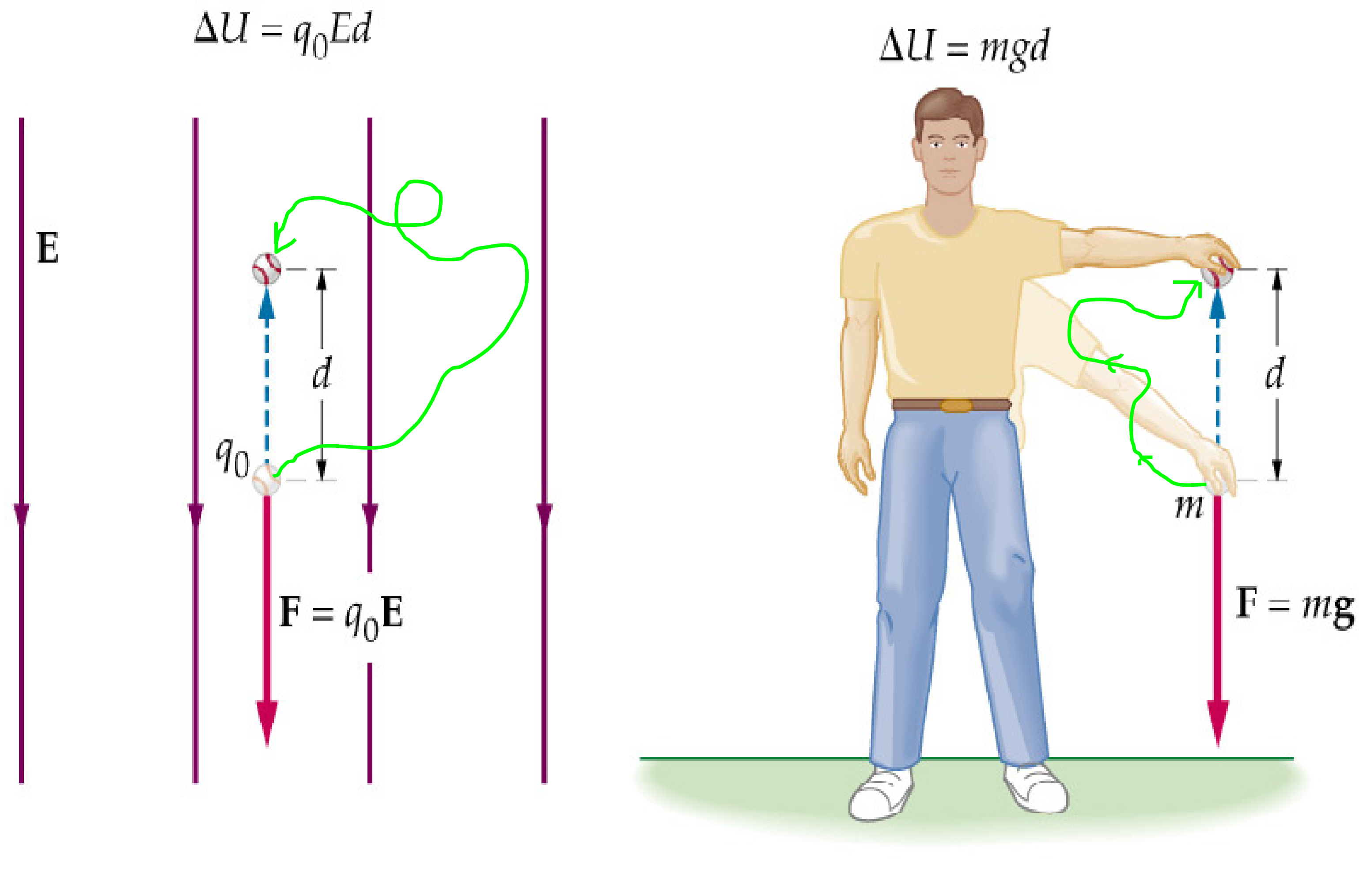
Potential energy decreases
2. A positive particle moves opposite to the direction of the electric field


Potential energy increases
3. A negative particle moves in the direction of the electric field


Potential energy increases
4. A negative particle moves opposite to the direction of the electric field


