Chapter 19
current, resistance and dc circuits

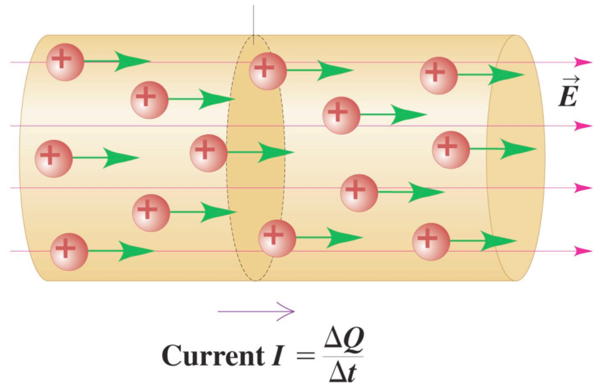
reading quiz

- the ‘conventional’ current direction is the direction of flow of
  A. electrons
  B. equivalent positive charges
  C. positrons

- Two wires made of pure copper have different resistances, these wires may differ in
  A. length
  B. resistivity
  C. both of the above

current

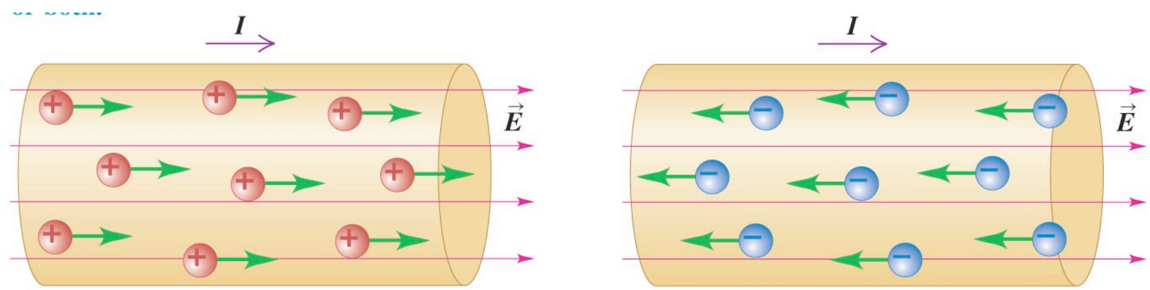
- previously we considered electrostatic situations in which no E-field could exist inside a conductor
- now we move to the case where an electric field is maintained within a conductor by an **external** source and the conductor forms a complete circuit
- this electric field applies a force to charges within the conductor



$$I = \frac{\Delta Q}{\Delta t}$$

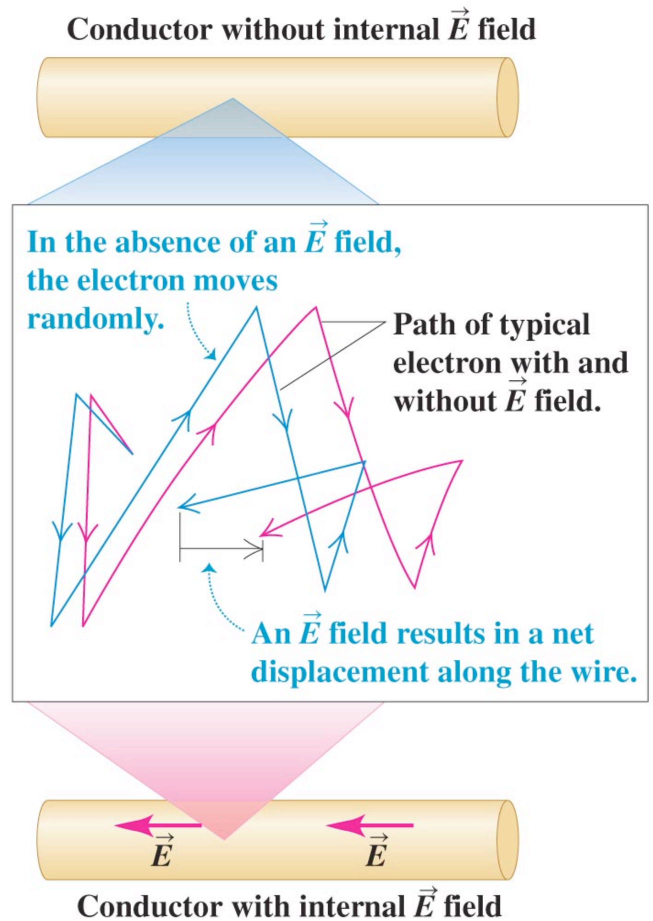
units are C/s or Amperes, A

- in a metal the moving charges are negative electrons, **but the *conventional current* is still defined as the direction positive charges would flow**



current

- rate of flow of charge is uniform throughout a conductor, else charge would accumulate in certain areas
- at non-zero temperature, electrons in a metal move a lot without any applied field
- although the drift velocity of electrons is rather slow O(10^{-4} m/s), the electric fields propagate through the metal at close to the speed of light, so we never notice a delay when 'switching on'



resistance and Ohm's law

- (conventional) current flows from a point of high potential to a point of lower potential along the direction of the E-field
- the current is proportional to the drift velocity of the charges in the conductor
- drift velocity is approximately proportional to the electric field magnitude E and hence to the potential difference V
- from this follows the empirical Ohm's law

$$V = IR$$

  - *resistance, R is measured in V/A, also known as Ω, Ohms*

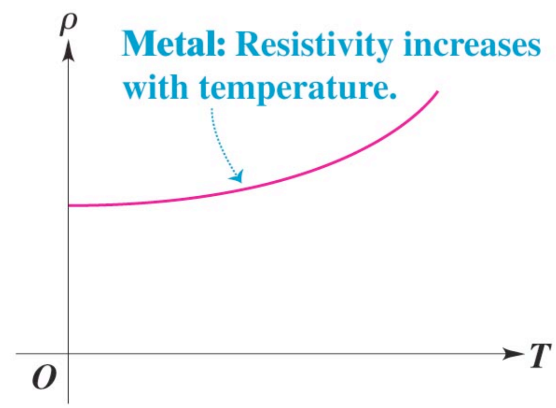
- Ohm's law is an approximate rule which holds for many materials carrying most currents

resistivity

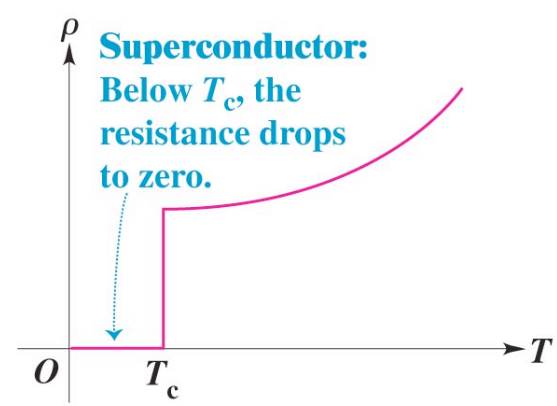
- the resistance of a cylindrical wire is found to increase with increasing length and decrease with decreasing cross-sectional area

$$R = \rho \frac{L}{A}$$

- ρ is the *resistivity* which is a property of the material at a given temperature



(a)



(b)

TABLE 19.1 Resistivities at room temperature

Substance	$\rho\ (\Omega \cdot m)$	Substance	$\rho\ (\Omega \cdot m)$
Conductors:		Mercury	95×10^{-8}
Silver	1.47×10^{-8}	Nichrome alloy	100×10^{-8}
Copper	1.72×10^{-8}	Insulators:	
Gold	2.44×10^{-8}	Glass	$10^{10} - 10^{14}$
Aluminum	2.63×10^{-8}	Lucite	$> 10^{13}$
Tungsten	5.51×10^{-8}	Quartz (fused)	75×10^{16}
Steel	20×10^{-8}	Teflon®	$> 10^{13}$
Lead	22×10^{-8}	Wood	$10^{8} - 10^{11}$

resistance quiz

- A constant potential difference of 24V is set up across an Ohmic resistor and a current of 0.24A measured to flow, what is the resistance of the resistor?

  A. 10 Ω
  B. 100 Ω
  C. 5.76 Ω

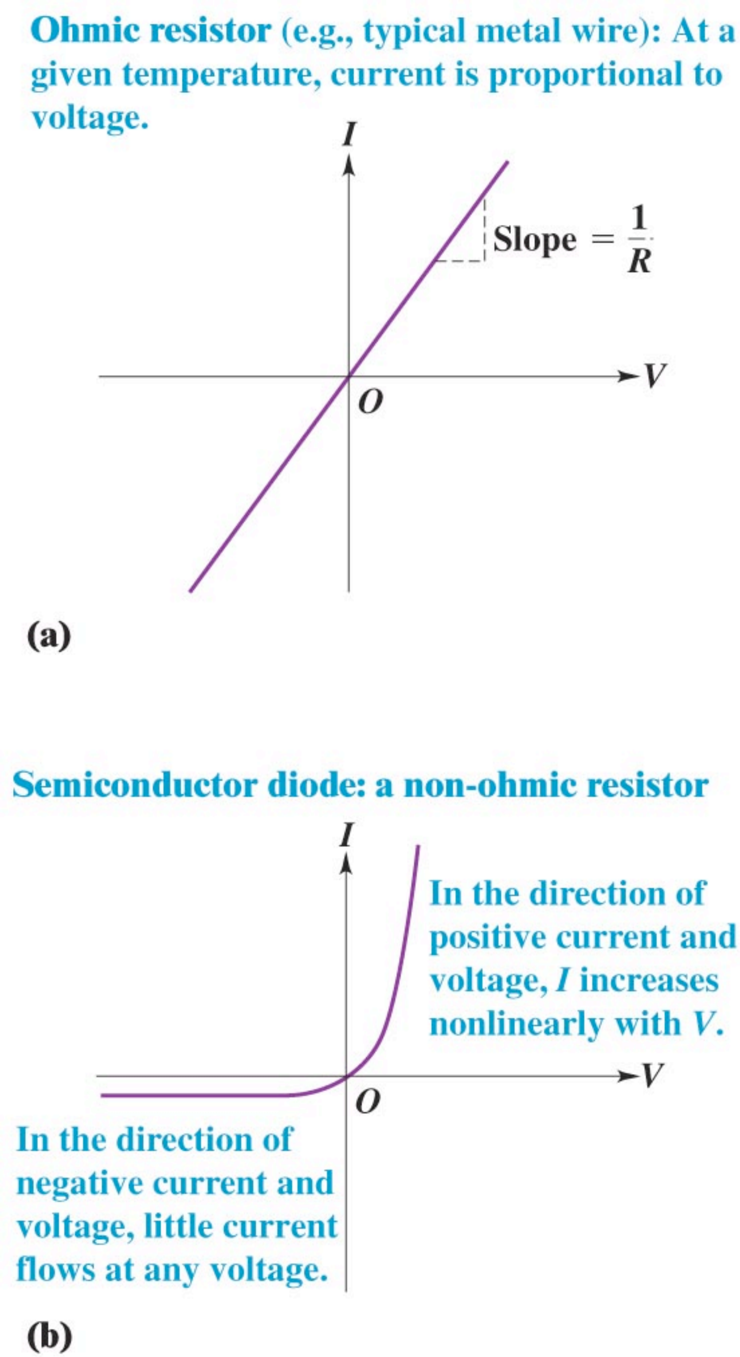
- The resistor is replaced with one of the same dimensions but made from a material of double the resistivity, with a 24V potential difference across it, what current flows?

  A. 0.12 A
  B. 0.48 A
  C. 50 A

resistance quiz

- A current of 1.6 A flows through a wire. The charge carriers are electrons - roughly how many pass through a cross-section of the wire every second?
  - A. 1
  - B. 10^{-19}
  - C. 10^{19}
  - D. 1600

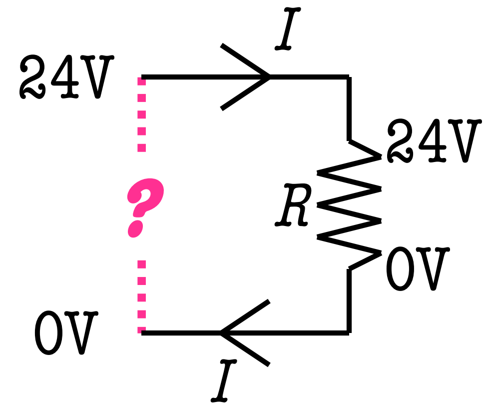
Ohm's law

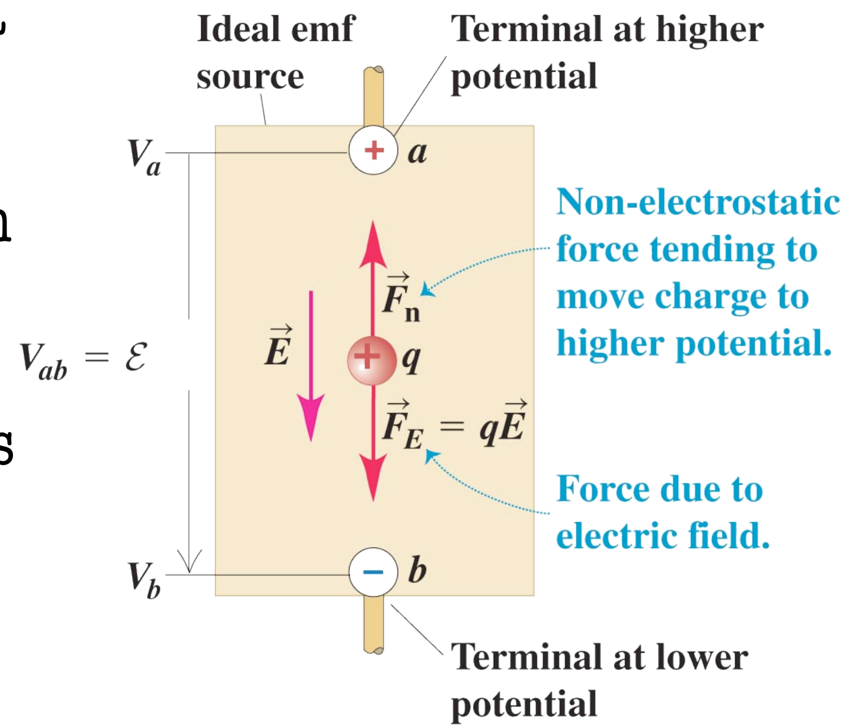


Ohmic resistor (e.g., typical metal wire): At a given temperature, current is proportional to voltage.
I
Slope = 1/R
V
O
(a)
Semiconductor diode: a non-ohmic resistor
I
In the direction of positive current and voltage, I increases nonlinearly with V.
V
O
In the direction of negative current and voltage, little current flows at any voltage.
(b)

electromotive ‘force’

