Chapter 22 - Alternating Current

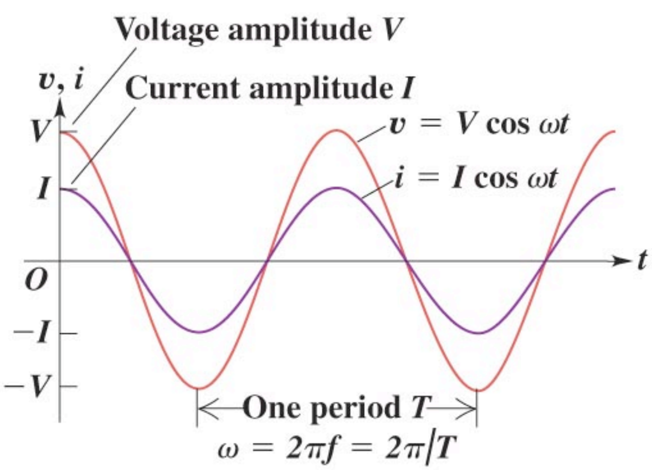
reading quiz

- which of the following is the correct relationship between peak voltage(V) and rms voltage($V_{\rm rms}$):
  - A. $V = V_{\rm rms}\sqrt{2}$
  - B. $V = V_{\rm rms}/\sqrt{2}$
  - C. $V = \sqrt{2V_{\rm rms}}$
  - D. $V = \sqrt{V_{\rm rms}/2}$

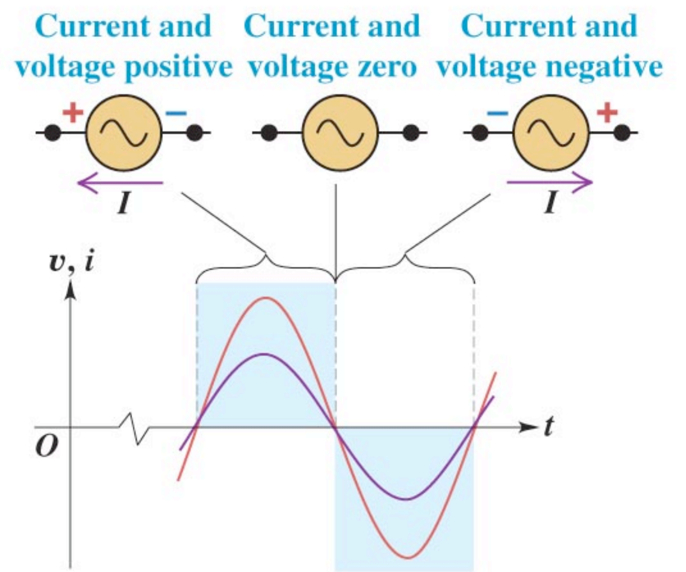
- The reactance of a capacitor is
  - A. proportional to the ac frequency
  - B. inversely proportional to the ac frequency
  - C. independent of the ac frequency
  - D. always zero

alternating current

- thus far we've mainly dealt with circuits under ***direct current*** conditions - that is where the supplied voltage or current does not change with time
- another common situation is where the voltage and current varies with time in a ***sinusoidal*** way



(a) Graphs of sinusoidal current and voltage versus time. The relative heights of the two curves are not significant.



(b) The graphs related to a schematic ac source.