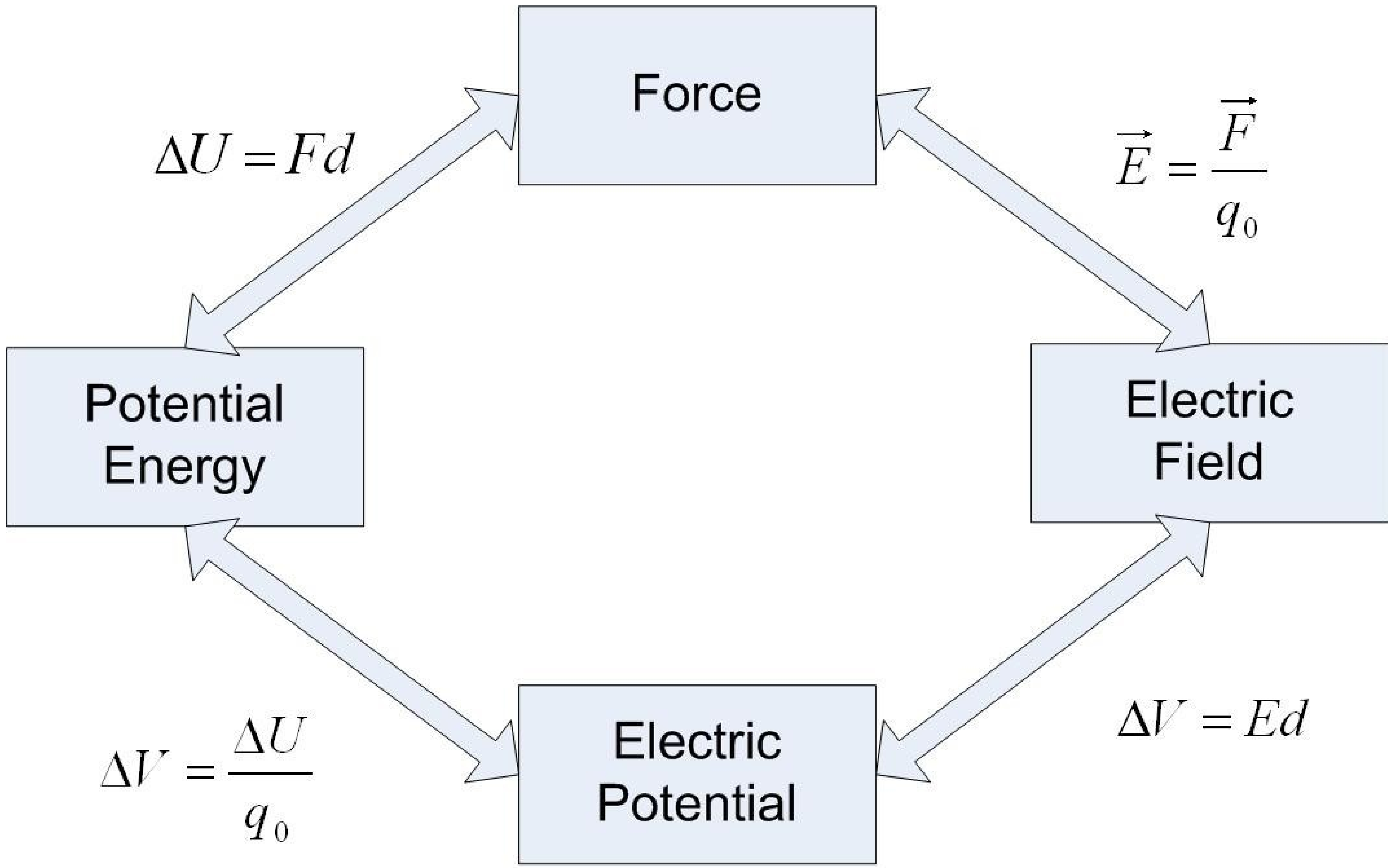
Potential energy decreases

Change in electric potential energy. (notice the direction of the blue arrow.)



(a) (b)

For a uniform field (E=const)



More terminology

Electric potential energy (U):

potential energy

electrostatic potential energy

Electric potential (V):

potential

potential difference

voltage (difference)

electrostatic potential

The zero of electrical potential

For calculating physical quantities it is the *difference* in potential which has significance, not the potential itself. Therefore, we are free to choose as having zero potential any arbitrary point which is convenient.

Typical choices are:
- the earth
- infinity

Part 2

Conservation of Energy

Conservation of energy

A consequence of the fact that electric force is conservative is that the total energy of an object is conserved (as long as non-conservative forces like friction can be ignored)

$$K_a + U_a = K_b + U_b$$

$$\frac{1}{2}mv_a^2 + qV_a = \frac{1}{2}mv_b^2 + qV_b$$

Problem

Problem 2. (20 points)

An electron with a speed of $5.0*10^8$ cm/s enters an electric field of magnitude $1.0*10^3$ N/C, traveling along the field lines in the direction that retards its motion.

(a) How far will the electron travel in the field before stopping momentary?

(b) What acceleration will the electron experience?

(c) How much time will have elapsed for the electron before stopping momentary?

(d) If the region with the electric field is only 8.0 mm long (too short for the electron to stop within it) what fraction of the electron's initial kinetic energy will be lost in that region?

a) $\frac{m_e V^2}{2} = q_e E d$, $d = \frac{m_e V^2}{2 q_e E} = 7.12 \cdot 10^{-2}\, m$

b) $a = \frac{F}{m_e} = \frac{q_e E}{m_e} = 1.76 \cdot 10^{14}\, m/s^2$

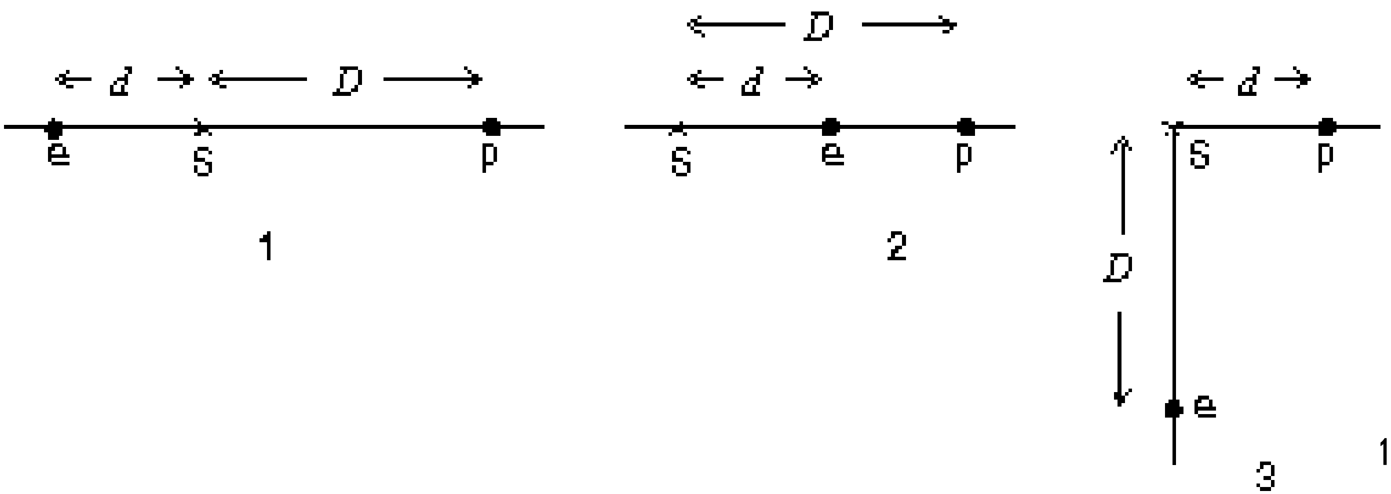
c) $v = at$, $t = \frac{v}{a} = 2.85 \cdot 10^{-8}\, s$