- current will always ‘flow’ from high potential to lower potential
- if we want a complete circuit with resistance we will need a device in which current can flow from lower to higher potential **against** the direction of the electric field



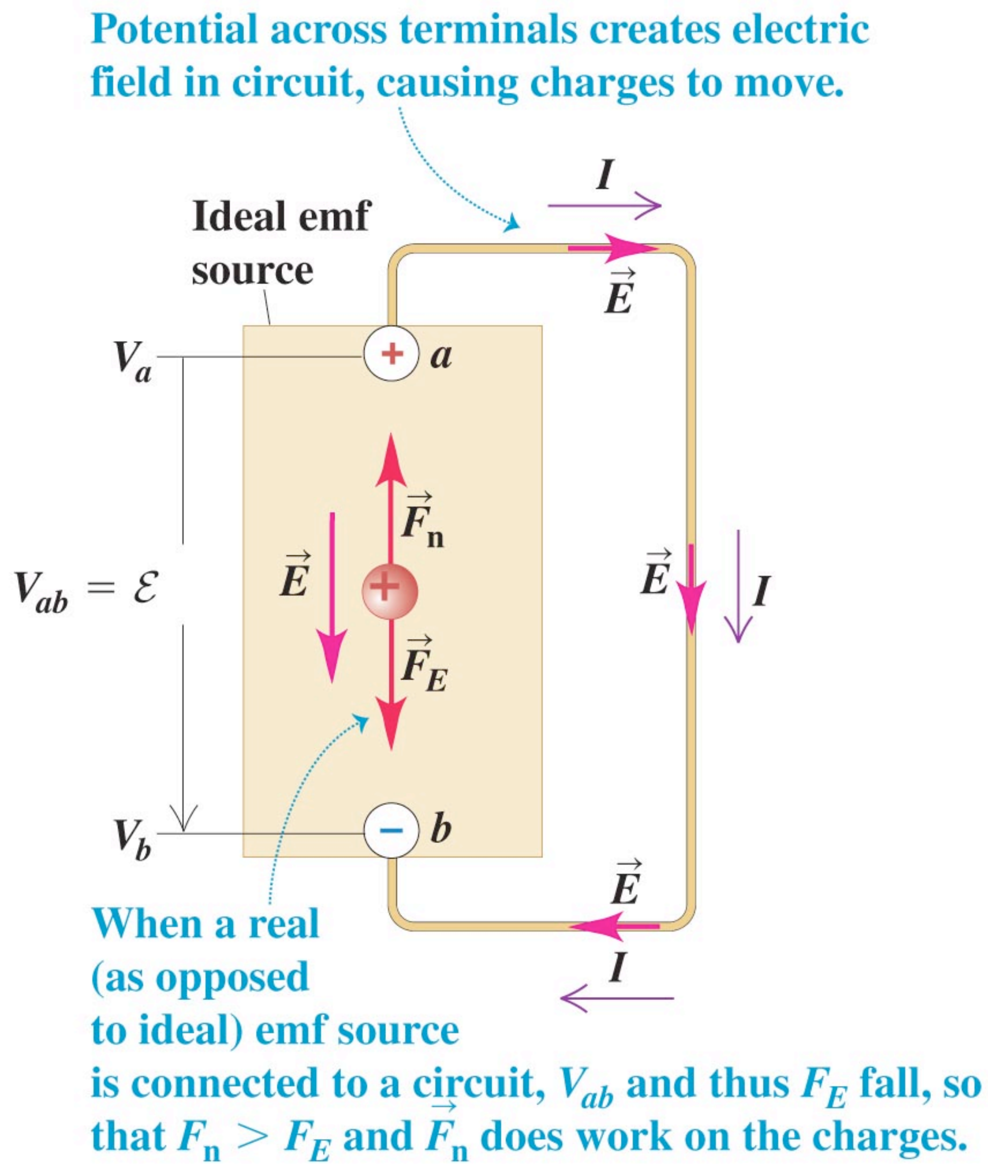


When the emf source is not part of a closed circuit, $F_n = F_E$ and there is no net motion of charge between the terminals.

- examples of these *emf* sources include **batteries**, electric generators, solar cells, thermocouples ...
- **ideal emf sources maintain a constant potential difference between the terminals**

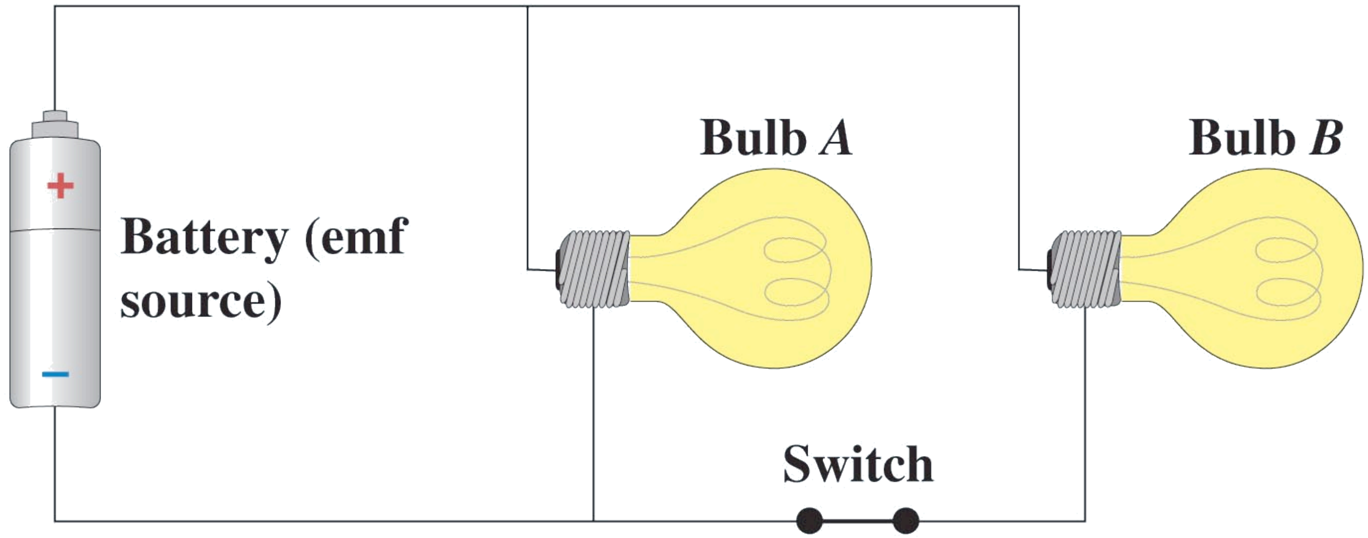
electromotive 'force' in a circuit

- now complete a circuit using a resistor



emfs supply constant voltage NOT constant current

- consider the circuit shown



- when the switch is opened what happens to bulb A
  - A. it gets brighter - more current through it
  - B. it gets dimmer - less current drawn from the battery
  - C. same brightness - bulb A receives the same current

real emf sources

- **real** sources of emf have an internal resistance experienced by the current as it flows through the source. we denote this by a lower case r
- hence the potential difference between the terminals is reduced and depends upon the current flowing

$$V = \mathcal{E} - Ir$$

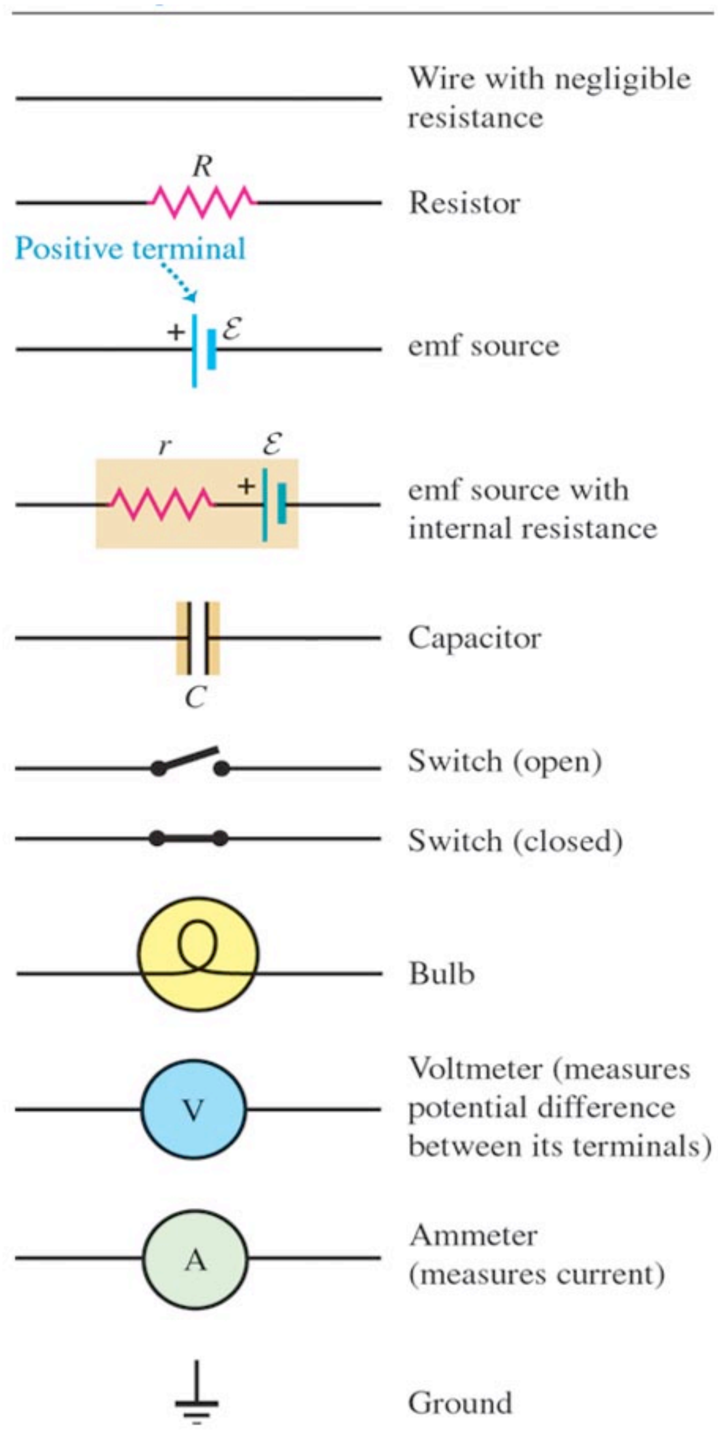
- hence connected in a circuit with a resistor we have

$$V = IR \qquad I = \frac{\mathcal{E}}{r + R}$$

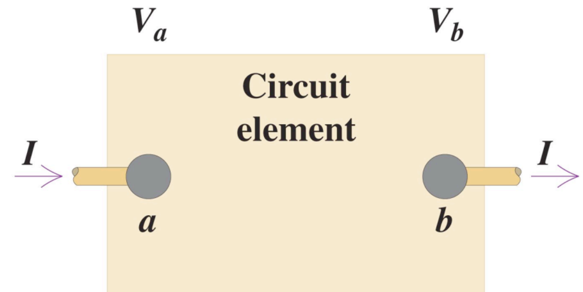
a dim flashlight

- as a flashlight battery ages its emf stays approximately constant, but its internal resistance increases. a fresh battery has an emf of 2.5V and negligible internal resistance. When the battery needs replacing its emf is still 2.5V but its internal resistance has increased to 1000 Ω. If this old battery is supplying 0.5 mA, what is its terminal voltage?

circuit symbols



energy and power in electrical circuits



- consider a circuit element, which might be a resistor, capacitor, battery or anything else
- when a charge q traverses the potential difference V_{ab} work qV_{ab} is done
- for a current $I = \dfrac{\Delta Q}{\Delta t}$ a charge ΔQ moves in a time Δt
- the work done on this charge is

$$\Delta W = \Delta Q\, V_{ab} = I V_{ab}\, \Delta t$$

- work done per unit time is called **power**

$$P = \frac{\Delta W}{\Delta t} = I V_{ab}$$

power usage of a resistor

- consider a resistor obeying Ohm's law which has a potential difference V_{ab} across it and a current I flowing through it