$$v = V \cos \omega t$$

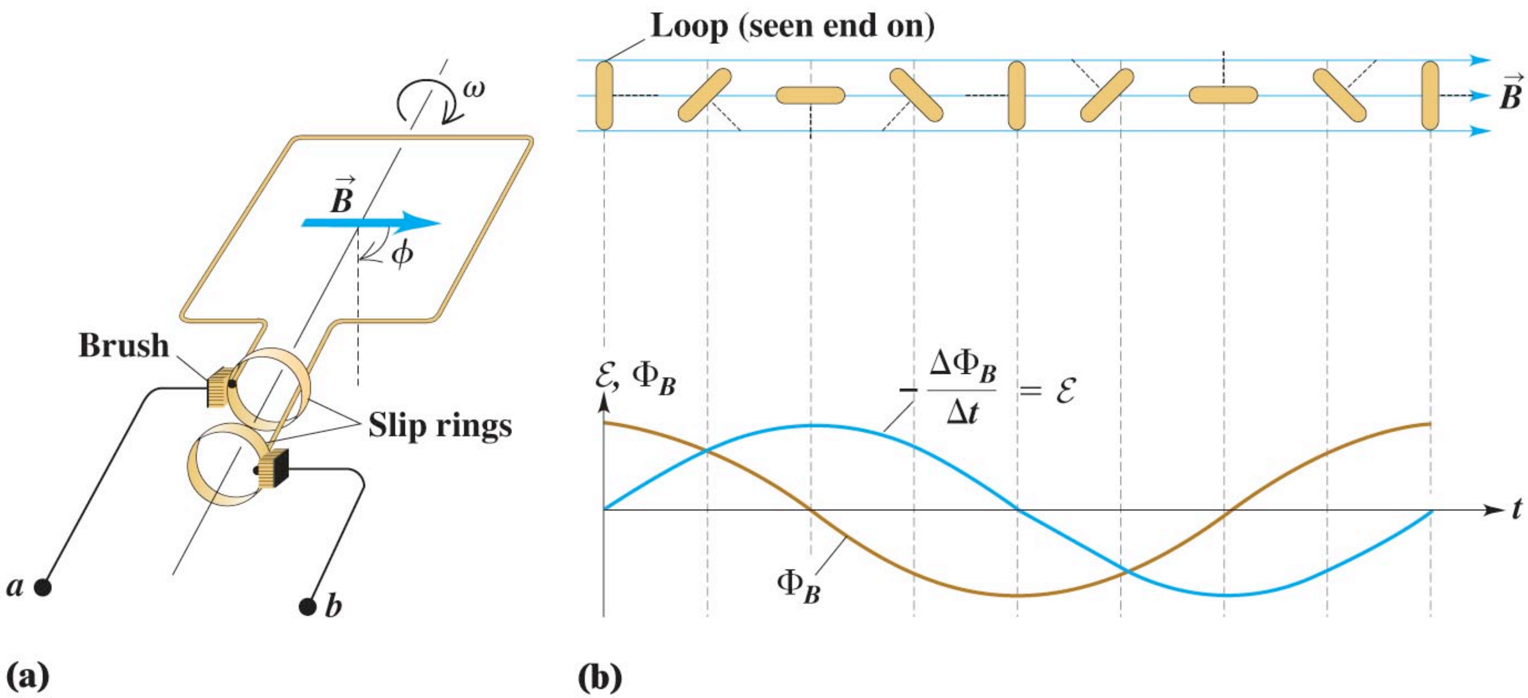
$$i = I \cos \omega t$$

V = peak voltage

I = peak current

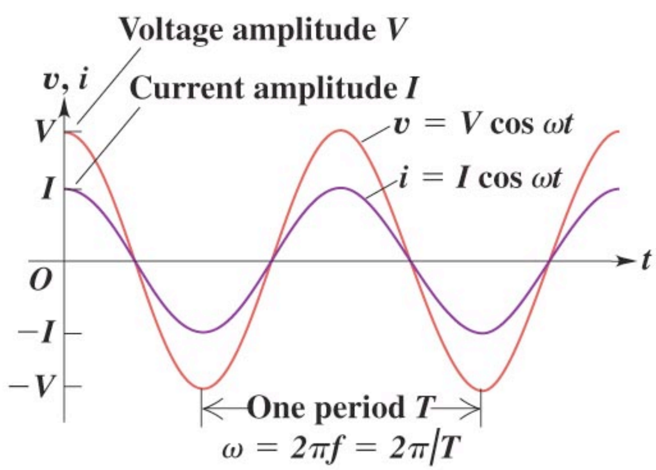
producing alternating current

- we saw one way last time



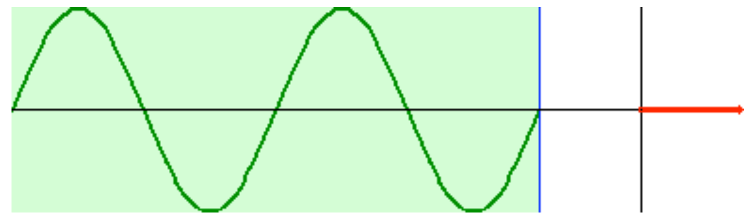
phasors

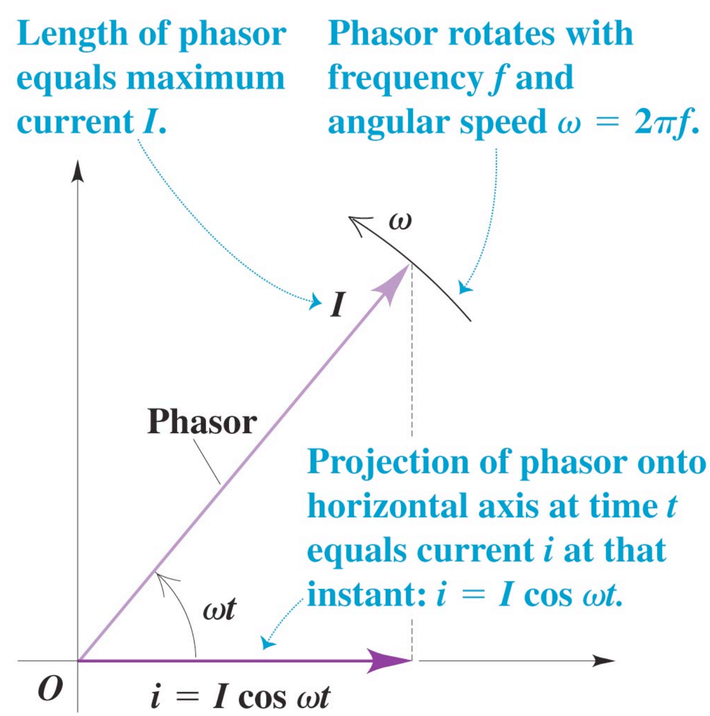
- one way to visualise sinusoidal behaviour (if you don't like graphs) is to use a '***phasor***'
- this is a vector that rotates at a fixed angular velocity



(a) Graphs of sinusoidal current and voltage versus time. The relative heights of the two curves are not significant.

$$v = V \cos \omega t$$
$$i = I \cos \omega t$$





alternating current

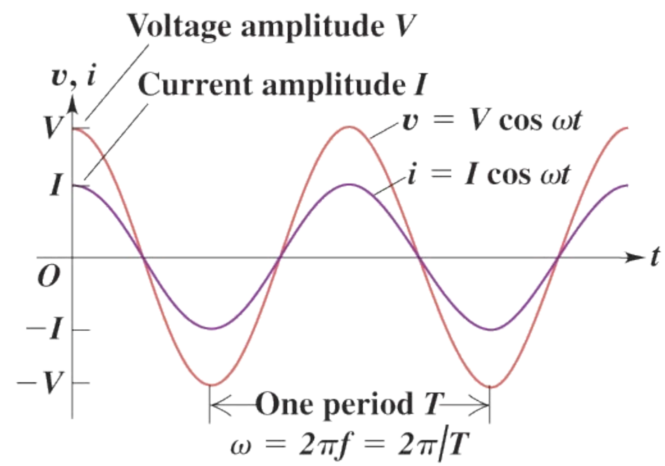
$$\omega = 2\pi f$$

- the current in a circuit behaves like $i = I\cos\omega t$, with a frequency f = 1 Hz. At t=0, the current is measured to be 1.0 A. What is the current 0.5 s later?
  - *A. 1.0 A*
  - *B. -1.0 A*
  - *C. 0.0 A*
  - *D. 0.5 A*

- what is the current at t=0.75 s ?
  - *A. 1.0 A*
  - *B. -1.0 A*
  - *C. 0.0 A*
  - *D. 0.5 A*

root-mean-square

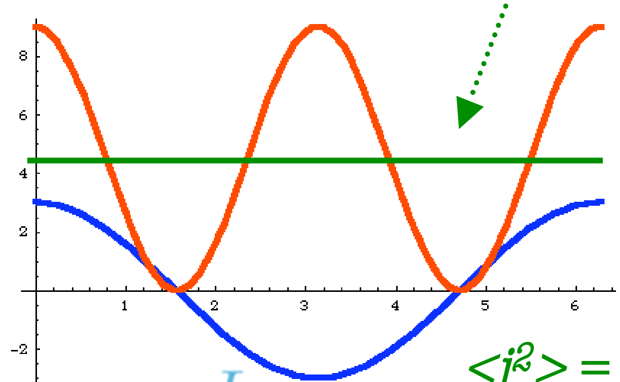
- notice that the simple average current is zero
- over a full cycle i is negative as much as positive



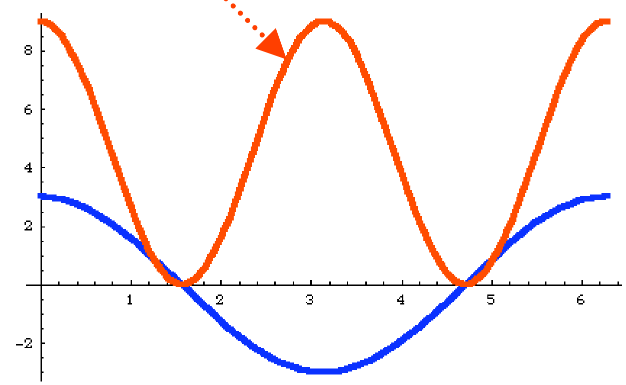
(a) Graphs of sinusoidal current and voltage versus time. The relative heights of the two curves are not significant.

- a more useful average is the ***root-mean-square***, or ***rms***
- defined to be the **square root** of the **average** of the **squared current**

$\sqrt{\langle i^2 \rangle} = 2.12$



$\langle i^2 \rangle = 4.5$

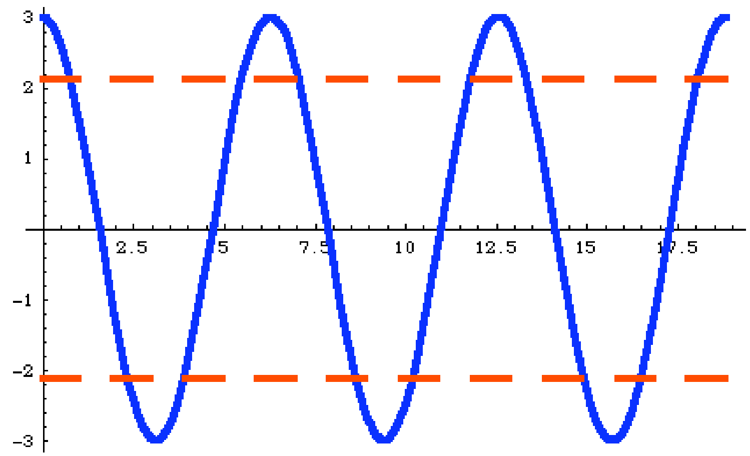


$I = 3$

in general $I_{\mathrm{rms}} = \dfrac{I}{\sqrt{2}}$

root-mean-square

- rms current and voltage is what is usually quoted as the current and voltage of an ac system
- e.g. the 120V household supply in the US is $V_{rms} = 120$ V
- the peak is $V = 120\text{ V} * \sqrt{2} = 170\text{ V}$
- good 'average description'



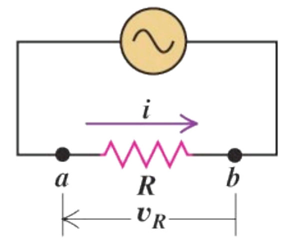
alternating current circuits

- how do simple circuits behave if we put alternating current through them?
- **circuit with a resistor**, Ohm's law describes the voltage drop across the resistor in terms of the current