d) $\Delta K = q_e E d_x$, fraction $\frac{q_e E d_x}{\frac{m_e V^2}{2}} = \frac{q_e E d_x}{q_e E d} = 0.112$

Net Potential

Three possible configurations for an electron e and a proton p are shown below. Take the zero of potential to be at infinity and rank the three configurations according to the potential at S, from most negative to most positive.

A) 1, 2, 3
B) 3, 2, 1
C) 2, 3, 1
D) 1 and 2 tie, then 3
E) 1 and 3 tie, then 2



Problem

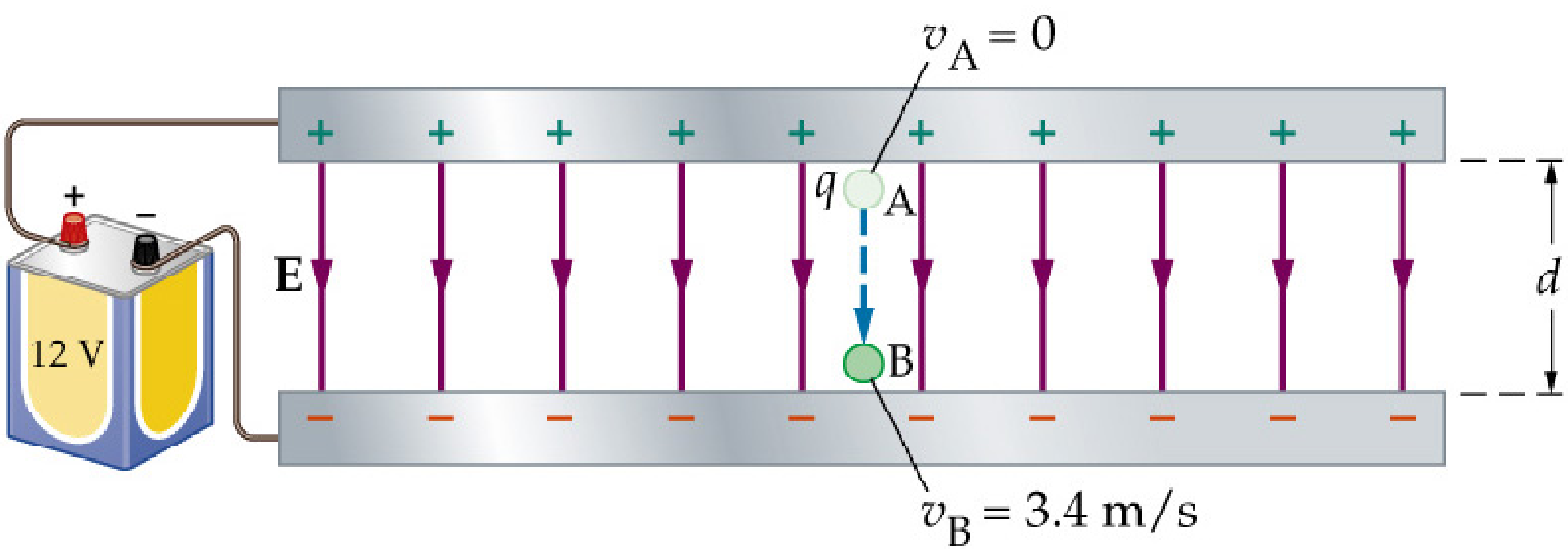
A charge q (q = 6.24 μC) is released from rest at the positive plate and reaches the negative plate with a speed of 3.4 m/s. The plates are connected to a 12-V battery

Calculate:

(a) the mass of the charge.

(b) its kinetic energy at point A and at point B.

(c) If d = 2cm, what is the electric field between the plates?



Problem

A uniform electric field with a magnitude of 1200 N/C points in the negative x direction.

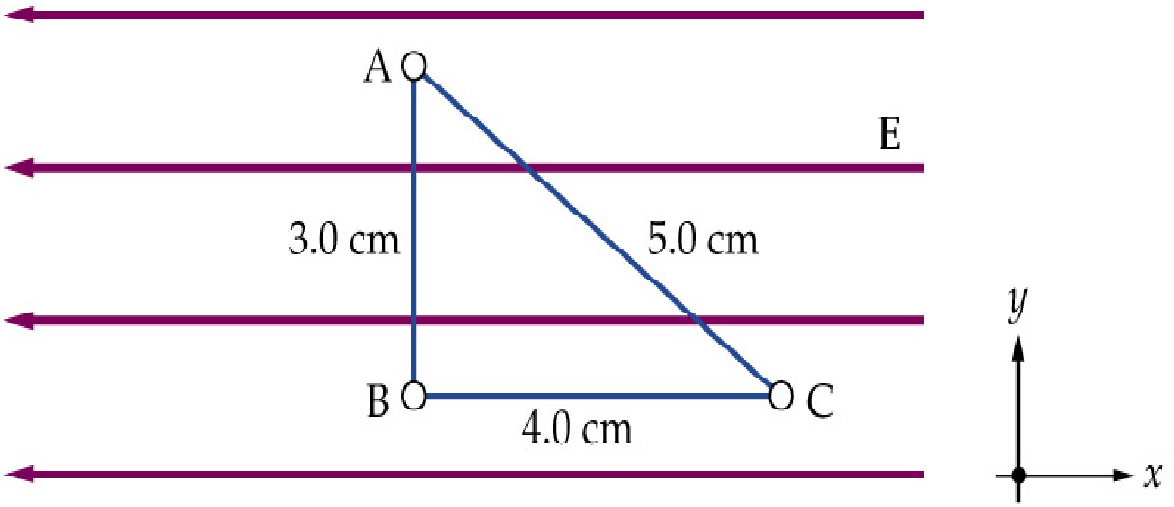
(b) What is the difference in electric potential $\Delta V = V_b - V_a$ between points a and b