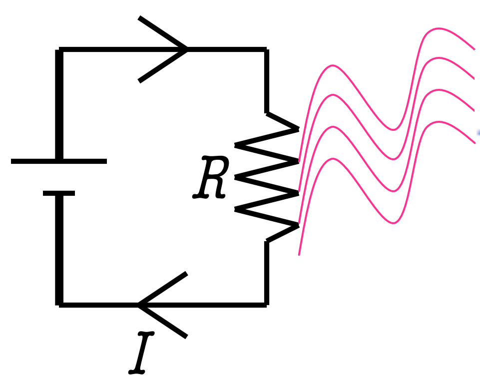
$$P = \frac{\Delta W}{\Delta t} = I V_{ab} \qquad V_{ab} = IR$$

$$P = I^2 R$$

- this power lost by the circuit is converted into heat in the resistor, ensuring energy is conserved

battery is supplying energy

$$P = \mathcal{E} I$$



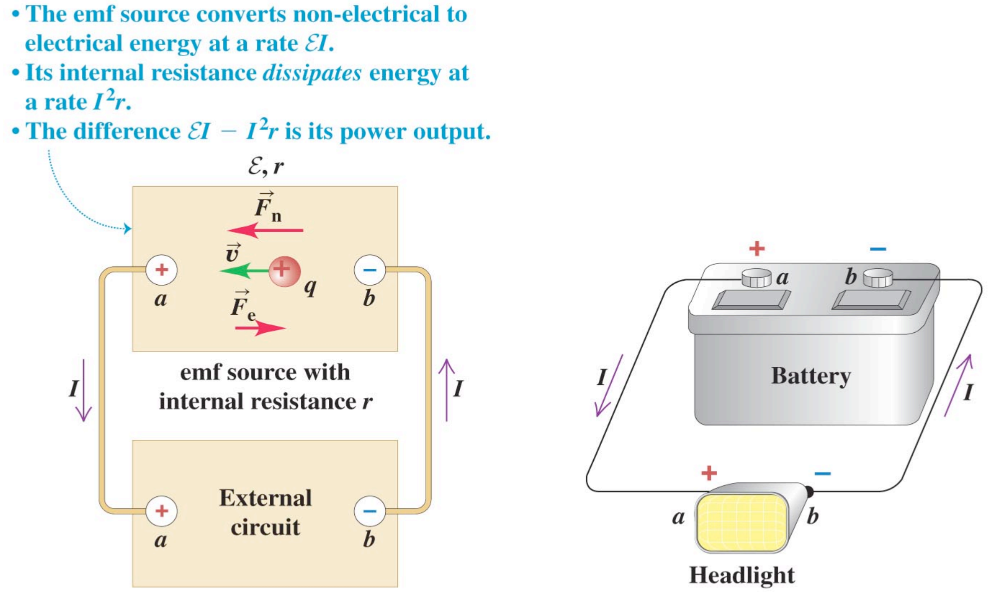
$$P = I^2 R$$

resistor loses energy as heat

power output of a **real** emf source

$$P = \frac{\Delta W}{\Delta t} = IV_{ab}$$

$$V = \mathcal{E} - Ir$$



(a) Diagrammatic circuit

(b) A real circuit of the type shown in (a)

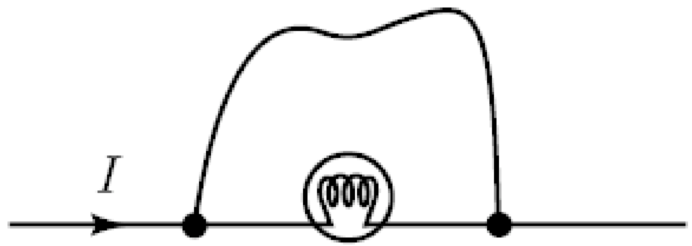
$$P = \mathcal{E}I - I^2r$$

rate of work *on* circulating charges

rate of dissipation of electrical energy in the source

electrical power delivered to the circuit (to be lost by resistors etc...)

Charge flows through a light bulb. Suppose a wire is connected across the bulb as shown. When the wire is connected,

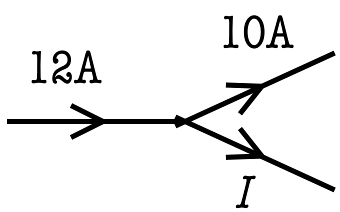


1. all the charge continues to flow through the bulb.
2. half the charge flows through the wire, the other half continues through the bulb.
3. all the charge flows through the wire.
4. none of the above

reading quiz

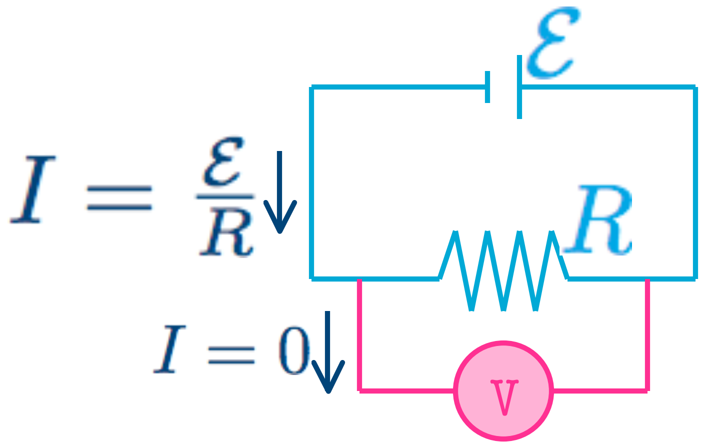
- Adding more resistors to a circuit can
  - A. *increase the resistance*
  - B. *decrease the resistance*
  - C. *do either of the above depending upon the arrangement*

- What is the unknown current in the circuit junction shown?
  - A. *2 A*
  - B. *- 2 A*
  - C. *22 A*

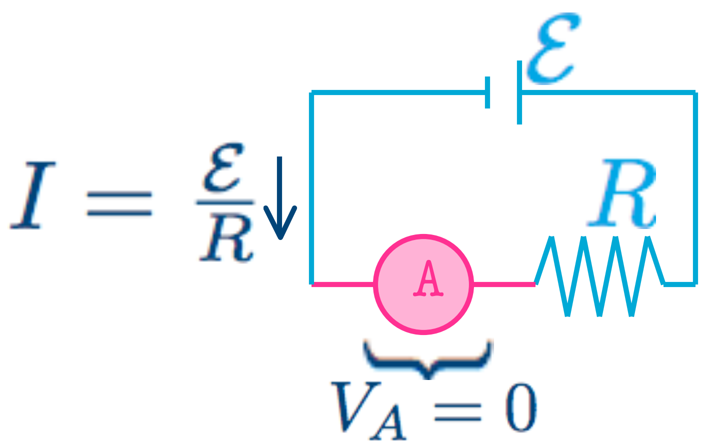


ideal voltmeters and ammeters

- it's handy to define idealised measurement tools
  - ideal voltmeter draws no current (infinite resistance)

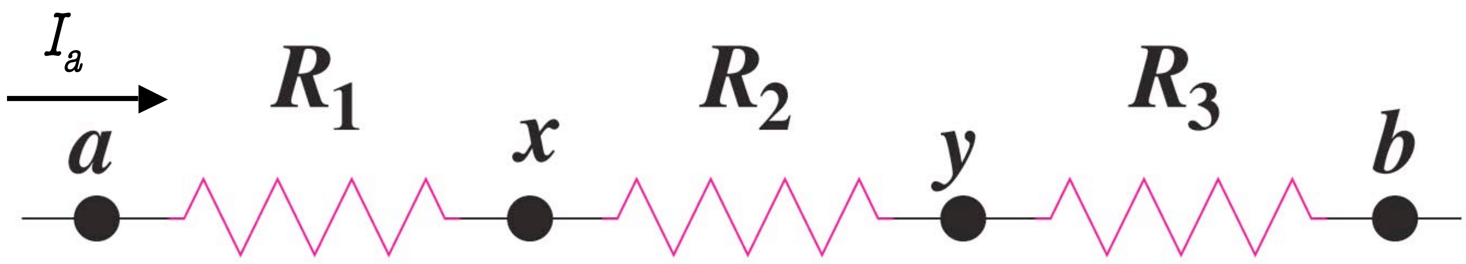


  - ideal ammeter causes no change in potential (zero resistance)



Resistors in series

- suppose we chain together a number of resistors



- What is the relation between the current at *a, x, y, b* ?

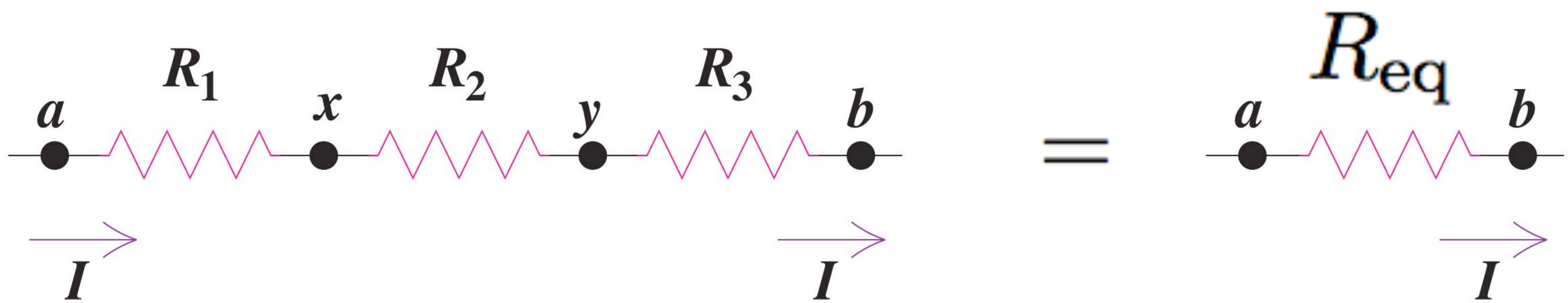
A. $I_a > I_x > I_y > I_b$
B. $I_a < I_x < I_y < I_b$
C. $I_a = I_x = I_y = I_b$

- What is the relation between the potential at *a, x, y, b* ?

A. $V_a > V_x > V_y > V_b$
B. $V_a < V_x < V_y < V_b$
C. $V_a = V_x = V_y = V_b$

Resistors in series

- suppose we chain together a number of resistors - this behaves like an 'equivalent resistance'



- current must be the same through each resistor

$$V_{ax} = IR_1$$

$$V_{xy} = IR_2$$

$$V_{yb} = IR_3$$

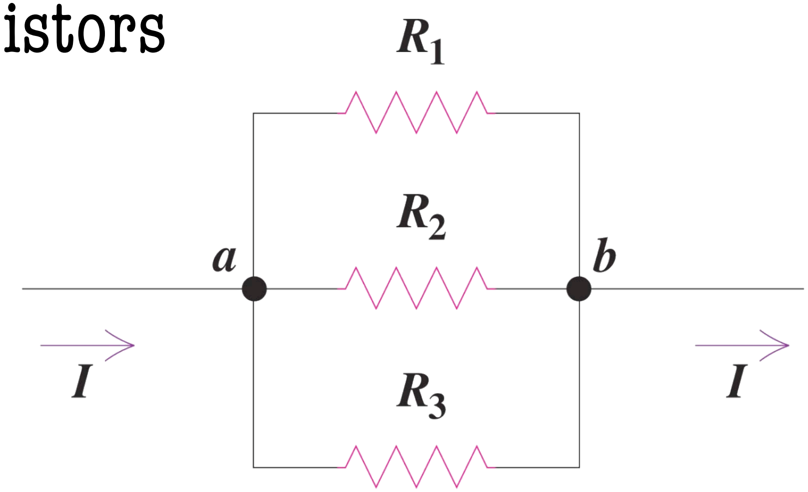
$$V_{ab} = V_{ax} + V_{xy} + V_{yb}$$

$$IR_{\mathrm{eq}} = IR_1 + IR_2 + IR_3$$

$$\boxed{R_{\mathrm{eq}} = R_1 + R_2 + R_3}$$

Resistors in parallel

- suppose we split a current between a number of resistors



how are the potential differences across the resistors related when $R_1 > R_2 > R_3$?