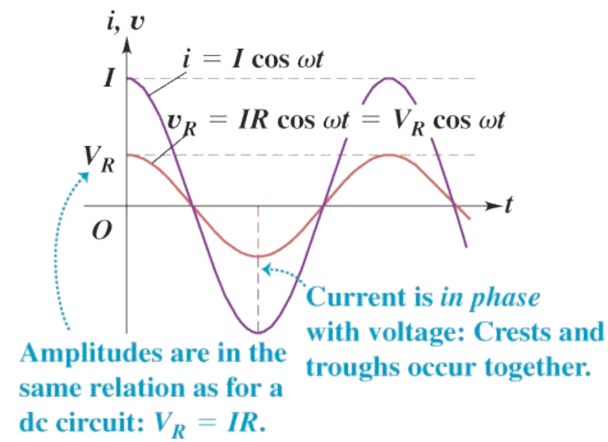
$$v_R = iR$$

$$v_R = IR\cos\omega t$$

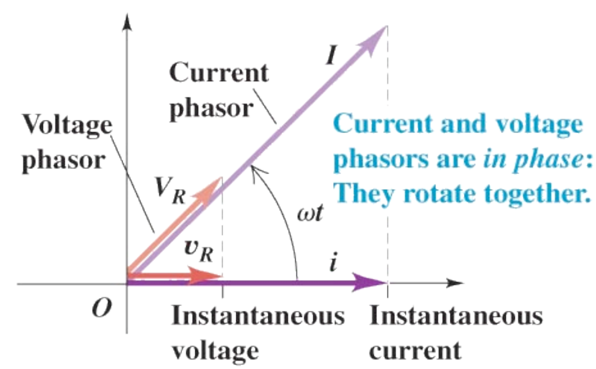
- time-dependence of voltage is the same as the current
  - "phasors stay in-step"



(a) Circuit with ac source and resistor



(b) Graphs of current and voltage versus time



(c) Phasor diagram

inductor

- **circuit with an inductor**, recall the version of Faraday's law for self-inductance

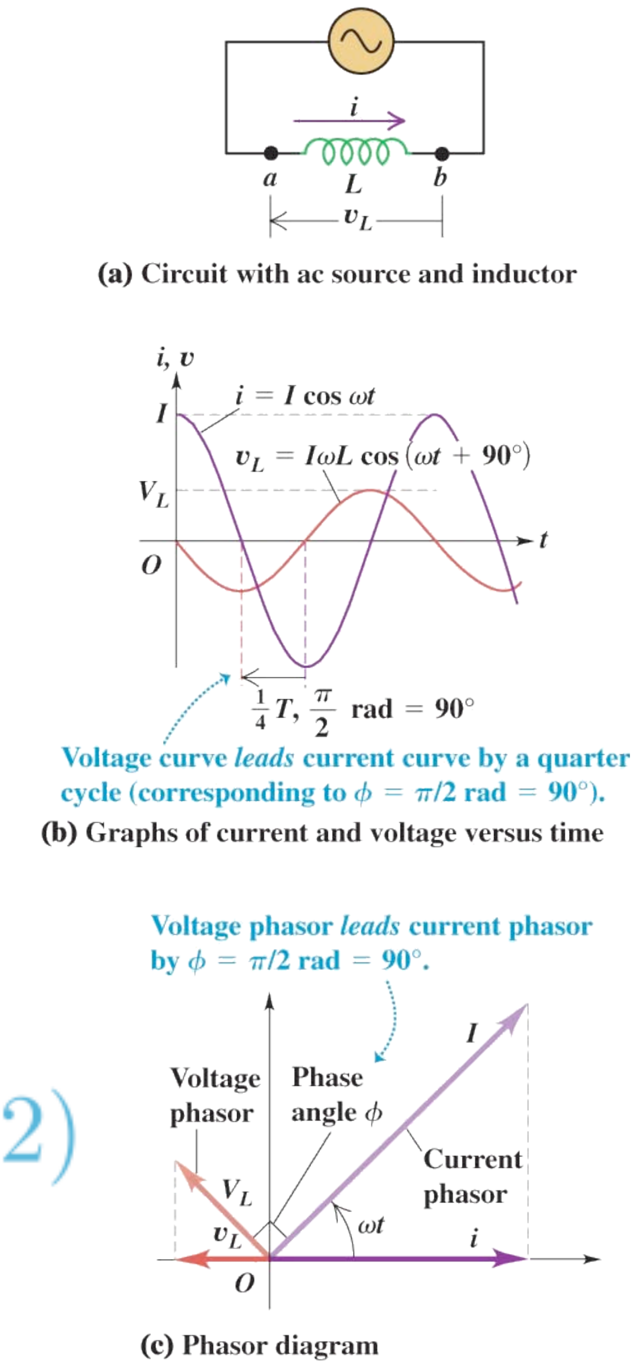
$$v_L = L\frac{\Delta i}{\Delta t}$$

- so the voltage is proportional to the ***slope*** of the current-time graph
  - current slope is biggest when *i=0* and smallest when *i=I*
  - voltage peaks one quarter of a cycle earlier than the current
  - "voltage leads current by 90° = π/2 rads"

$$v_L = -I\omega L\sin(\omega t)$$

$$v_L = -I\omega L\cos(\omega t + \pi/2)$$

" inductive reactance $\boldsymbol{X_L = \omega L}$ "



(a) Circuit with ac source and inductor

(b) Graphs of current and voltage versus time

(c) Phasor diagram

inductor

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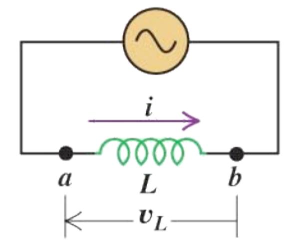


(a) Circuit with ac source and inductor

so the peak (or rms) voltage across the inductor can be written

$$V_L = I\,X_L$$

which looks like Ohm's law with reactance in place of resistance

- high frequency current signals will have a higher reactance and hence cause smaller voltages across the inductor
  - *inductors "choke" off high frequency signals*

capacitor

- **circuit with an capacitor**, recall that the charge on a capacitor is related to the voltage across it

$$v_c = q/C$$

- the rate at which the voltage changes then is related to the rate at which the charge changes, i.e. the current

$$\frac{\Delta v_c}{\Delta t} = \frac{1}{C}\frac{\Delta q}{\Delta t} = \frac{i}{C}$$

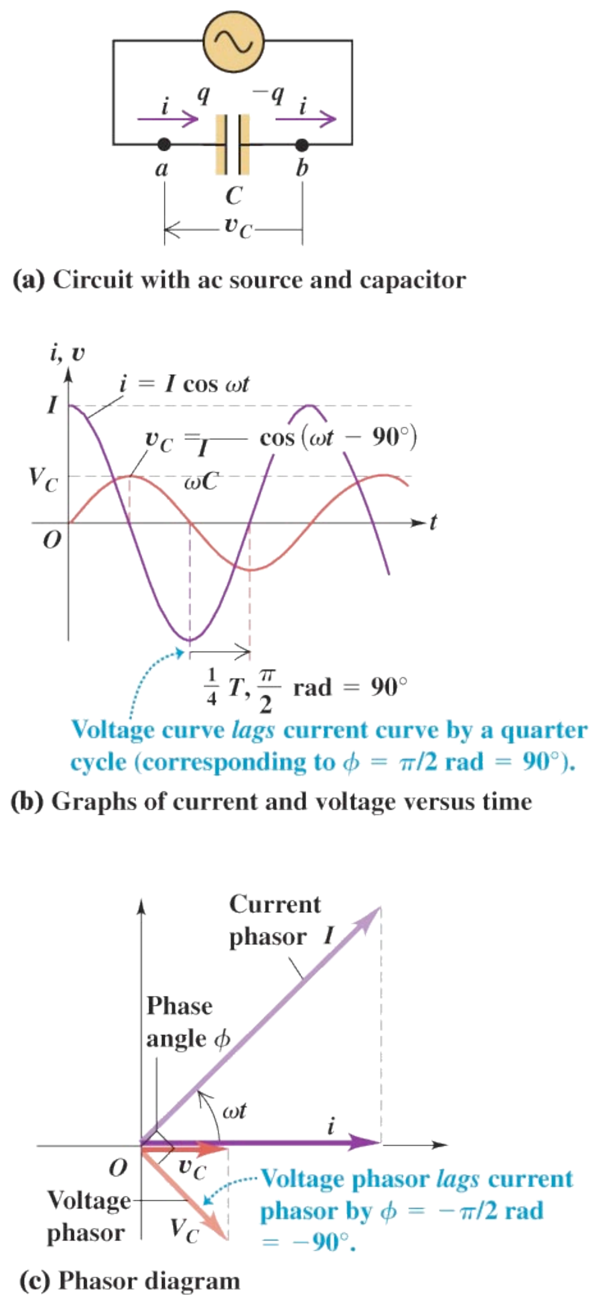
- the only way this can hold true is if

$$v_C = \frac{1}{\omega C}\sin\omega t$$

$$v_C = \frac{1}{\omega C}\cos(\omega t - \pi/2)$$

  - "voltage lags current by 90° = π/2 rads"