(c) What is the difference $\Delta V = V_b - V_c$?

(d) What is the difference $\Delta V = V_c - V_a$?

(e) If a particle with mass of 3.5 g and a charge +0.045μC is released from rest at point A, in what direction it will move?

(f) What speed will be after moving through a distance of 5 cm?



Let's discuss this!

Part 3

The Electric Potential of Point Charges

The electric potential of point charges

The difference in electric potential energy and electric potential between two points can be written as

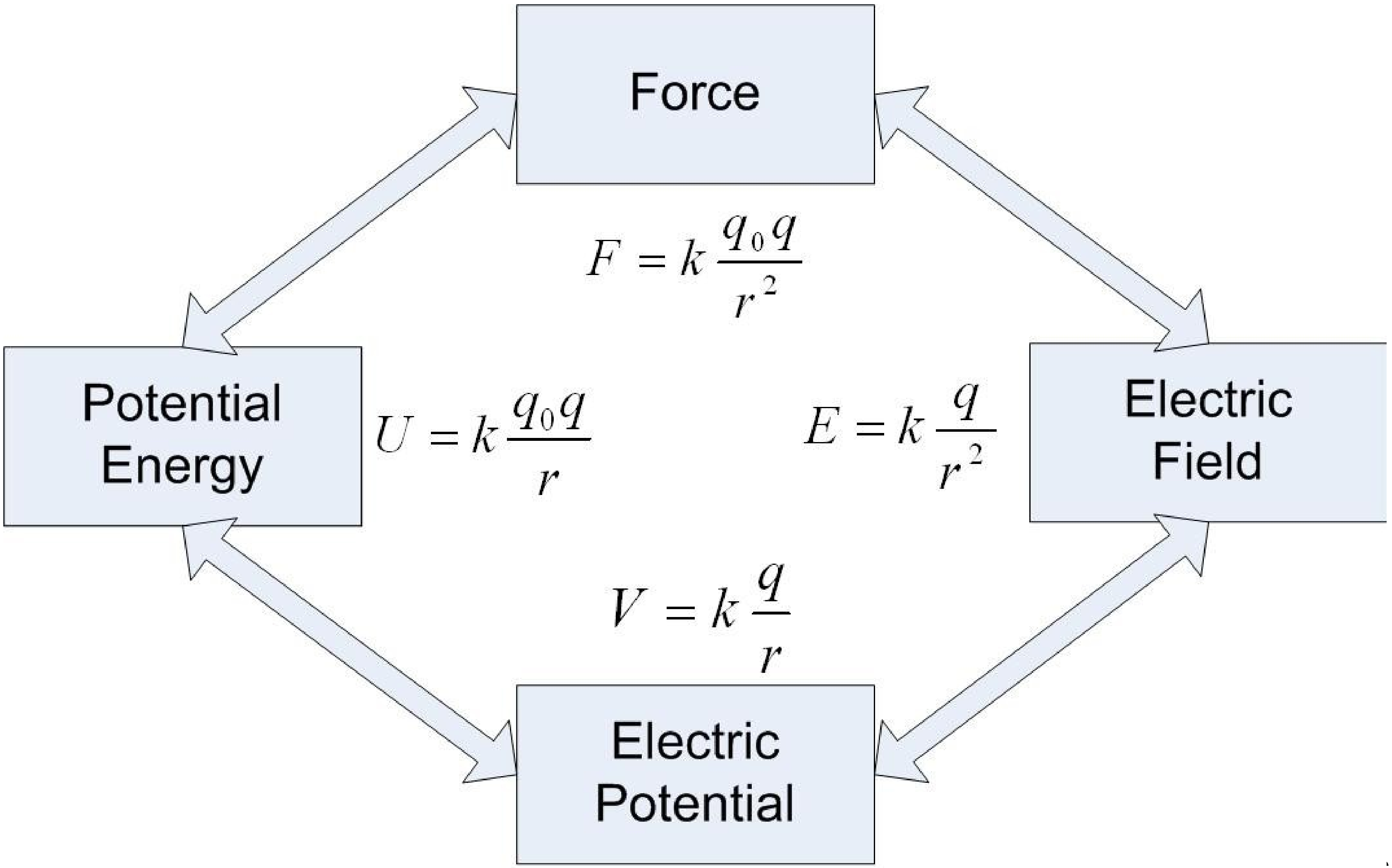
$$U_a - U_b = k\frac{q_0 q}{r_a} - k\frac{q_0 q}{r_b} \qquad V_a - V_b = k\frac{q}{r_a} - k\frac{q}{r_b}$$

Since the potential can be set to zero at any location, we choose the electric potential to be zero infinitly far from a given origin ($V_b \rightarrow 0$ as $r_b \rightarrow \infty$)

Thus

$$U = k\frac{q_0 q}{r} = q_0 V \qquad V = k\frac{q}{r}$$

For a point electric charge



Many Charges and Superposition

If we wish to know the potential at a *given* point in space which results from all surrounding charges, we simply add up the potential at that point due to each charge:

Note that because potential is a scalar, the summation is not difficult. We just need to insert the sign of each charge.

$$\varsigma_A = \kappa \frac{\theta_1}{\rho_1} + \kappa \frac{\theta_2}{\rho_2} + \kappa \frac{\theta_3}{\rho_3} + \ldots$$

How many grains form a pile of sand?