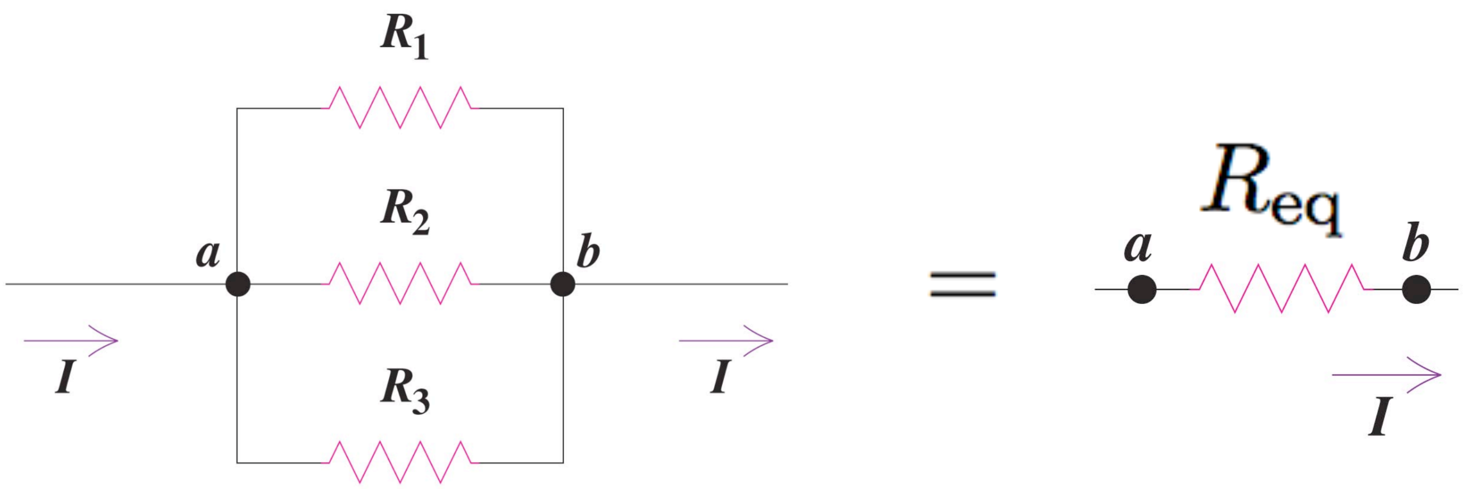
A. $V_1 = V_2 = V_3$

B. $V_1 < V_2 < V_3$

C. $V_1 > V_2 > V_3$

Resistors in parallel

- suppose we split a current between a number of resistors - this behaves like an ‘equivalent resistance’



potential across each resistor is the same

$$V_{ab} = I_1 R_1$$

$$V_{ab} = I_2 R_2$$

$$V_{ab} = I_3 R_3$$

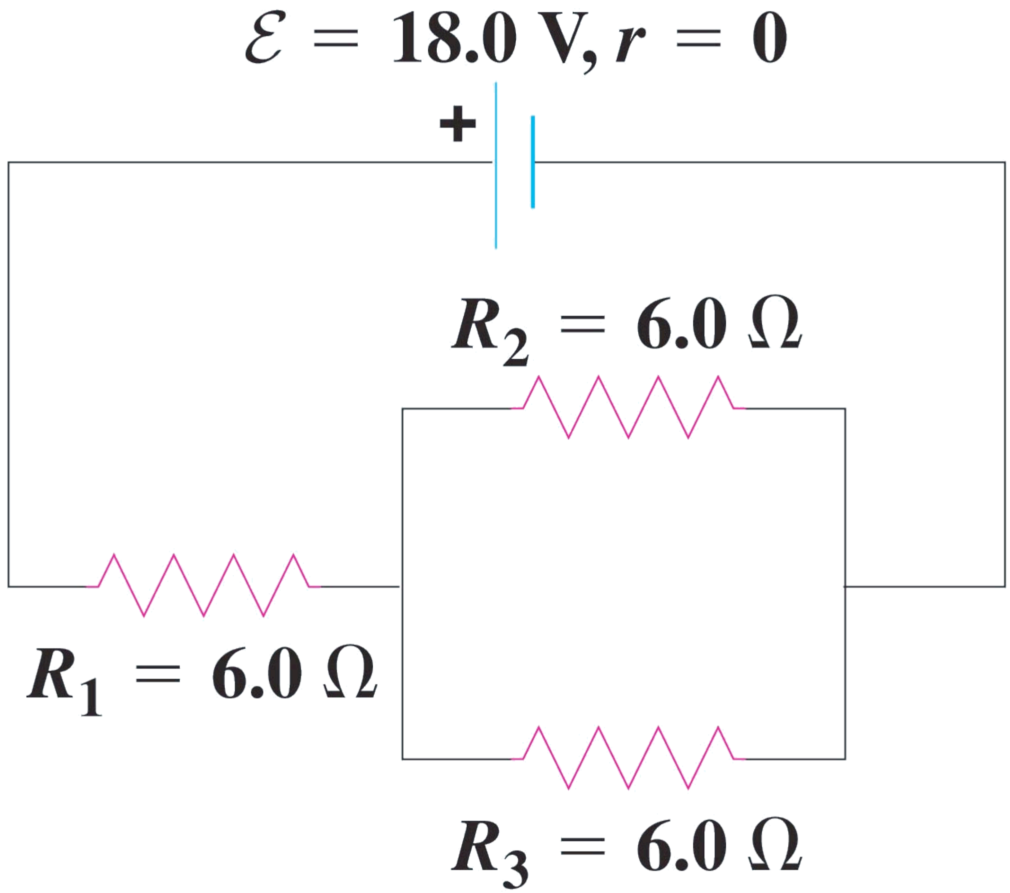
$$I = I_1 + I_2 + I_3$$

$$\frac{V_{ab}}{R_{eq}} = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

example 19.9

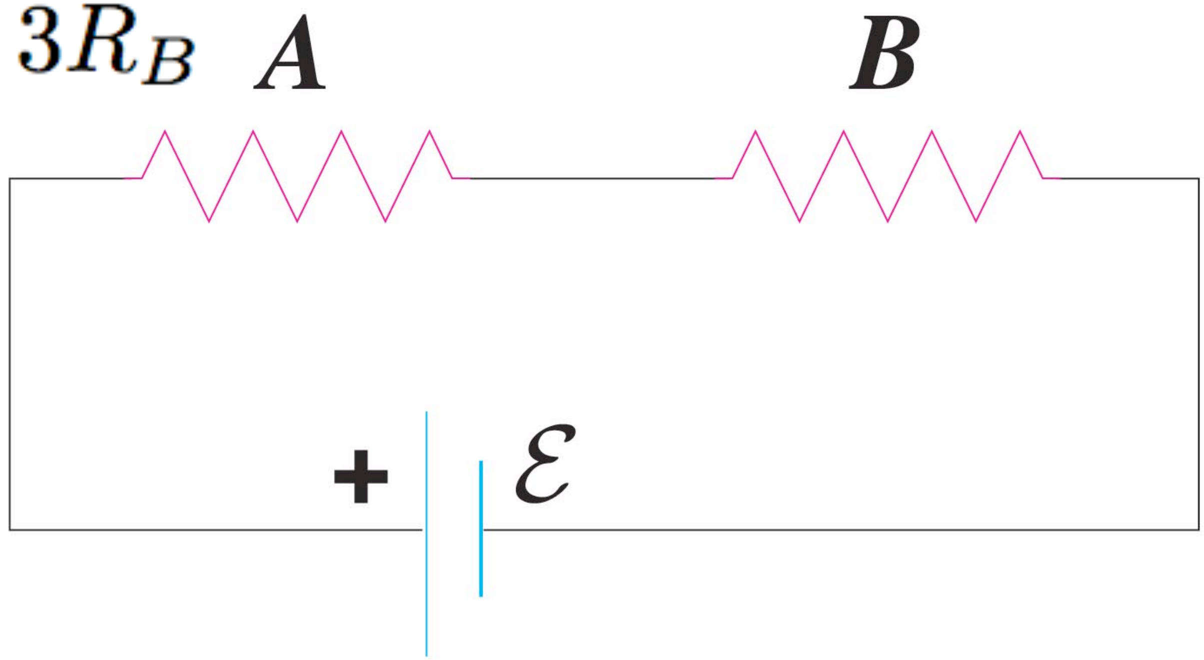
- find the equivalent resistance and the current in each resistor



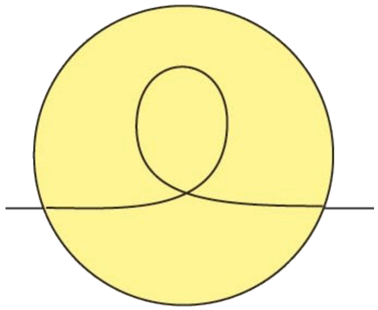
resistors in series

- which statement is true?
  - *A. current through A is three times the current though B*
  - *B. current through B is three times the current though A*
  - *C. voltage drop across A is three times the voltage drop across B*
  - *D. the potential difference is the same across A and B*

$$R_A = 3R_B$$



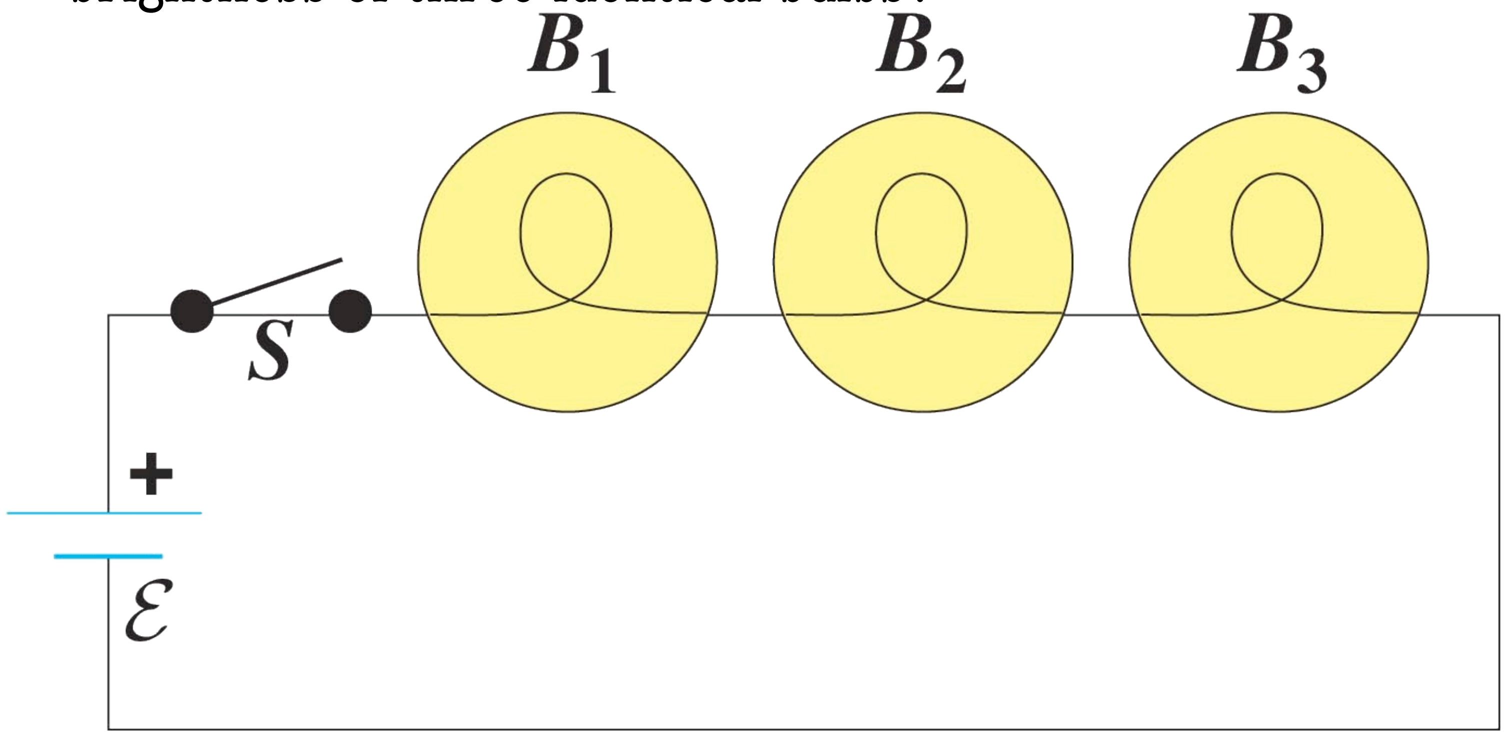
bulbs



- bulbs act like resistors
- their brightness is proportional to the power loss in them, $I^2R = V^2/R$
- so the larger the current, the brighter the bulb, (or the larger the potential drop the brighter the bulb)

three identical bulbs

- with the switch closed, what is the correct ordering of the brightness of three identical bulbs?



A. $B_1 > B_2 > B_3$

B. $B_1 < B_2 < B_3$

C. $B_1 = B_2 = B_3$

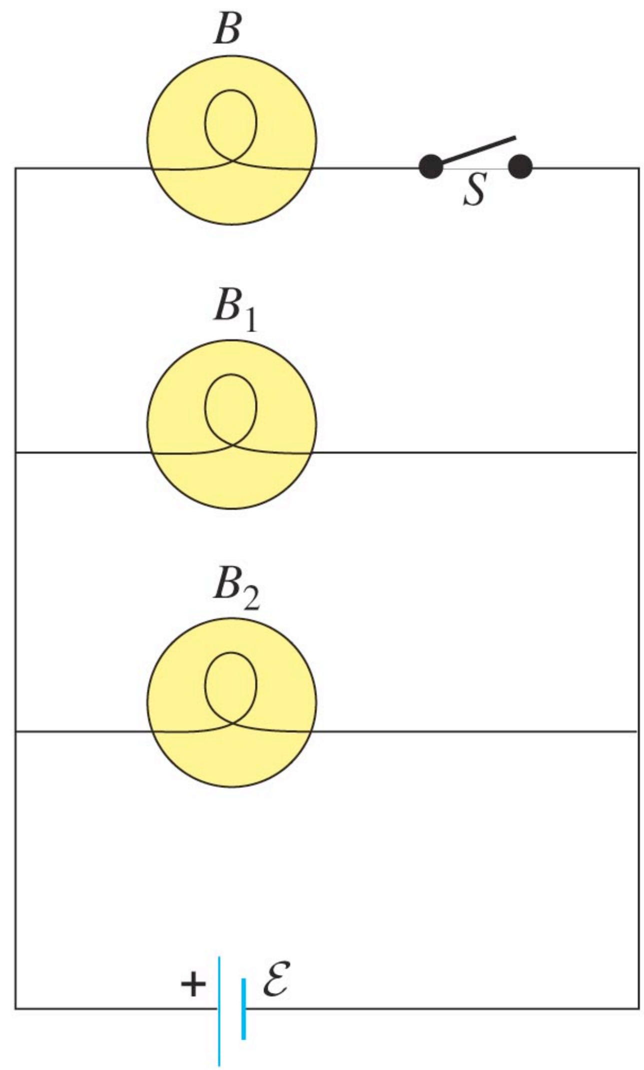
three identical bulbs again

- how does the brightness of bulbs B_1 and B_2 change when the switch is closed?