" capacitive reactance $\boldsymbol{X_C = 1/(\omega C)}$ "



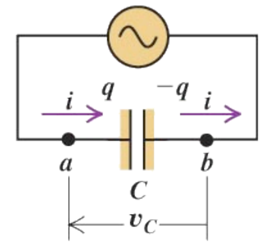
(a) Circuit with ac source and capacitor

(b) Graphs of current and voltage versus time

(c) Phasor diagram

capacitor

“ capacitive reactance $X_C = 1/(\omega C)$ ”



(a) Circuit with ac source and capacitor

so the peak (or rms) voltage across the capacitor can be written

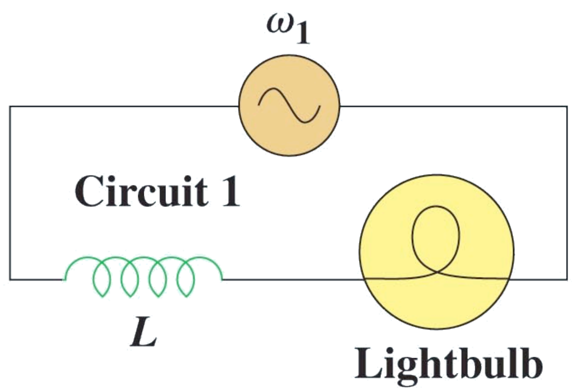
$$V_C = I\,X_C$$

which looks like Ohm’s law with reactance in place of resistance

- low frequency current signals will have a higher reactance and hence cause smaller voltages across the capacitor
  - *capacitors block low frequency currents*

reactance & current

$X_C = 1/(\omega C)$
$X_L = \omega L$
$V = I X$



keeping the source voltage constant we **reduce** the angular frequency ω_1

what happens to the brightness of the bulb:

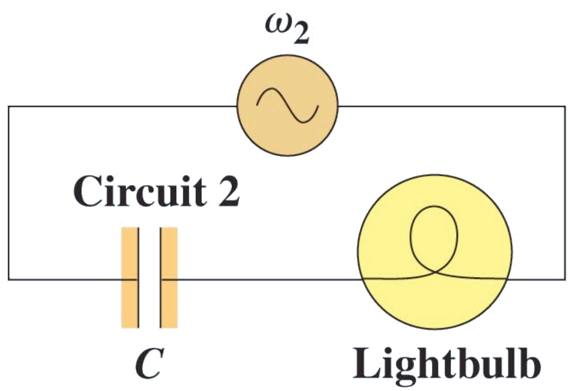
A. gets dimmer

B. gets brighter

C. stays the same

reactance & current

$X_C = 1/(\omega C)$
$X_L = \omega L$
$V = I X$



keeping the source voltage constant we **reduce** the angular frequency ω_2

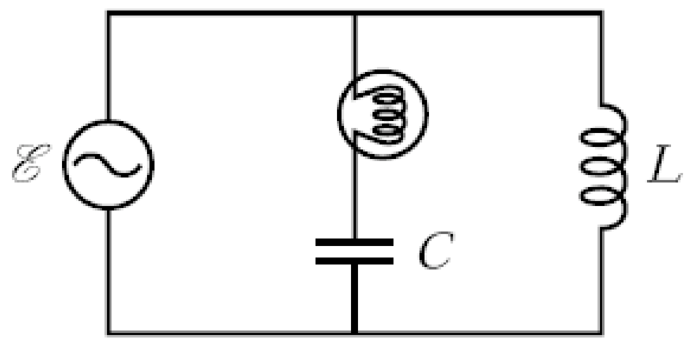
what happens to the brightness of the bulb:

A. gets dimmer

B. gets brighter

C. stays the same

The light bulb has a resistance R, and the emf drives the circuit with a frequency w. The light bulb glows most brightly at



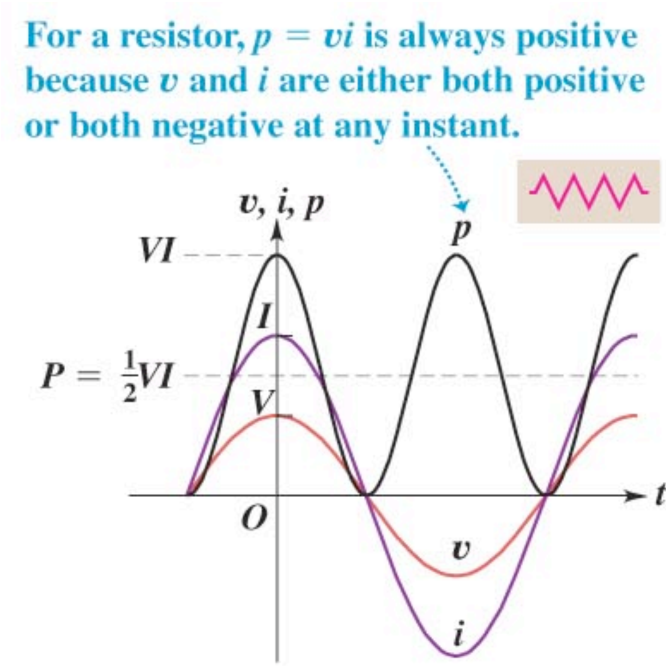
1. very low frequencies.
2. very high frequencies.
3. the frequency $w = 1/\sqrt{LC}$.

$X_C = 1/(\omega C)$
$X_L = \omega L$
$V = I X$

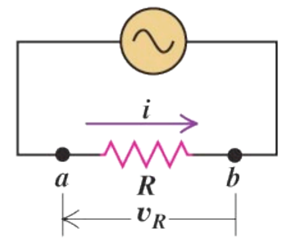
power in ac circuits

- **circuit with a resistor**

- the instantaneous power supplied by the voltage source $p = v\,i$



(a) Pure resistor



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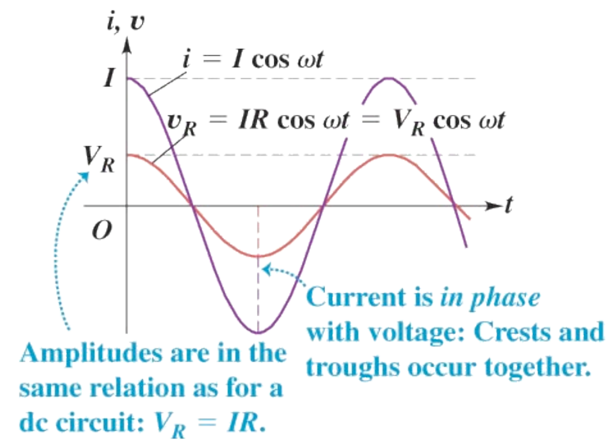


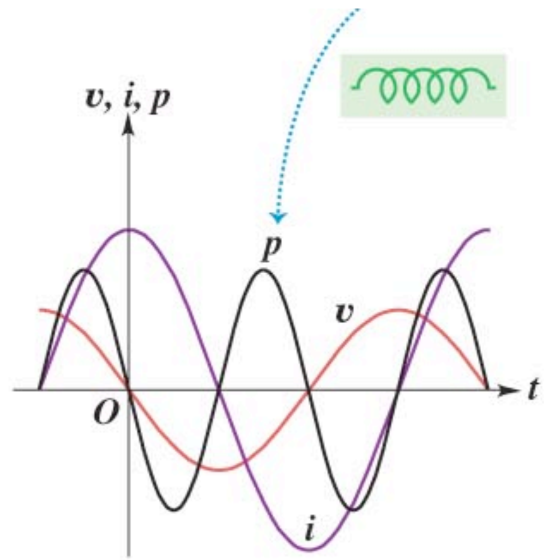
(b) Graphs of current and voltage versus time