?

One proton

A proton is released from rest in a region of space with non-zero electric field. As the proton moves, does the electric potential energy of the proton increase, decrease or stay the same? Explain.

?

Two protons

Two protons are released from rest when they are D nm apart. After being released

(a) their kinetic energies gradually decrease to zero as they move apart

(b) their kinetic energies increase as they move apart

(c) their electric potential energy gradually decreases to zero as they move apart

(d) their electric potential energy increase as they move apart

Two protons (more)

?

Two protons are released from rest when they are D nm apart.

(a) What is the maximum speed they will reach?

(b) What is the maximum acceleration they will achieve?

(c) When does this acceleration occur?

(d) Will the answers to questions a-c be different if we consider two electrons?

(e) What if we have an electron and a proton?

The Electron Volt

It is often convenient to work with a unit of energy called the electron volt.

One electron volt is defined as the amount of energy an electron (with charge *e*) gains when accelerated through a potential difference of -1 *V*:

$1\ eV = (1.6 \times 10^{-19}\ C)V = 1.6 \times 10^{-19}\ J$

Part 4

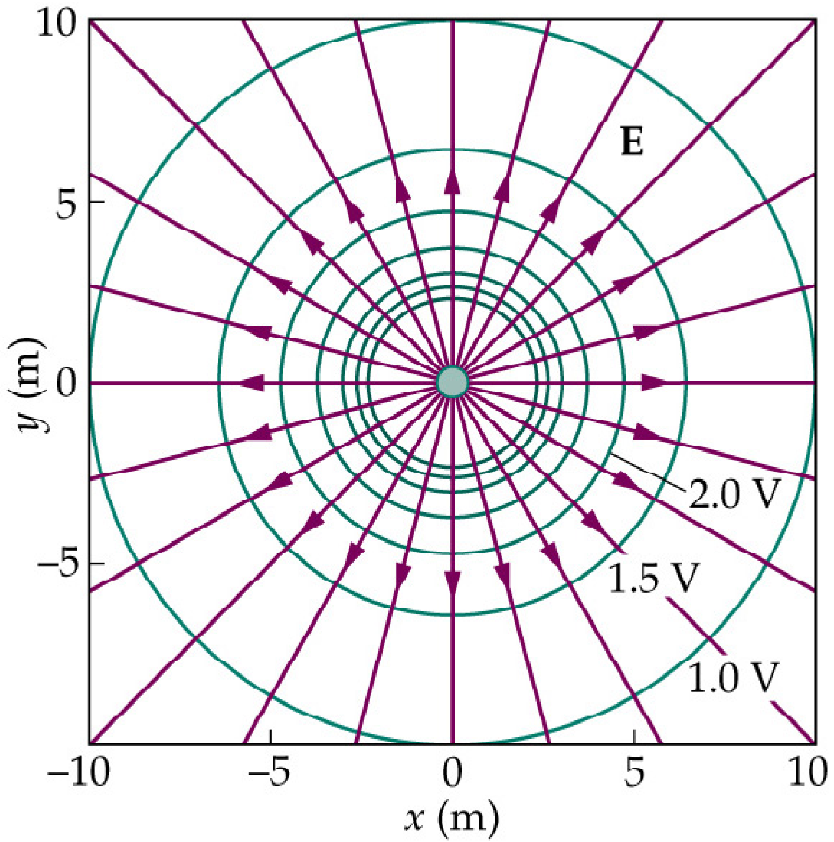
Equipotential Surfaces

Equipotential Surfaces

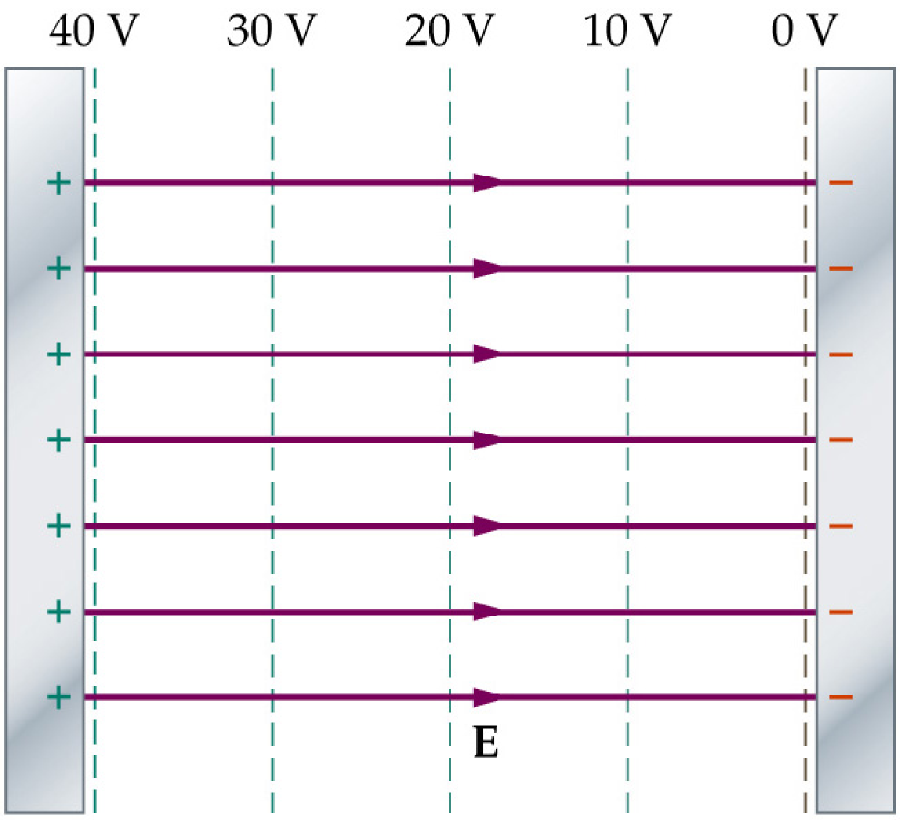
A surface in space for which the potential is the same everywhere (like the surface of a conductor) is called an equipotential surface.