A. B_1 & B_2 get dimmer

B. B_1 & B_2 get brighter

C. B_1 & B_2 stay the same



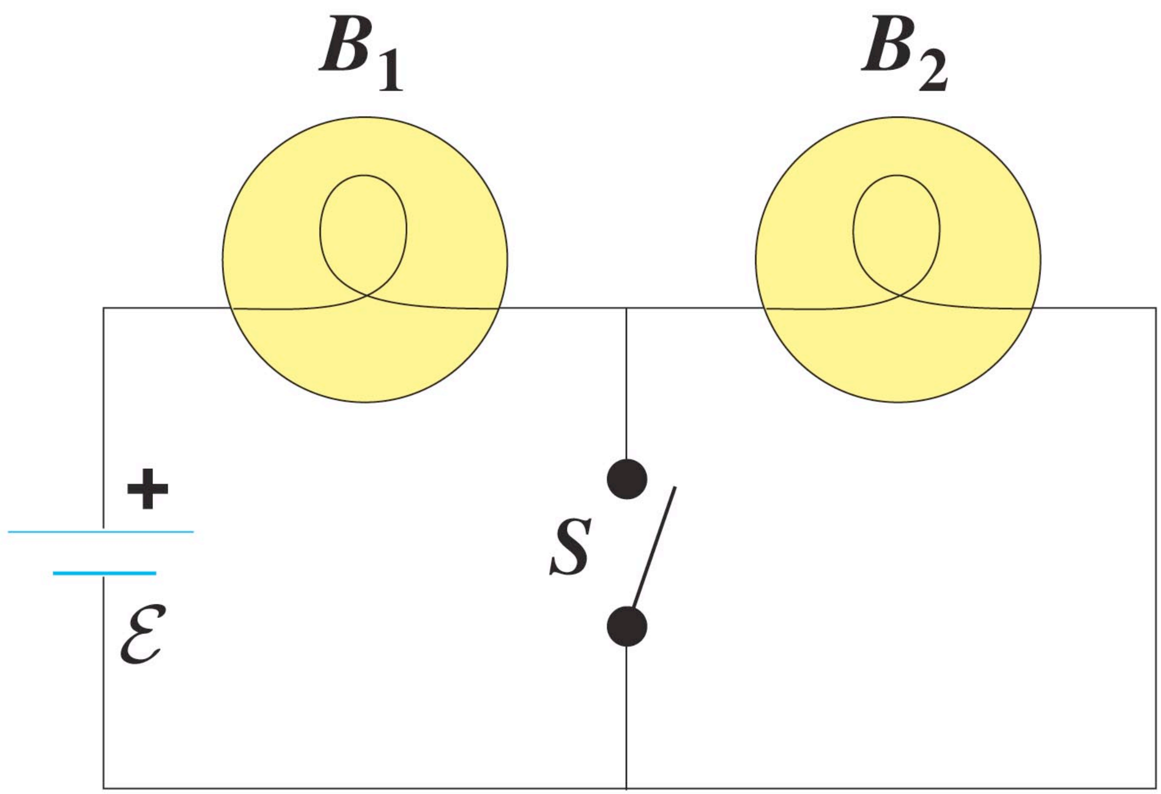
two identical bulbs

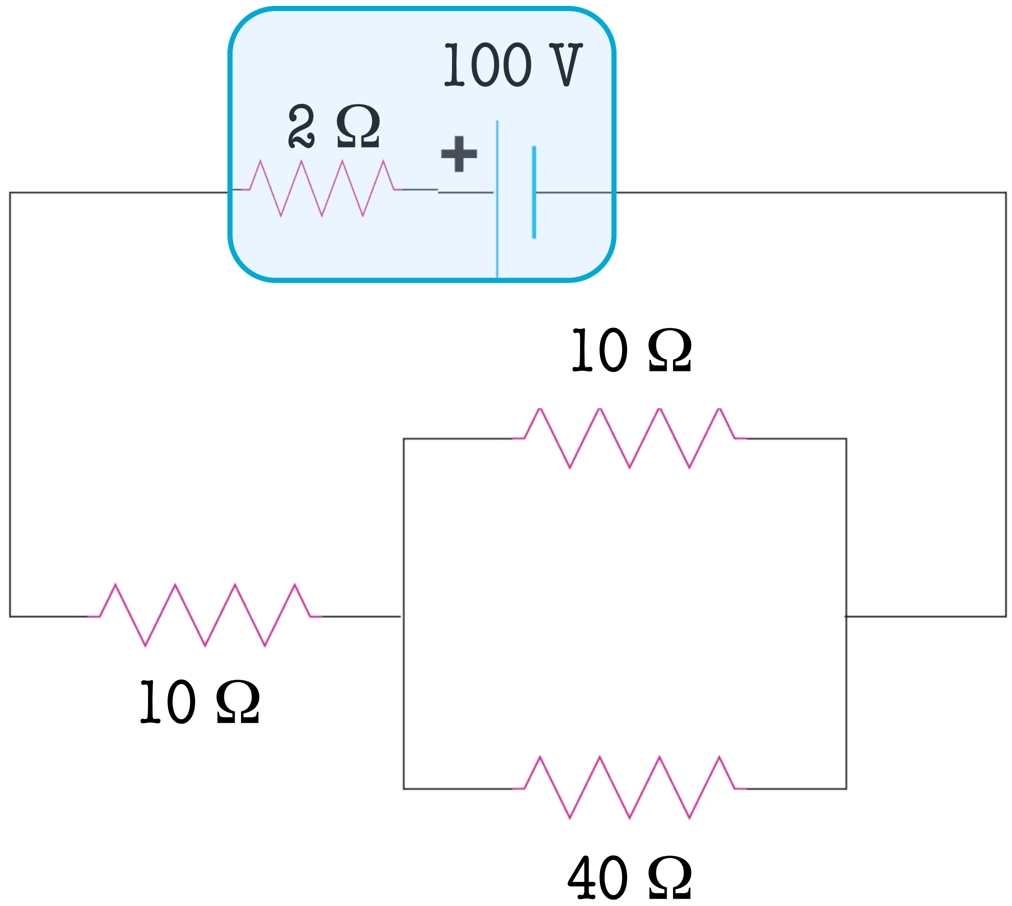
- how does the brightness of bulb B_1 change when the switch is closed?

A. B_1 gets dimmer

B. B_1 gets brighter

C. B_1 stays the same

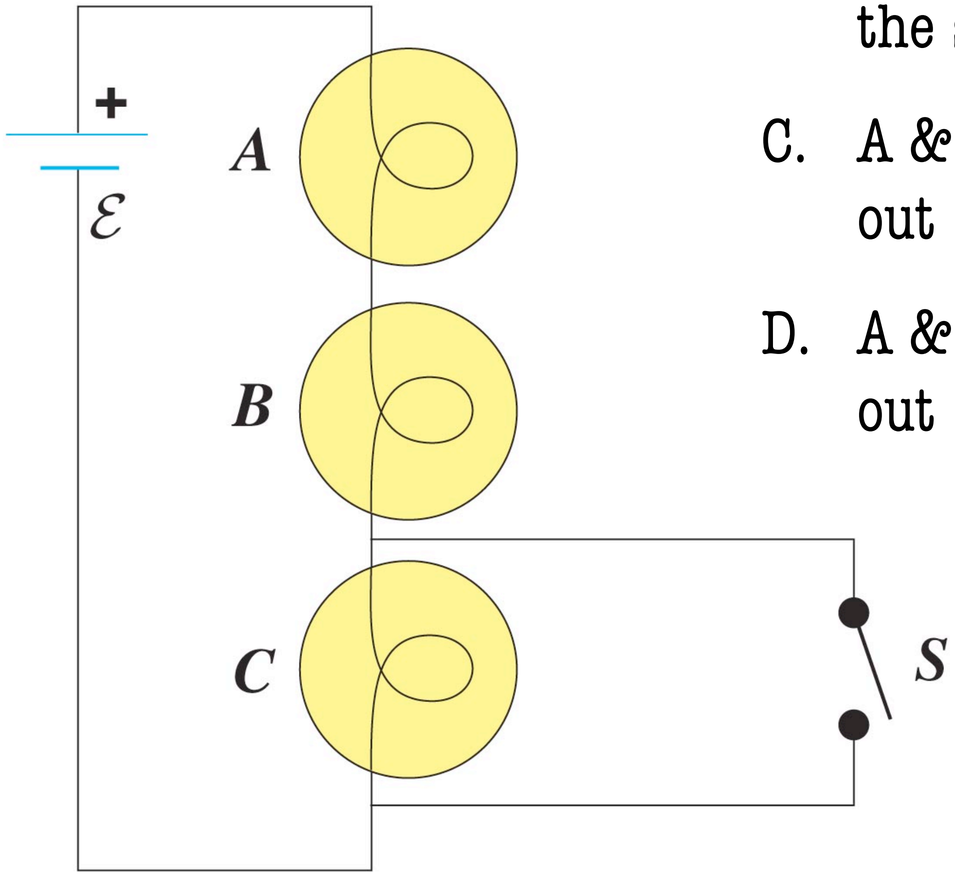




find the current in the 40 Ω resistor.

three identical bulbs again

- what happens to A, B and C when the switch is closed?



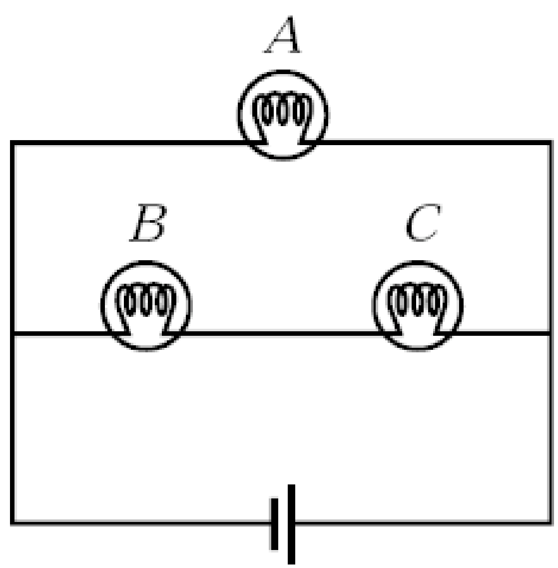
A. A & B get dimmer, C gets brighter

B. A & B get brighter, C stays the same

C. A & B get brighter, C goes out

D. A & B stay the same, C goes out

The three light bulbs in the circuit all have the same resistance. Given that brightness is proportional to power dissipated, **the brightness of bulbs *B* and *C* together**, compared with the brightness of bulb *A*, is



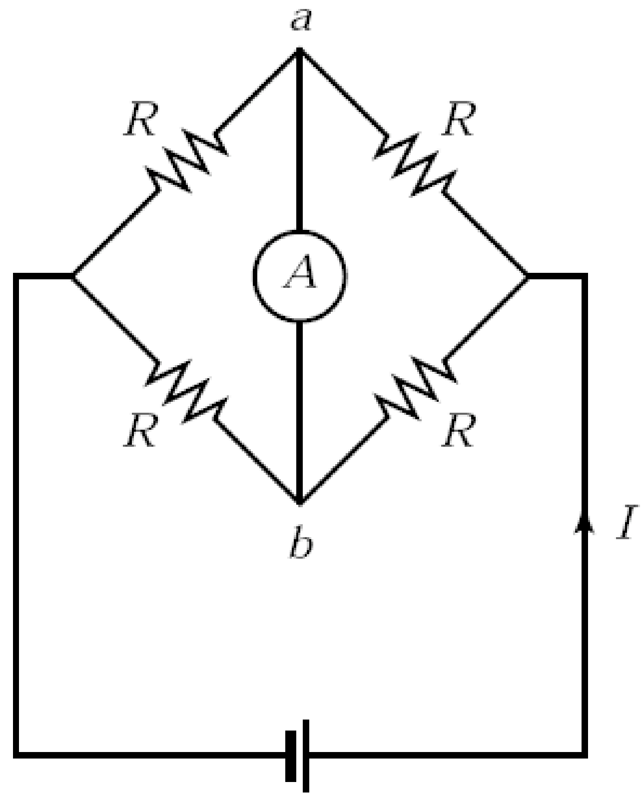
think carefully about this one

A. twice as much.
B. the same.
C. half as much.

$$P = V^2/R$$

SOMEWHAT HARDER:

An ammeter A is connected between points a and b in the circuit below, in which the four resistors are identical. The current through the ammeter is



A. I / 2.
B. I / 4.
C. zero.
D. need more information

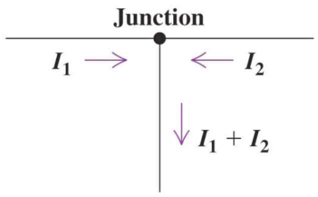
Kirchhoff's rules

- sometimes circuits can't simply by analyzed using parallel and series equivalent resistances
- Kirchhoff's rules codify the facts we already know about conservation of charge and energy in a way that makes solving circuits easy
  - junction rule: the sum of the currents into a junction is zero