- average power $P = VI/2 = V_{rms}\, I_{rms} = (I_{rms})^2\, R$

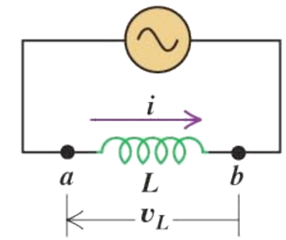
power in ac circuits

- **circuit with an inductor**

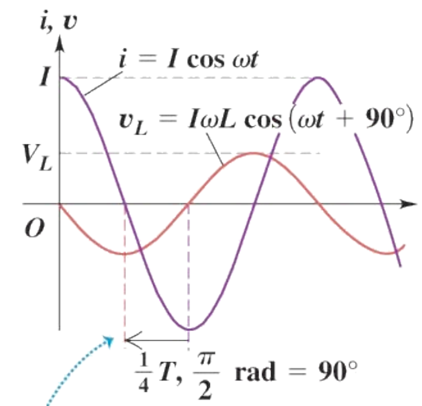
- the instantaneous power supplied by the voltage source $p = v\,i$



(b) Pure inductor



(a) Circuit with ac source and inductor



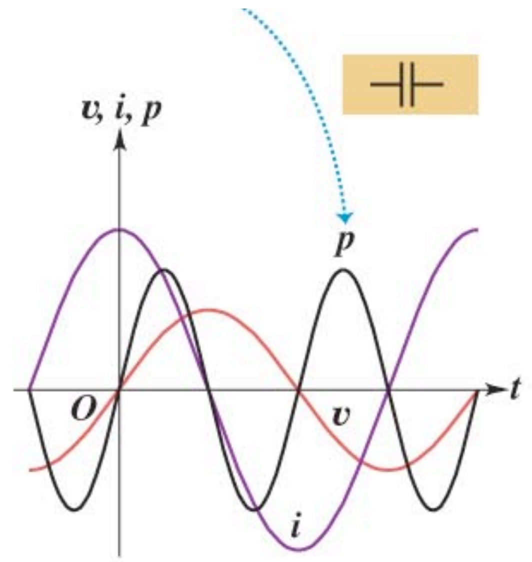
(b) Graphs of current and voltage versus time

- voltage leads by a quarter cycle
- average power $P = 0$!

power in ac circuits

- **circuit with a capacitor**

- the instantaneous power supplied by the voltage source $p = v\,i$



(c) Pure capacitor

- voltage lags by a quarter cycle
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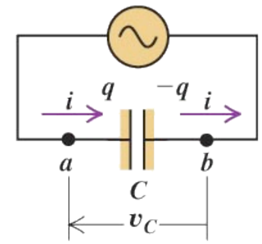

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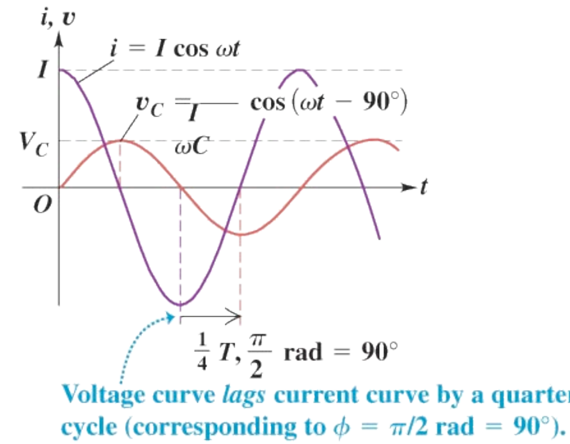
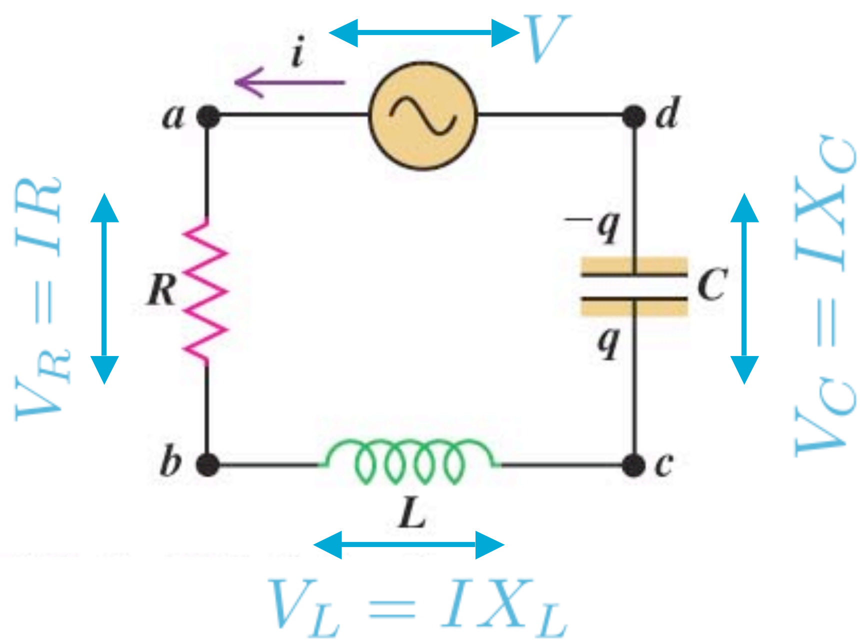
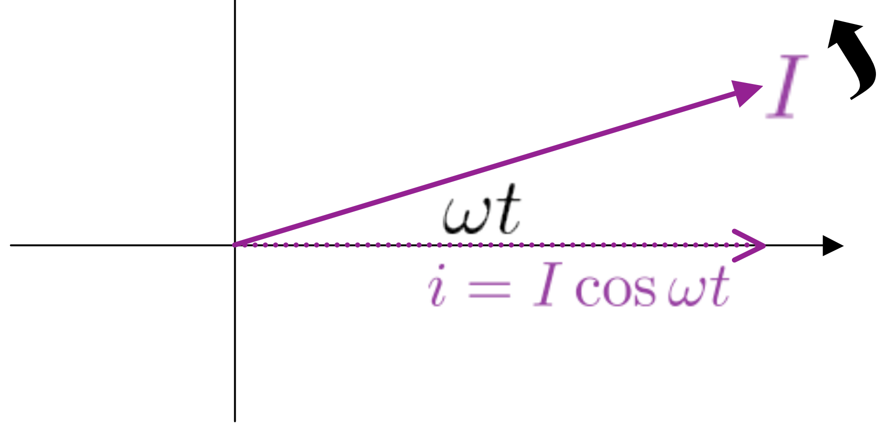


(b) Graphs of current and voltage versus time

series RLC

- $v = v_R + v_C + v_L$
- these are time-varying voltages we should add the phasors - just like adding vectors





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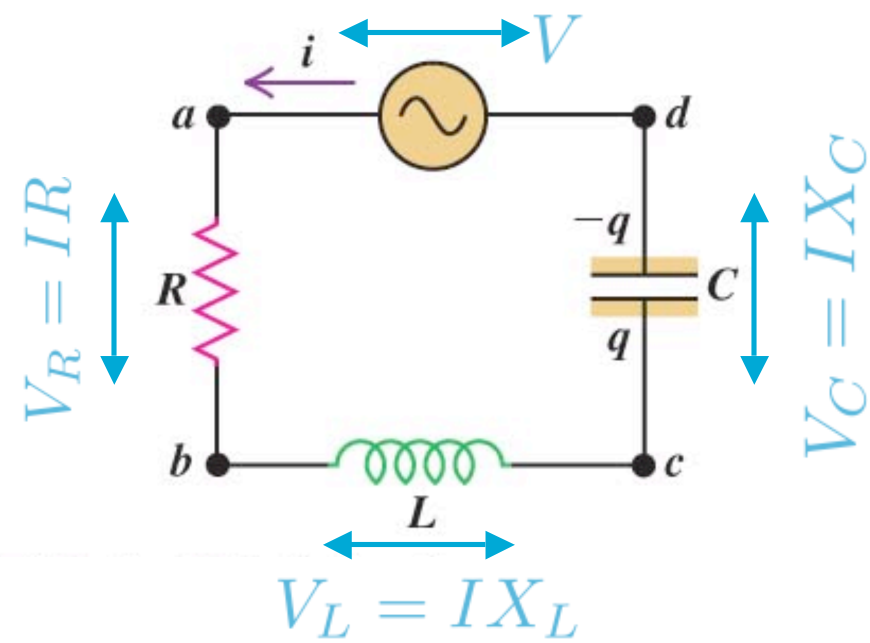