Since no work is required to move a charge on an equipotential surface, *the electric field at every point on an equipotential surface is perpendicular to the surface.*

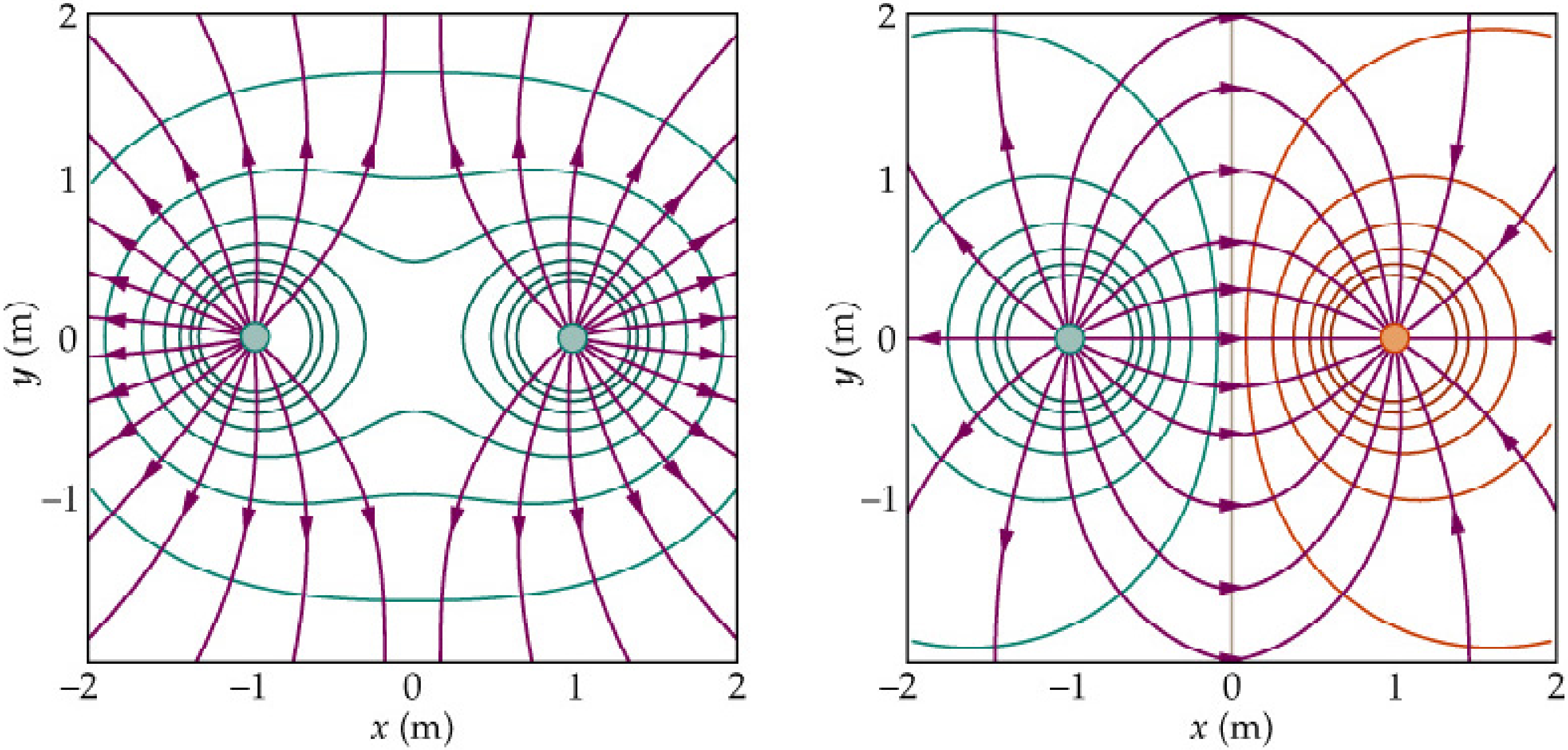
Equipotentials for a point charge



Equipotential surfaces for a uniform electric field



Equipotential surfaces for two point charges



(a) (b)

Part 5

Capacitance & Capacitors

Capacitance and Capacitors

Storing energy as potential energy:

stretching a spring

pulling a bowstring

We can also store energy as potential energy in an electric field.

Capacitor is a device that is used to do that.

A Simple Capacitor

Two charges (with equal but opposite charges of magnitude q)

Why is that?