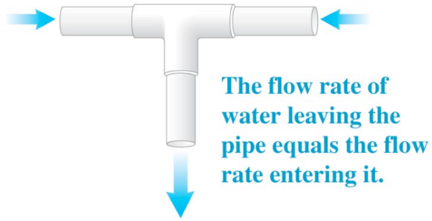
$$\sum I = 0$$

  - loop rule: the sum of the potential differences in a loop is zero

$$\sum V = 0$$



(a) Kirchhoff's junction rule

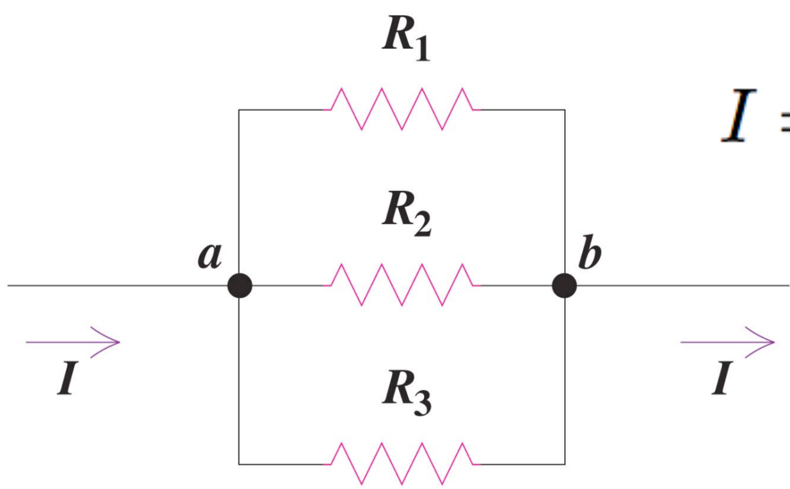


(b) Water-pipe analogy for Kirchhoff's junction rule

Kirchhoff's junction rule

the sum of the currents into a junction is zero $\sum I = 0$

already used this:



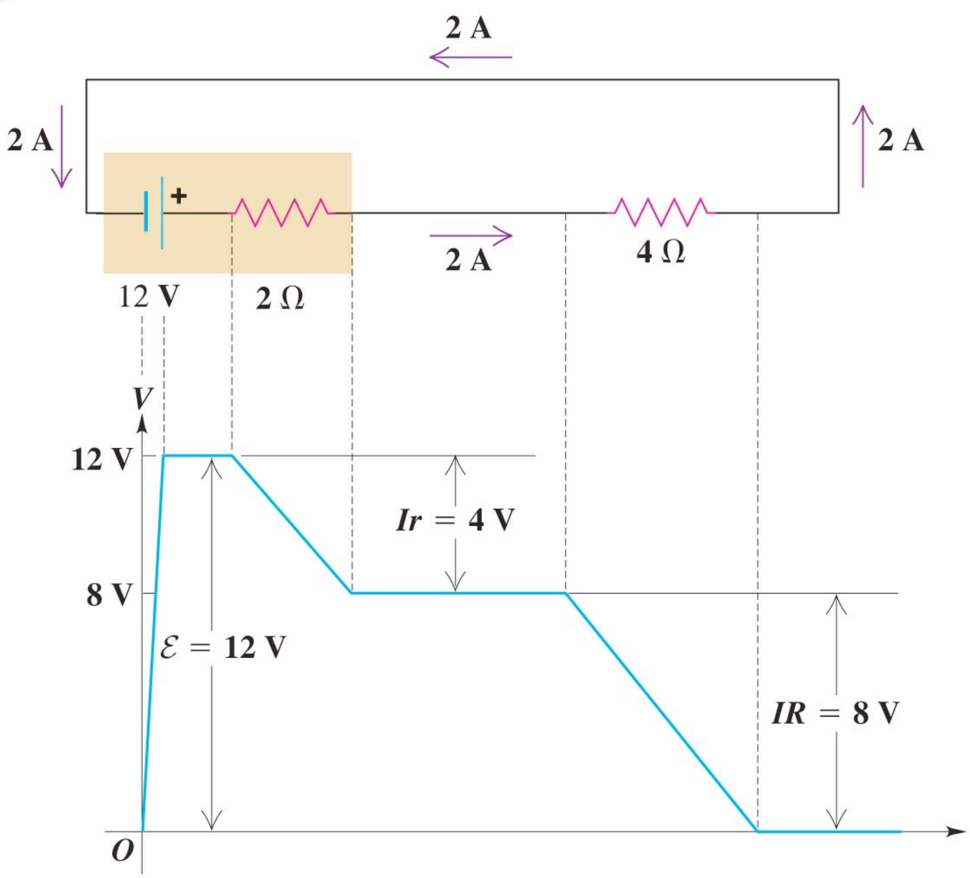
$$I = I_1 + I_2 + I_3$$

Kirchhoff's loop rule

the sum of the potential differences in a loop is zero

$$\sum V = 0$$

already seen this:



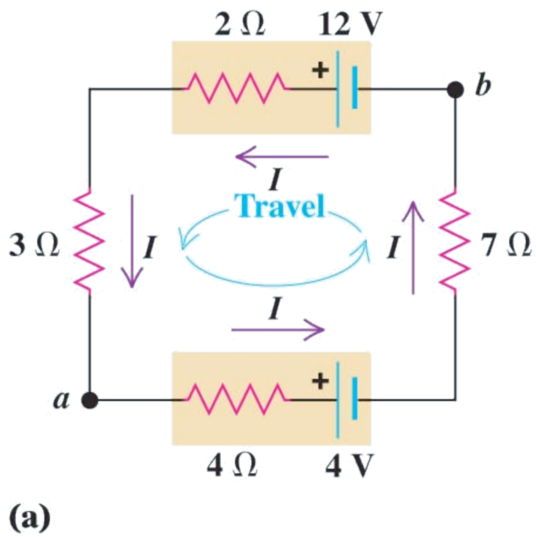
$$V_{\mathrm{emf}} = +12\,\mathrm{V}$$

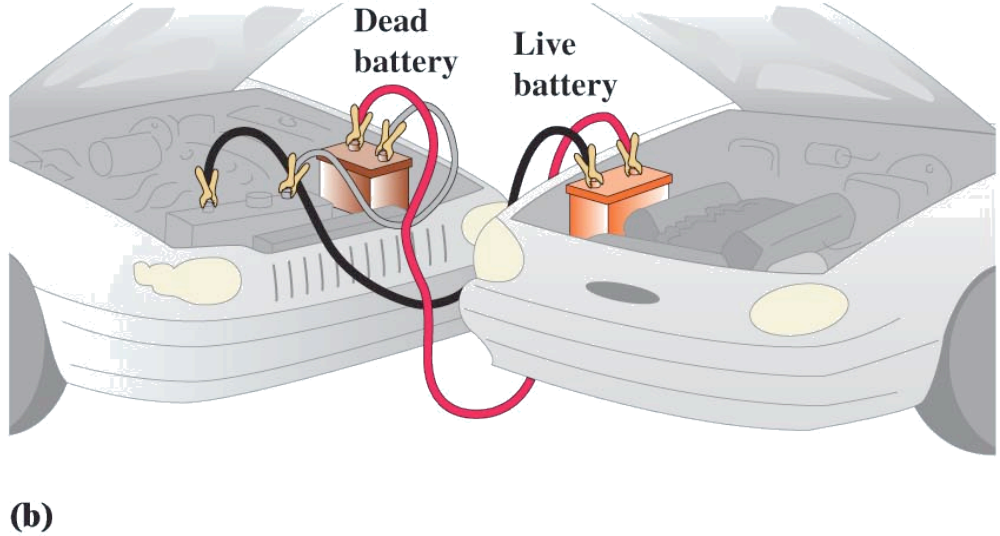
$$V_{\mathrm{int.r}} = -Ir = -4\,\mathrm{V}$$

$$V_R = -IR = -8\,\mathrm{V}$$

$$V_{\mathrm{emf}} + V_{\mathrm{int.r}} + V_R = +12 - 4 - 8\,\mathrm{V} = 0\,\mathrm{V}$$

example 19.10 - jump start your car

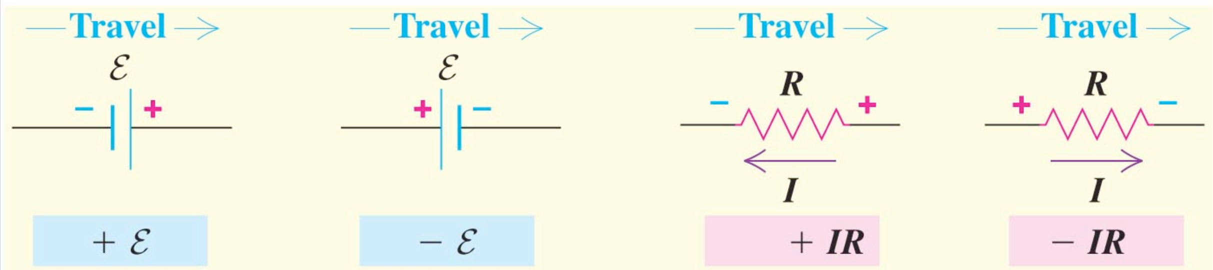




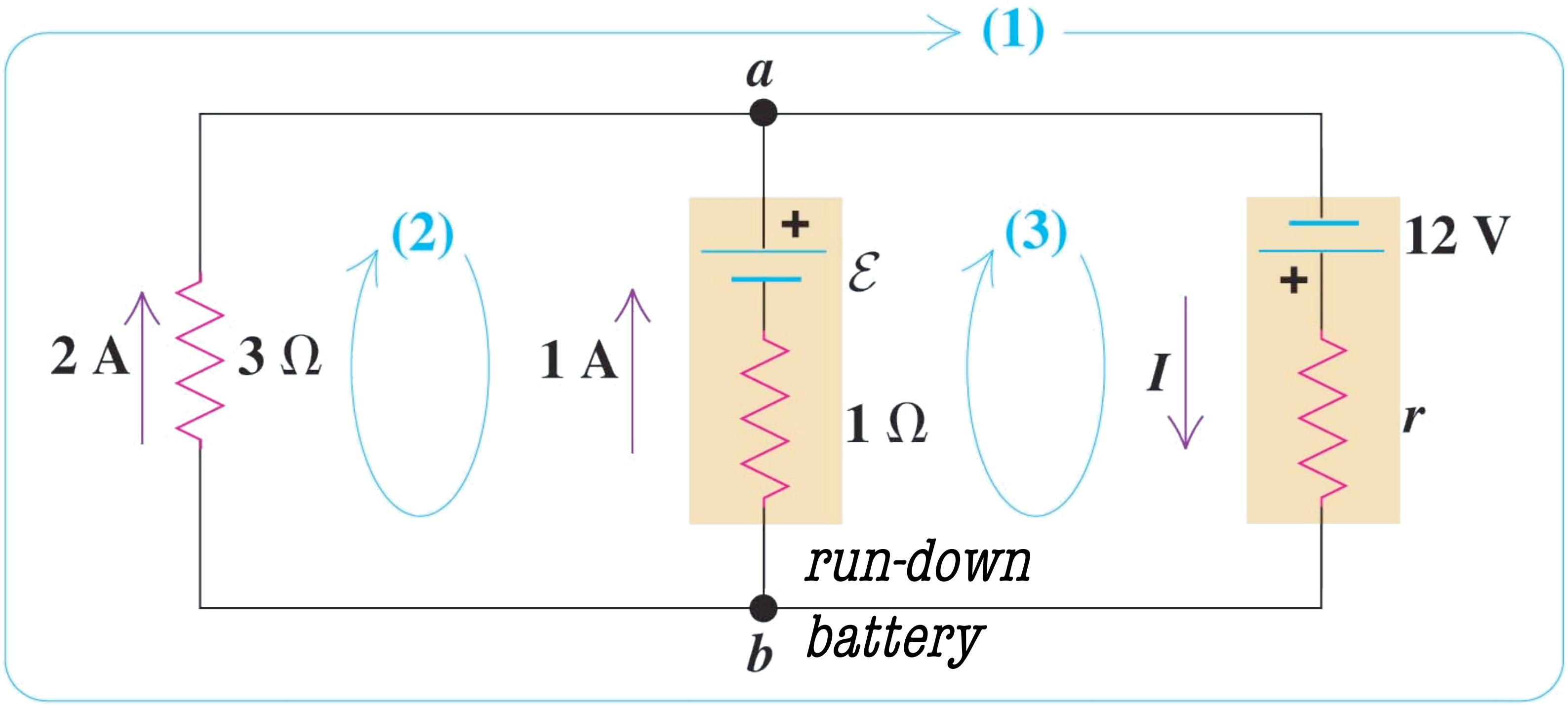
let's evaluate counterclockwise starting from *a:*

$$-I(4\,\Omega) - 4.0\,\text{V} - I(7\,\Omega) + 12\,\text{V} - I(2\,\Omega) - I(3\,\Omega) = 0$$

$$8\,\text{V} - I(16\,\Omega) = 0 \qquad I = \frac{8\,\text{V}}{16\,\Omega} = 0.5\,\text{A}$$



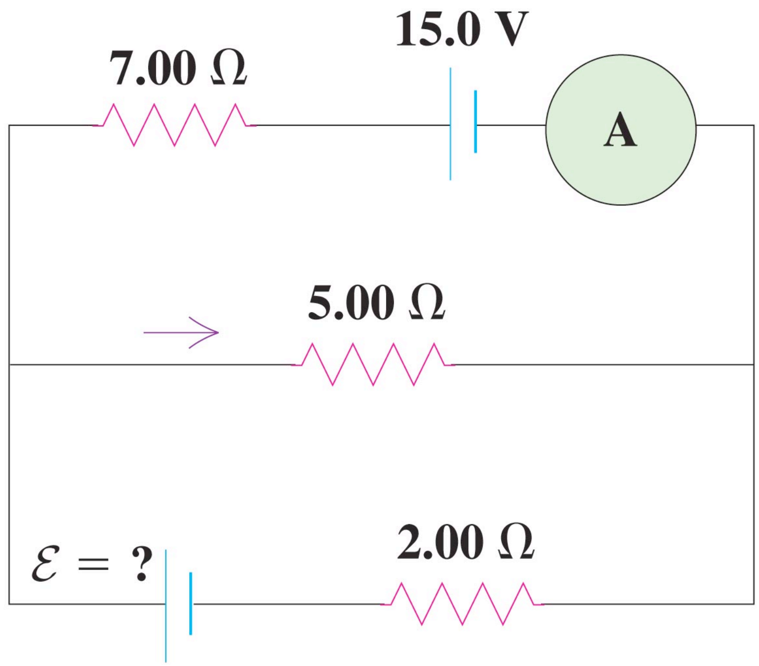
example 19.11 - recharging a battery



find $I, r, \mathcal{E}$ and the electrical power for each of the emf's

are the emf's supplying power to the circuit or draining power?