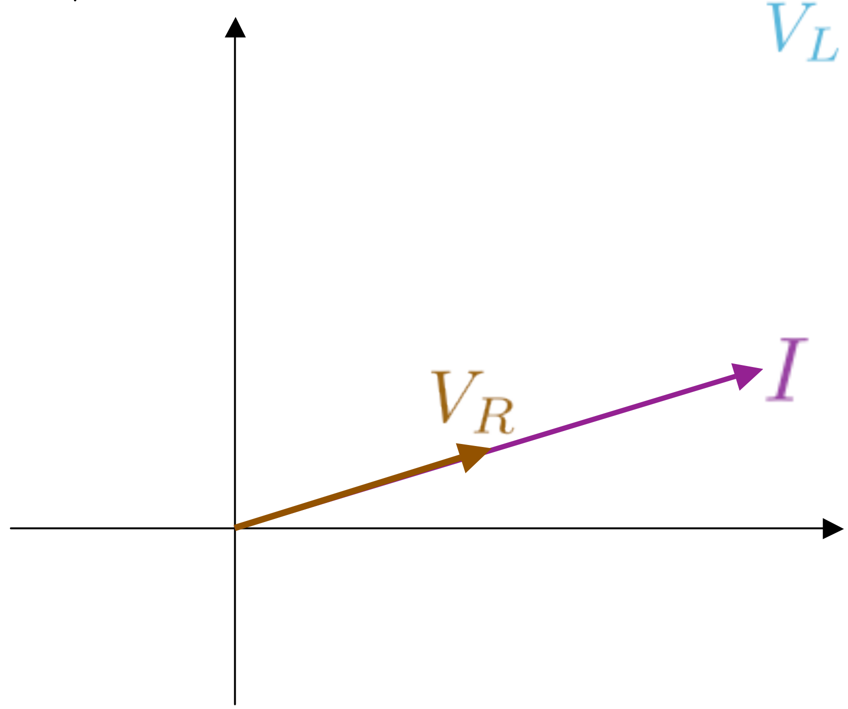

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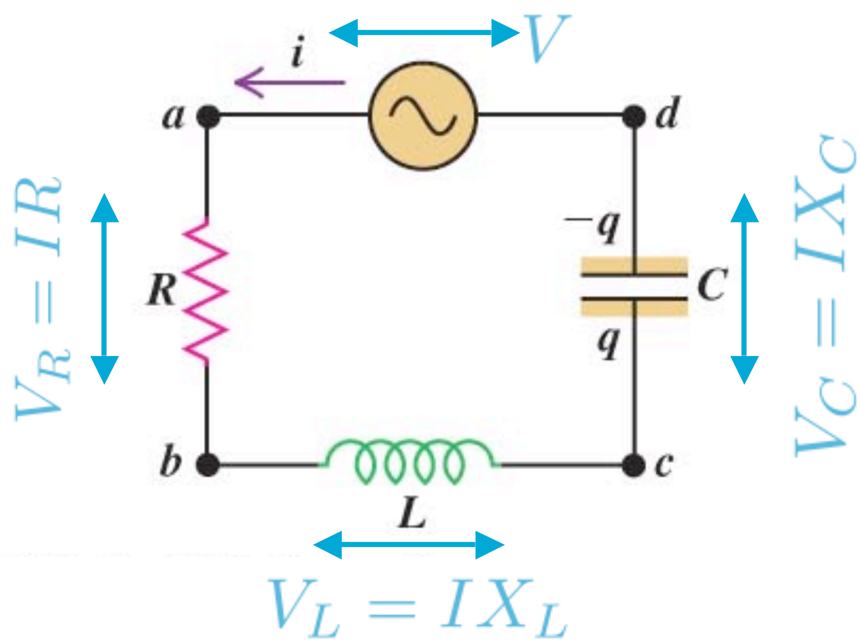

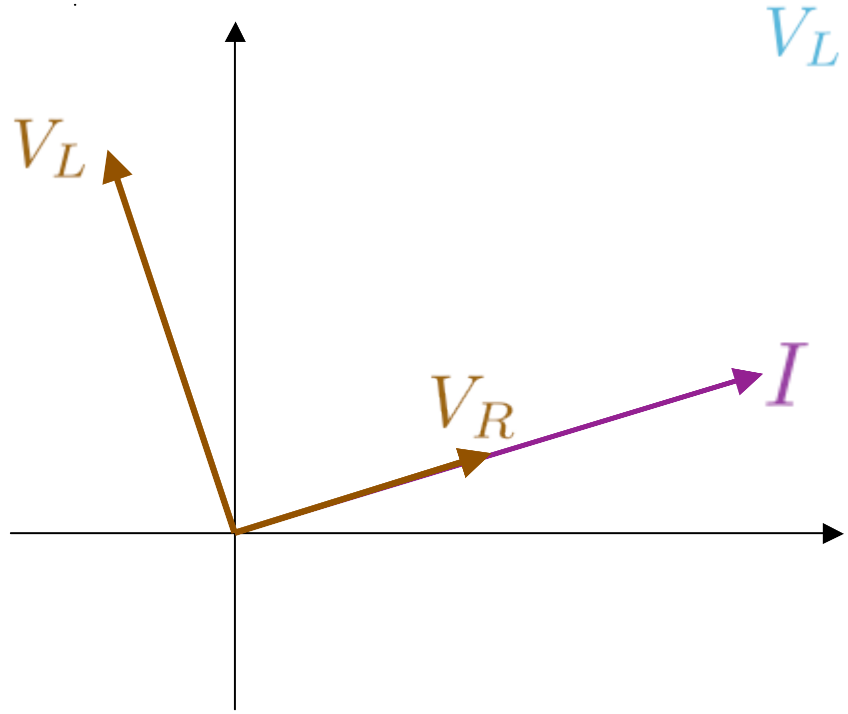

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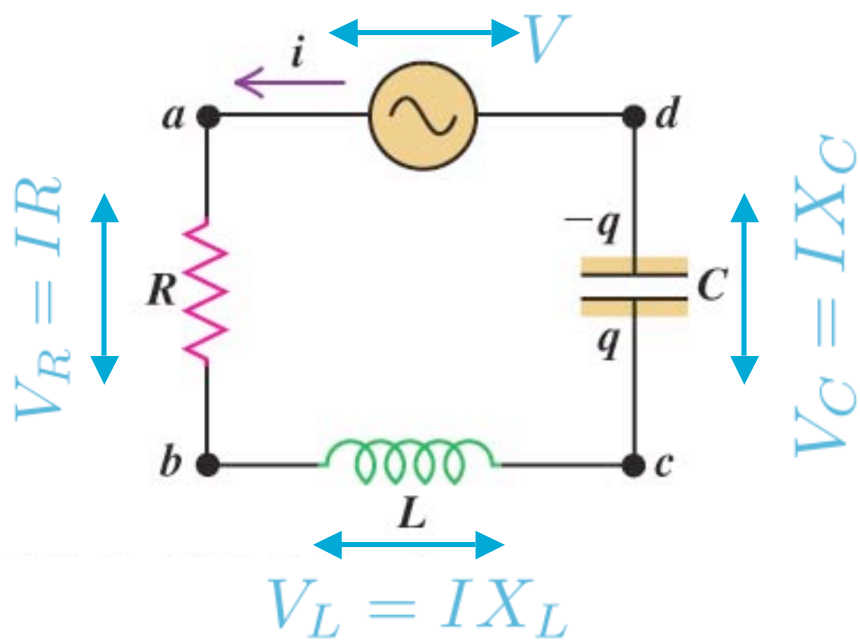