Connection: energy -> potential difference

$$V_a - V_b = k\frac{q_1}{r_a} - k\frac{q_2}{r_b} = 2k\frac{q}{r}$$

Definition for Capacitance

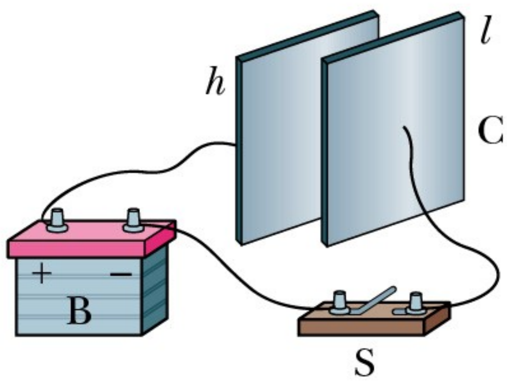
The charge and the potential difference V for a capacitor are proportional to each other

$$q = CV$$

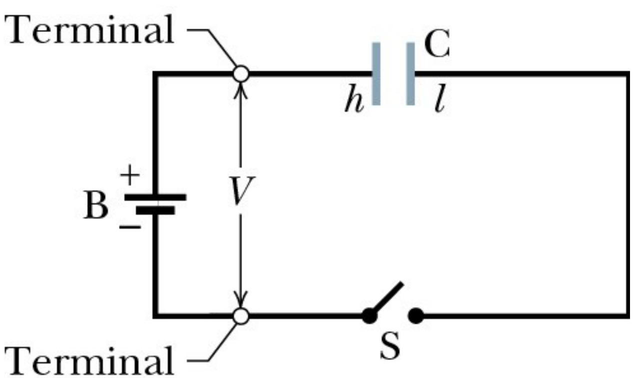
The constant *C* is called the **capacitance** of the capacitor. Its value depends only on the geometry of the plates and not on their charge or potential difference

SI unit: coulomb/volt = farad, F

Charging a capacitor



(a)

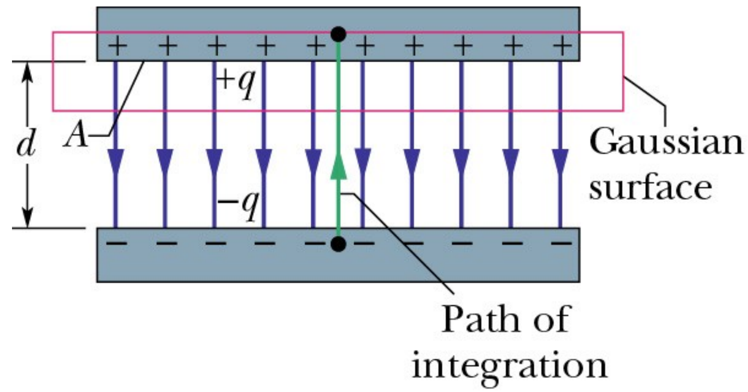


(b)

Two conductors (conducting plates) connected to a battery.

Conductors: negative charge can move rather freely.

Capacitance for different geometries



A parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

A – the plate area

d – the plate separation

ε_0 – the permittivity constant

$\varepsilon_0 = 8.85*10^{-12} F/m = 8.85*10^{-12} C^2/N \cdot m^2$

There are definitions for other geometries

Gauss's law is the toll to calculate.

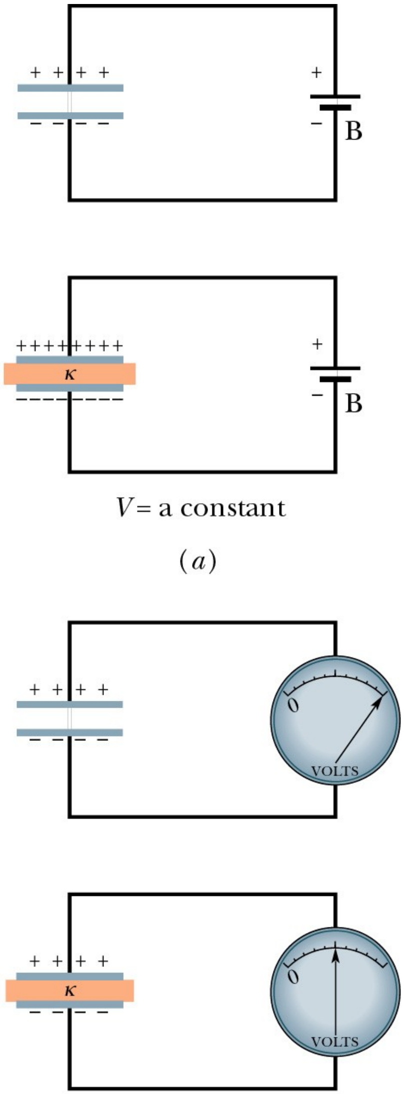
Energy stored in an electric field

Electric potential energy of a charged capacitor

$$U = \frac{1}{2} qV = \frac{q^2}{2C} = \frac{1}{2} CV^2$$

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates

Capacitor with dielectric



V = a constant

(a)

q = a constant

(b)

Dielectric: insulating material (plastic, paper …)