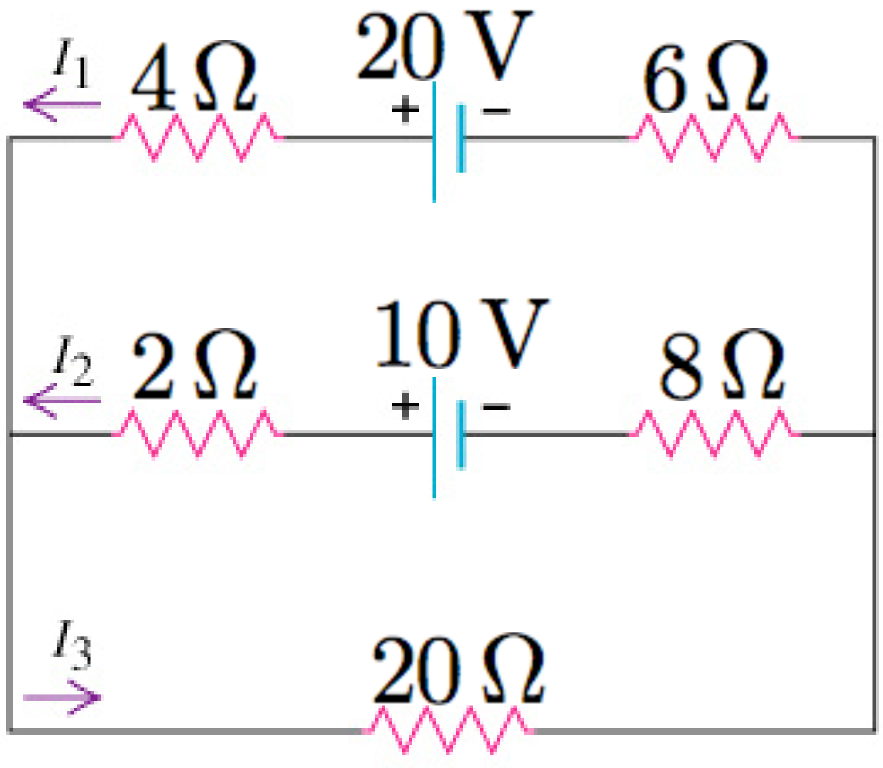
problem 61



5.00 Ohm resistor is consuming power at 20.0 W, what current does the ammeter read?

What is the emf of the lower source?

hardest problem you can get



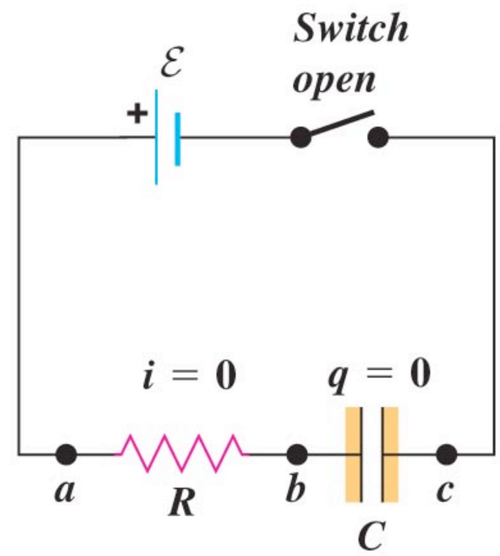
find the three currents

simultaneous equations!

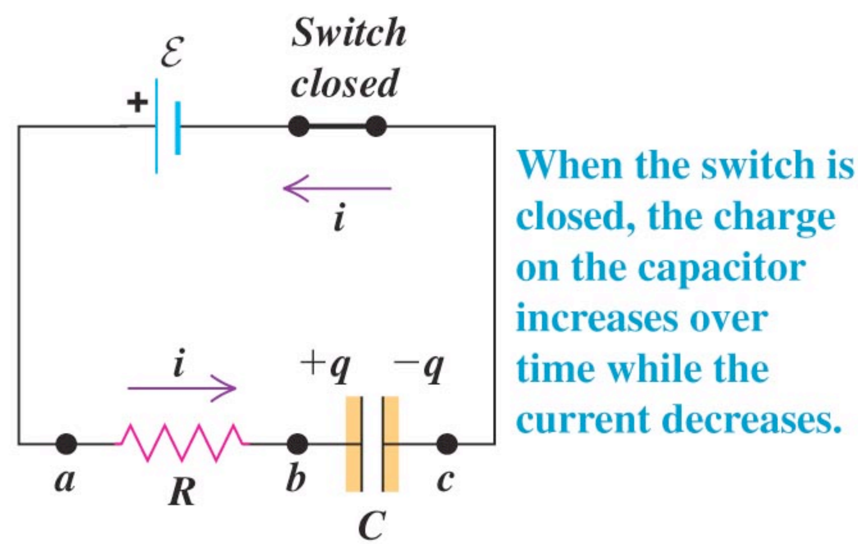
RC circuits

- using the components we've considered so far we can build circuits where the current is not constant with time

$$\mathcal{E} = iR + \frac{q}{C}$$



(a) Capacitor initially uncharged



(b) Charging the capacitor

RC circuits

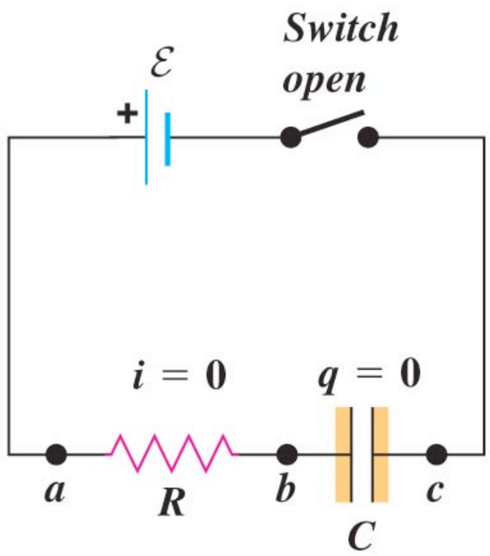
$$\mathcal{E} = iR + \frac{q}{C}$$

$$q(t=0) = 0$$

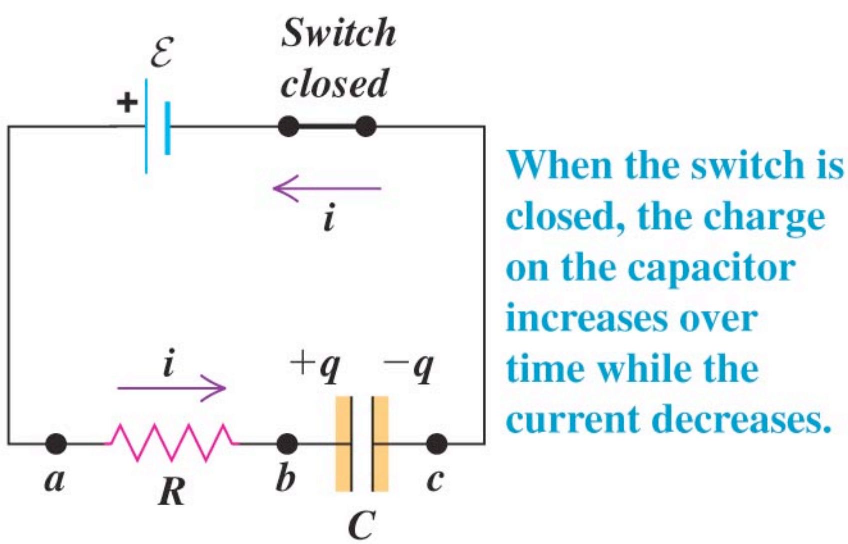
$$i(t=0) = \frac{\mathcal{E}}{R}$$

$$i(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$$

$$q(t) = Q_f (1 - e^{-t/RC})$$



(a) Capacitor initially uncharged



(b) Charging the capacitor

RC circuits

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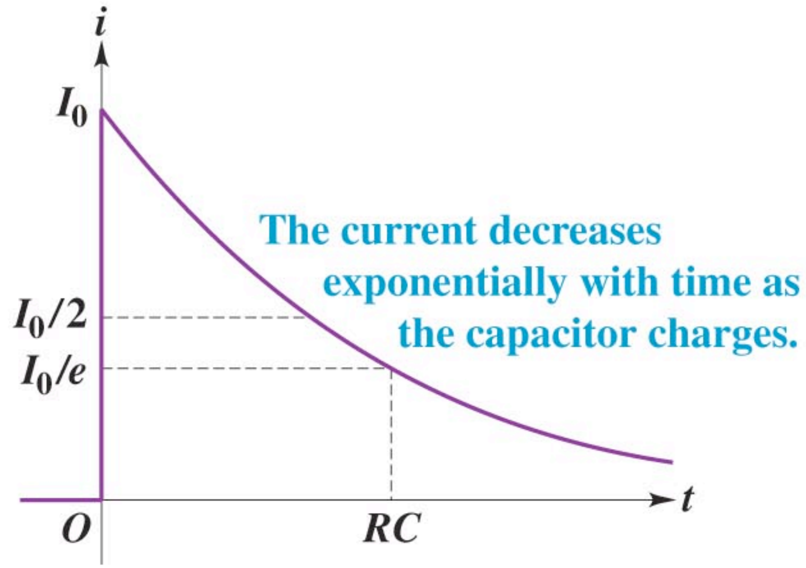
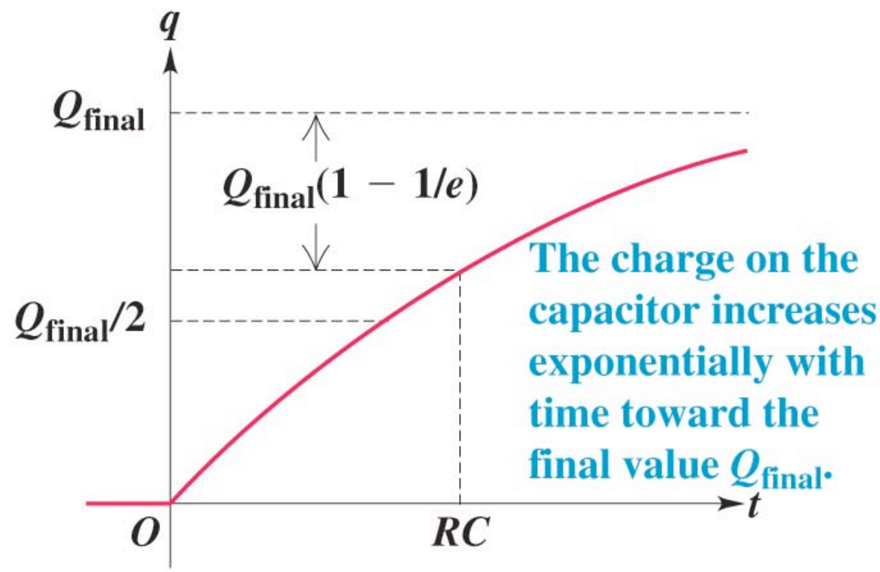


(a) Graph of current versus time

$$i(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$$

$$q(t) = Q_f(1 - e^{-t/RC})$$



(b) Graph of capacitor charge versus time

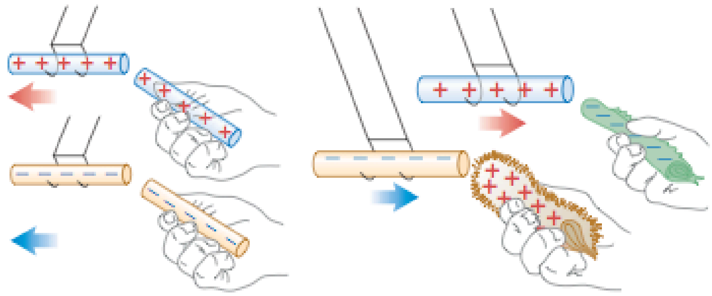
review - chapter 17

Electric Charge; Conductors and Insulators

(**Sections 17.1–17.3**) The fundamental entity in electrostatics is electric charge. There are two kinds of charge: positive and negative. Like charges repel each other; unlike charges attract. **Conductors** are materials that permit electric charge to move within them. **Insulators** permit charge to move much less readily. Most metals are good conductors; most nonmetals are insulators.

All ordinary matter is made of atoms consisting of protons, neutrons, and electrons. The protons and neutrons form the nucleus of the atom; the electrons surround the nucleus at distances much greater than its size. Electrical interactions are chiefly responsible for the structure of atoms, molecules, and solids.