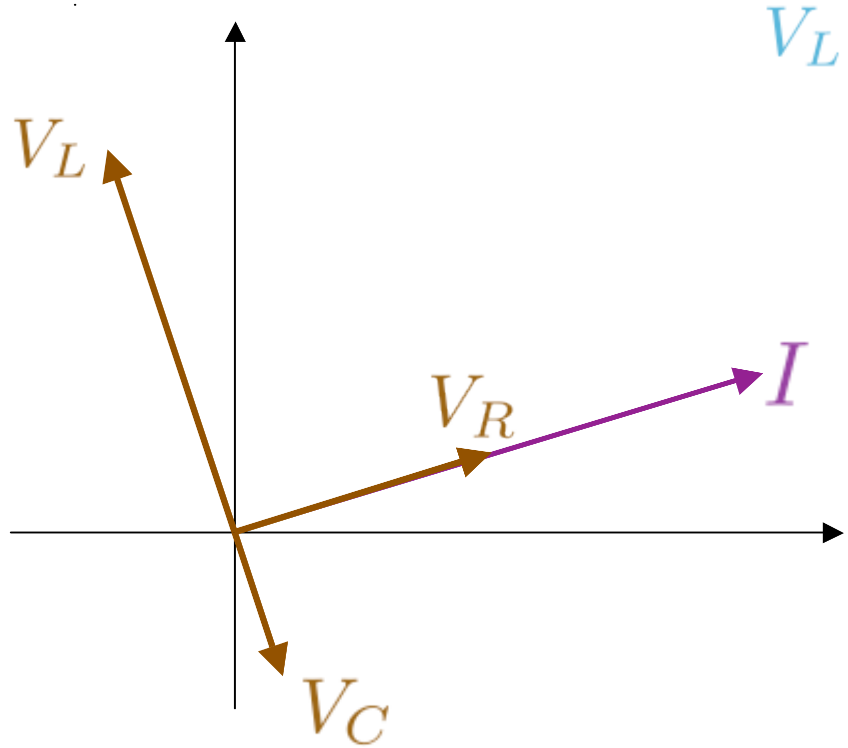

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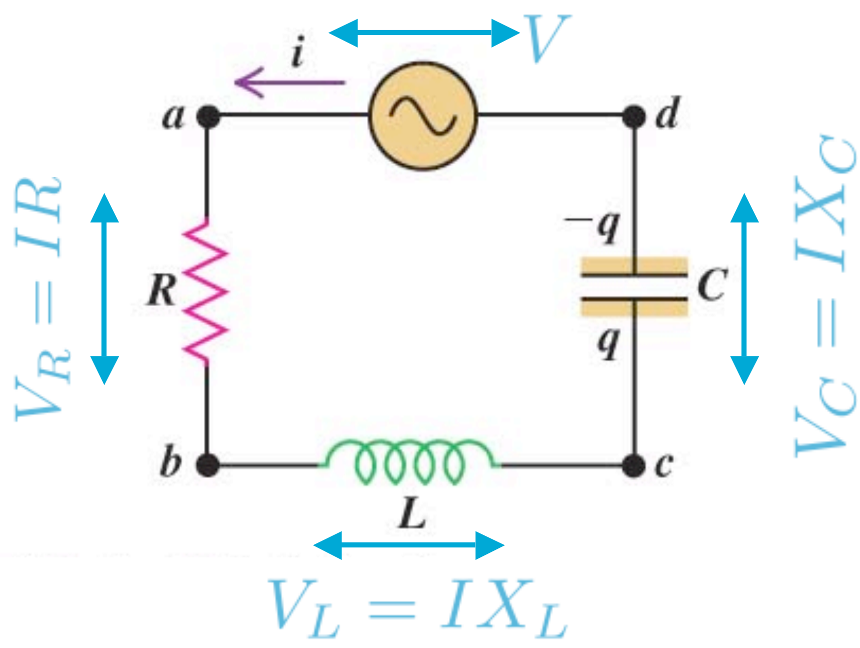