$$C = \kappa C_0$$

k is the dielectric constant

air	1.00054
paper	3.5
Silicon	12.0
Water	80.4
Titania ceramic	130.0

capacitors in series

agnitude of charge on each
te equal by charge conservation

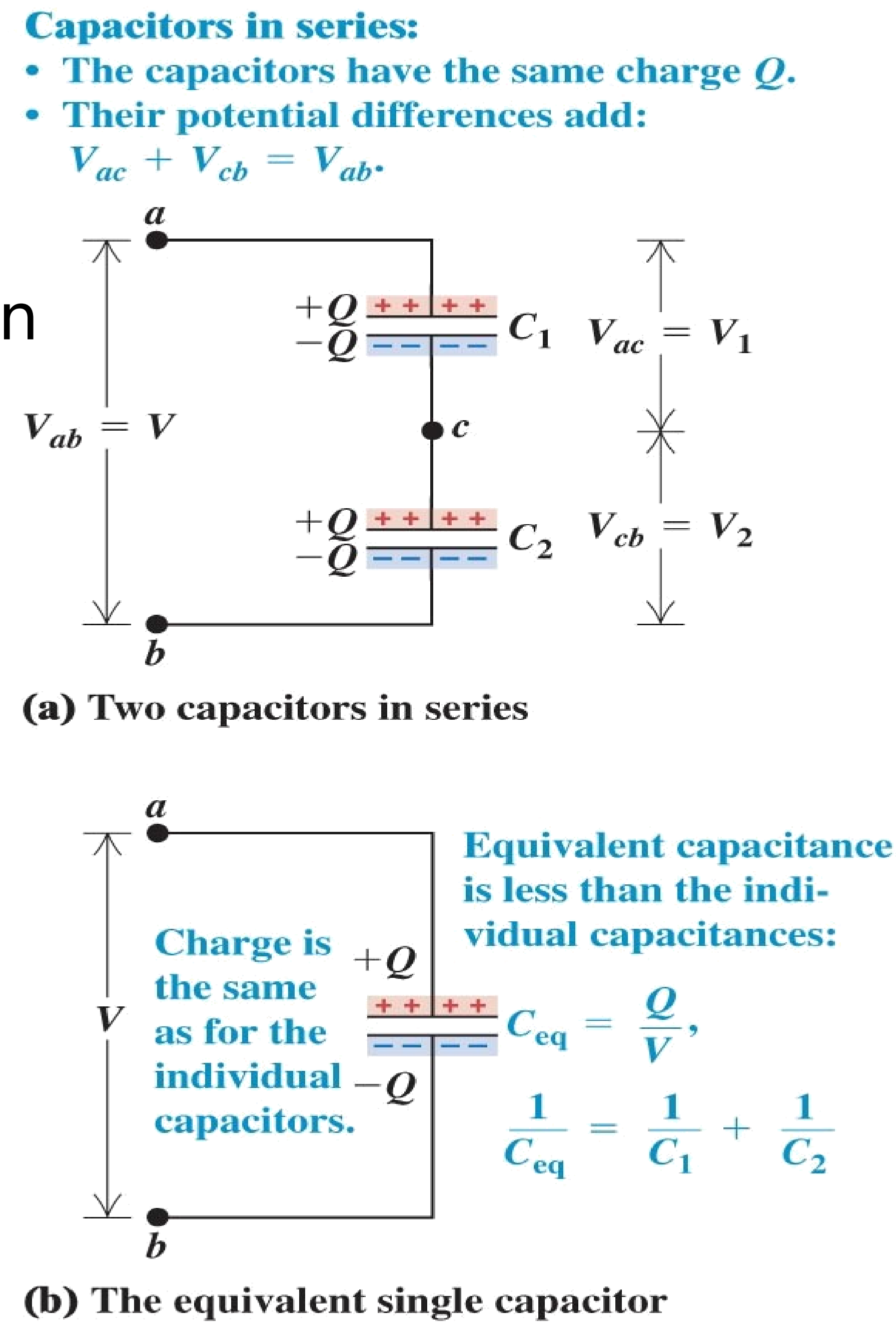
$$V_{ac} = V_1 = \frac{Q}{C_1} \qquad V_{cb} = V_2 = \frac{Q}{C_2}$$

$$V_{ab} = V_{ac} + V_{cb}$$

$$V_{ab} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$V_{ab} = \frac{Q}{C_{eq}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



capacitors in parallel

- potential difference across each capacitor is the same

$$V_{ab} = \frac{Q_1}{C_1} \qquad V_{ab} = \frac{Q_2}{C_2}$$

- total charge on the upper plates is

$$Q = Q_1 + Q_2$$

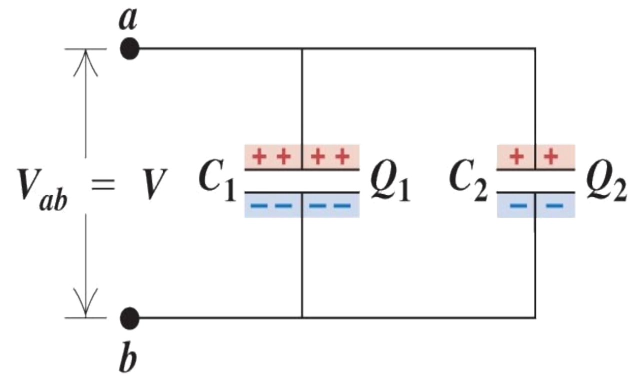
$$Q = C_1 V_{ab} + C_2 V_{ab}$$

$$C_{\text{eq}} V_{ab} = C_1 V_{ab} + C_2 V_{ab}$$

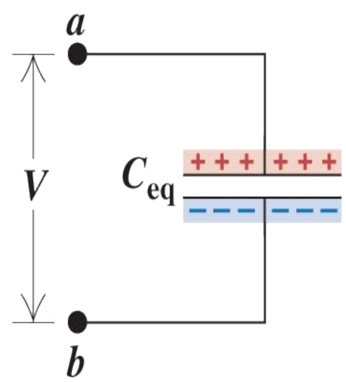
$$\boxed{C_{\text{eq}} = C_1 + C_2}$$

Capacitors in parallel:

- The capacitors have the same potential V.
- The charge on each capacitor depends on its capacitance: $Q_1 = C_1 V$, $Q_2 = C_2 V$.



(a) Capacitors connected in parallel



Charge is the sum of the individual charges:
$Q = Q_1 + Q_2$
Equivalent capacitance:
$C_{\text{eq}} = C_1 + C_2$.

(b) The equivalent single capacitor