Electric charge is conserved: It can be transferred between objects, but isolated charges cannot be created or destroyed. Electric charge is quantized: Every amount of observable charge is an integer multiple of the charge of an electron or proton.

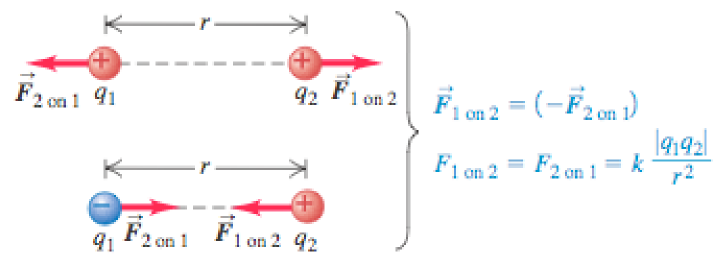


Coulomb's Law

(**Section 17.4**) **Coulomb's law** is the basic law of interaction for point electric charges. For point charges q_1 and q_2 separated by a distance r, the magnitude F of the force each charge exerts on the other is

$$F = k\frac{|q_1 q_2|}{r^2}. \quad (17.1)$$

The force on each charge acts along the line joining the two charges. It is repulsive if q_1 and q_2 have the same sign, attractive if they have opposite signs. The forces form an action–reaction pair and obey Newton's third law.

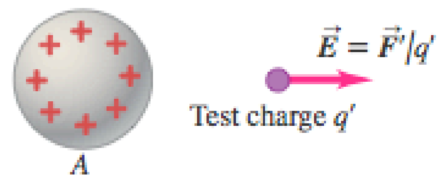


review - chapter 17

Electric Field and Electric Forces

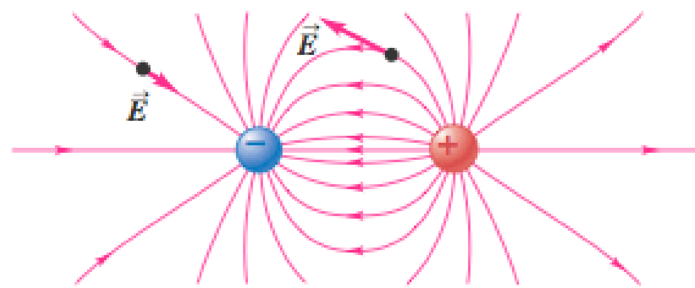
(Sections 17.5 and 17.6) Electric field, a vector quantity, is the force per unit charge exerted on a test charge at any point, provided that the test charge is small enough that it does not disturb the charges that cause the field. The principle of superposition states that the electric field due to any combination of charges is the vector sum of the fields caused by the individual charges. From Coulomb's law, the magnitude of the electric field produced by a point charge is

$$E = k\frac{|q|}{r^2}. \qquad (17.4)$$



Electric Field Lines

(Section 17.7) Field lines provide a graphical representation of electric fields. A field line at any point in space is tangent to the direction of $\vec{E}$ at that point, and the number of lines per unit area (perpendicular to their direction) is proportional to the magnitude of $\vec{E}$ at the point. Field lines point away from positive charges and toward negative charges.

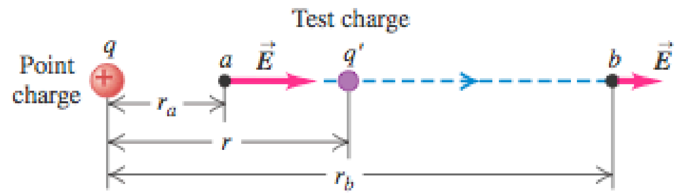


review - chapter 18

Electric Potential Energy

(Section 18.1) The work W done by the electric-field force on a charged particle moving in a field can be represented in terms of potential energy U: $W_{a \to b} = U_a - U_b$ (Equation 18.2). For a charge q' that undergoes a displacement $\vec{s}$ parallel to a uniform electric field, the change in potential energy is $U_a - U_b = q'Es$ (Equation 18.5). The potential energy for a point charge q' moving in the field produced by a point charge q at a distance r from q' is

$$U = k\frac{qq'}{r}. \qquad (18.8)$$



Potential

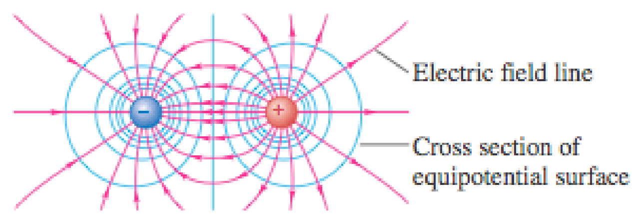
(Section 18.2) Potential, a scalar quantity denoted by V, is potential energy per unit charge. The potential at any point due to a point charge is

$$V = \frac{U}{q'} = k\frac{q}{r}. \qquad (18.12)$$

A positive test charge tends to "fall" from a high-potential region to a low-potential region.

Equipotential Surfaces

(Section 18.3) An **equipotential surface** is a surface on which the potential has the same value at every point. At a point where a field line crosses an equipotential surface, the two are perpendicular. When all charges are at rest, the surface of a conductor is always an equipotential surface, and all points in the interior of a conductor are at the same potential.



review - chapter 18

Capacitors

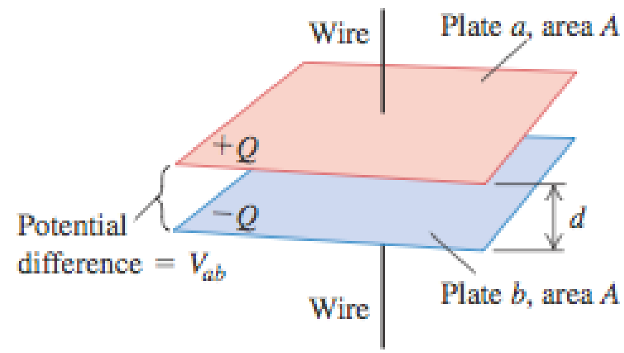
(Sections 18.5 and 18.6) A **capacitor** consists of any pair of conductors separated by vacuum or an insulating material. The **capacitance** C is defined as $C = Q/V_{ab}$ (Equation 18.14). A **parallel-plate capacitor** is made with two parallel plates, each with area A, separated by a distance d. If they are separated by vacuum, the capacitance is $C = \epsilon_0(A/d)$ (Equation 18.16).

When capacitors with capacitances C_1, C_2, C_3, . . . are connected in series, the equivalent capacitance C_{eq} is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots. \qquad (18.17)$$

When they are connected in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots. \qquad (18.18)$$

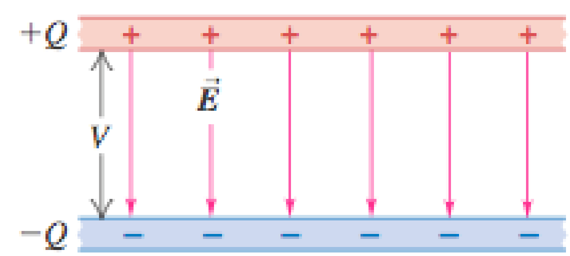


Electric Field Energy

(Section 18.7) The energy U required to charge a capacitor C to a potential difference V and a charge Q is equal to the energy stored in the capacitor and is given by

$$U = W_{total} = \left(\frac{V}{2}\right)Q = \frac{Q^2}{2C} = \frac{1}{2}CV^2. \qquad (18.19)$$

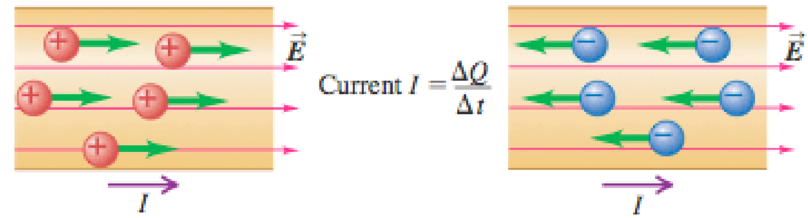
This energy can be thought of as residing in the electric field between the conductors; the energy density u (energy per unit volume) is $u = \frac{1}{2}\epsilon_0 E^2$ (Equation 18.20).



review - chapter 19

Current

(Section 19.1) Current is the amount of charge flowing through a conductor per unit time. The SI unit of current is the ampere, equal to 1 coulomb per second $(1\ \text{A} = 1\ \text{C}/\text{s})$. If a net charge ΔQ flows through a wire in time Δt, the current through the wire is $I = \Delta Q/\Delta t$ (Equation 19.1).

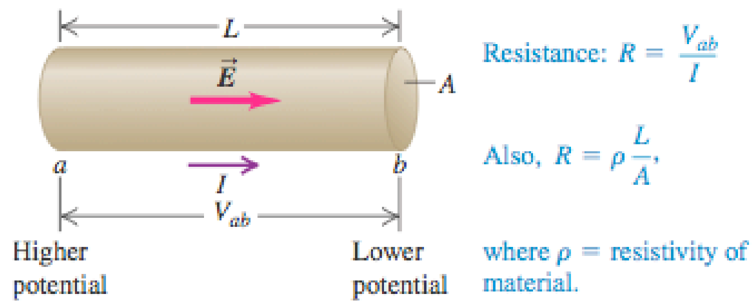


Resistance and Ohm's Law

(Section 19.2) In a conductor, the **resistance** R is the ratio of voltage to current: $R = V/I$ (Equation 19.2). The SI unit of resistance is the **ohm** (Ω), equal to 1 volt per ampere. In materials that obey **Ohm's law,** the potential difference V between the ends of a conductor is proportional to the current I through the conductor; the proportionality factor is the resistance R.

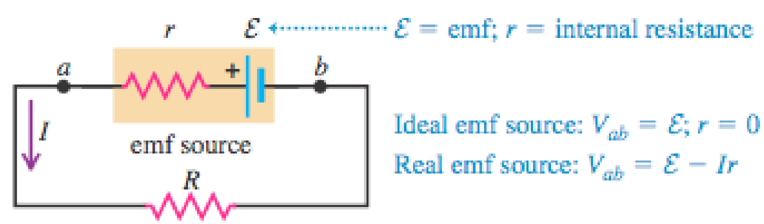
For a given conducting material, resistance R is proportional to length and inversely proportional to cross-sectional area. For a specific material, this relationship can be expressed as $R = \rho(L/A)$ (Equation 19.3), where ρ is the **resistivity** of that material.

Resistance and resistivity vary with temperature; for metals, they usually increase with increasing temperature.



Electromotive Force and Circuits

(Section 19.3) A **complete circuit** is a conductor in the form of a loop providing a continuous current-carrying path. A complete circuit carrying a steady current must contain a source of electromotive force (emf), symbolized by $\mathcal{E}$. An ideal source of emf maintains a constant potential difference $V_{ab} = \mathcal{E}$ (Equation 19.5), but every real source of emf has some internal resistance r. The terminal potential difference V_{ab} then depends on current: $V_{ab} = \mathcal{E} - Ir$ (Equation 19.7).



review - chapter 19

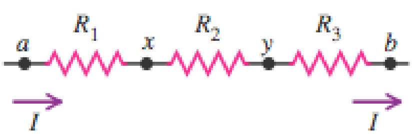
Energy and Power in Electric Circuits

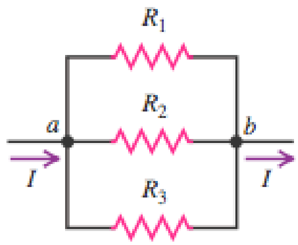
(Section 19.4) A circuit element with a potential difference V and a current I puts energy into a circuit if the current direction is from lower to higher potential in the device and takes energy out of the circuit if the current is opposite. The power P (rate of energy transfer) is $P = VI$ (Equation 19.9). A resistor R always takes energy out of a circuit, converting it to thermal energy at a rate given by $P = V_{ab}I = I^2R = V_{ab}^2/R$ (Equation 19.10).

Resistors in Series and in Parallel

(Section 19.5) When several resistors $R_1, R_2, R_3, \cdots$ are connected in series, the **equivalent resistance** R_{eq} is the sum of the individual resistances: $R_{eq} = R_1 + R_2 + R_3 + \cdots$. (Equation 19.12). When several resistors are connected in parallel, the equivalent resistance R_{eq} is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots. \qquad (19.13)$$





formula sheet

- both sides of one sheet of ‘Letter’
- any formulae and diagrams you like
- **no words**

- to be handed in with your test
- points deducted if you break the rules above

example solutions

A parallel-plate capacitor having rectangular plates of area *A* and separation *d* is charged up by connecting to a potential difference by a battery. The battery is disconnected, leaving the plates charged. If the plates are moved closer to each other,

1. the capacitance *A. Increases, B. Decreases, C. Stays the same*
2. the potential difference between the plates *A. Increases, B. Decreases, C. Stays the same*
3. the energy stored in the capacitor *A. Increases, B. Decreases, C. Stays the same*

for a parallel plate capacitor $C = \varepsilon_0 A/d$

d decreases $\Rightarrow$ C increases

1. A

$Q = CV \Rightarrow V = Q/C$, Q is fixed since the capacitor is not attached to anything (+Q, −Q)

C increases $\Rightarrow$ V decreases

2. B

$U = \frac{1}{2}CV^2 = Q^2/2C$, Q is fixed, C increases $\Rightarrow$ U decreases

3. B

"explanations" might get you partial credit even for wrong answers

example solutions

- what is the equivalent capacitance of this system if C_1 = 8.0 pF and C_2 = 24.0 pF ?
- what is the potential difference across each of the capacitors if the battery supplies 12.0 V ?