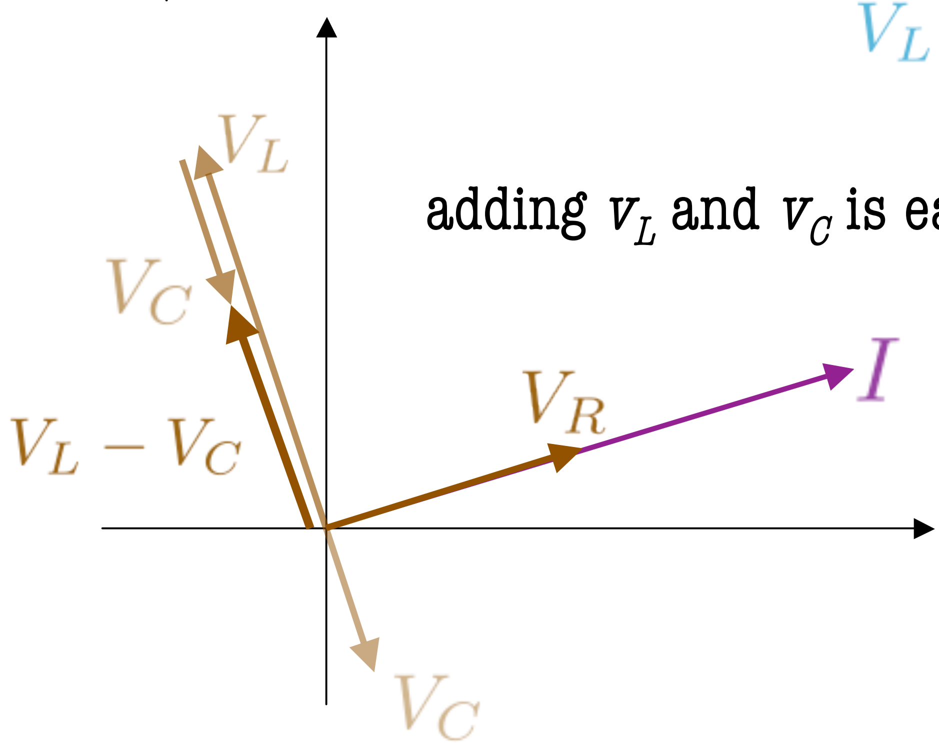

adding v_L and v_C is easy

series RLC

- $v = v_R + v_C + v_L$
- these are time-varying voltages
  we should add the phasors
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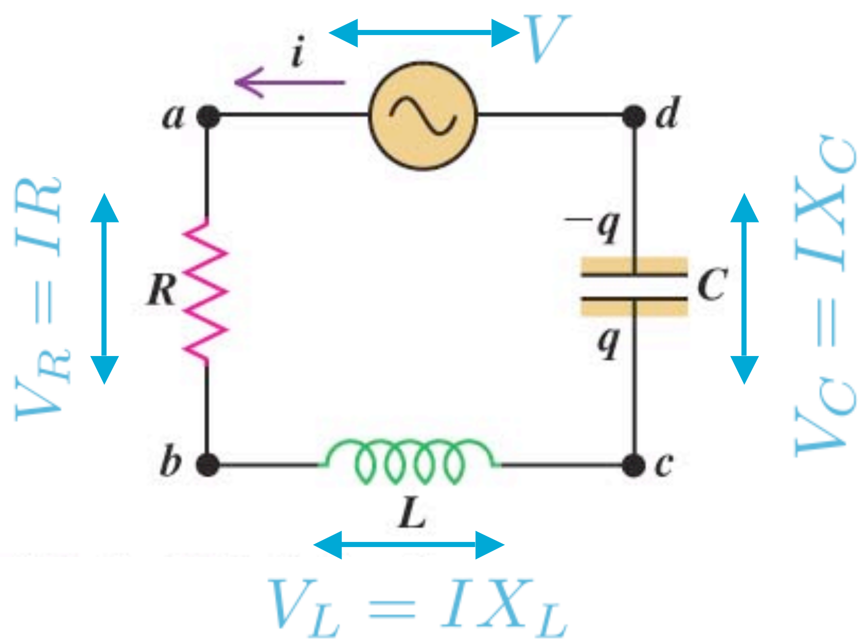
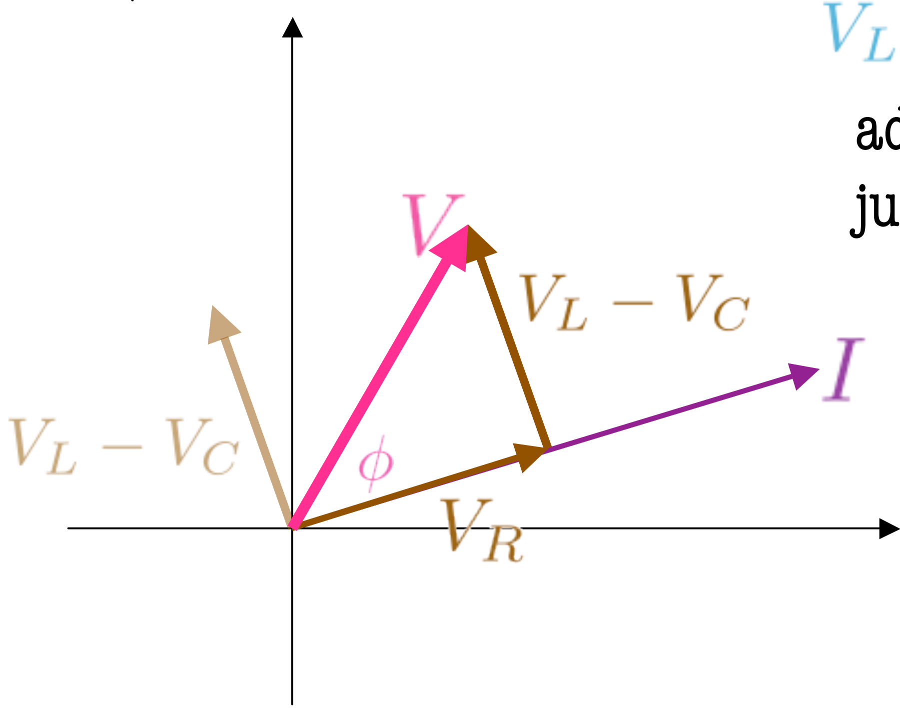
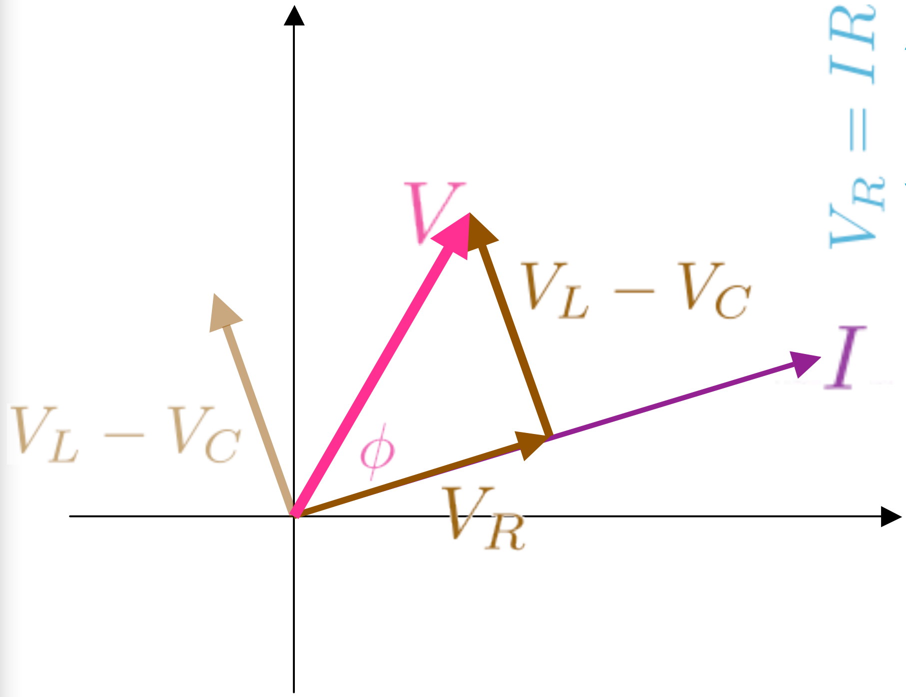
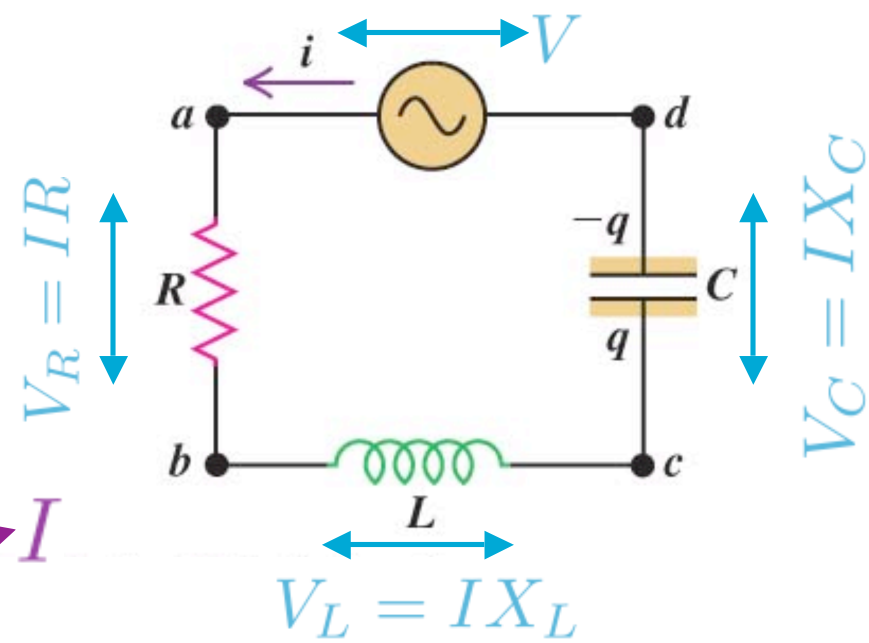


adding $v_L + v_C$ to v_R is
just adding vectors



series RLC





we define the **impedance** of the circuit to be *Z=V/I*

by pythagoras $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

the phase lag is given by

so $Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

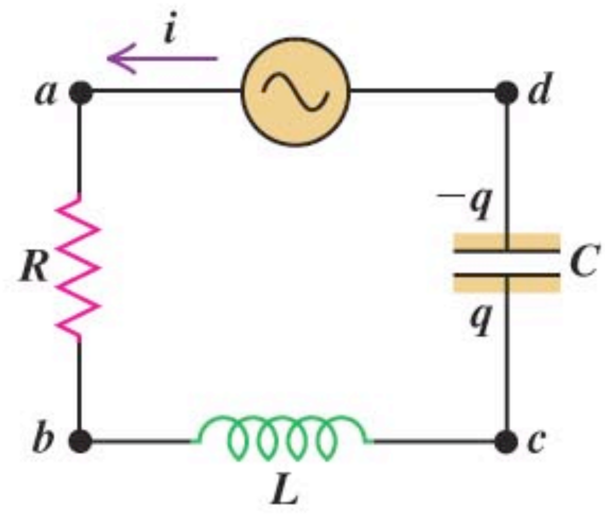
$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$

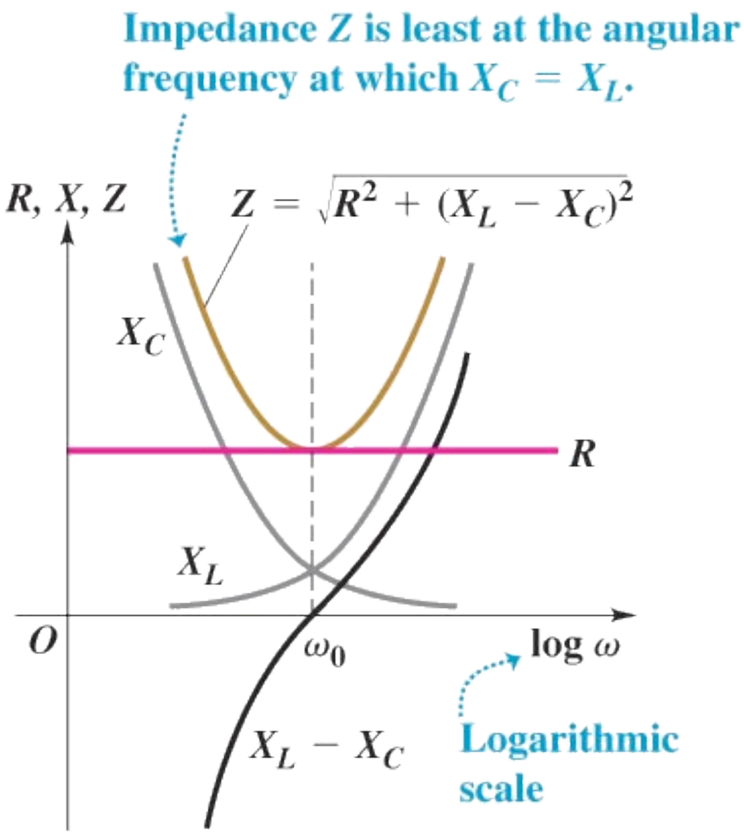
resonant RLC circuit

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

takes its smallest possible value ($Z=R$) if

$$\omega L = \frac{1}{\omega C} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$





for a fixed V, the peak current is maximised when Z is minimised