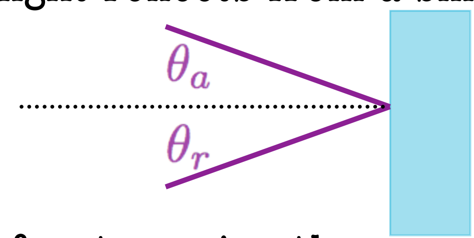
Chapter 24 - Geometric Optics

geometric optics

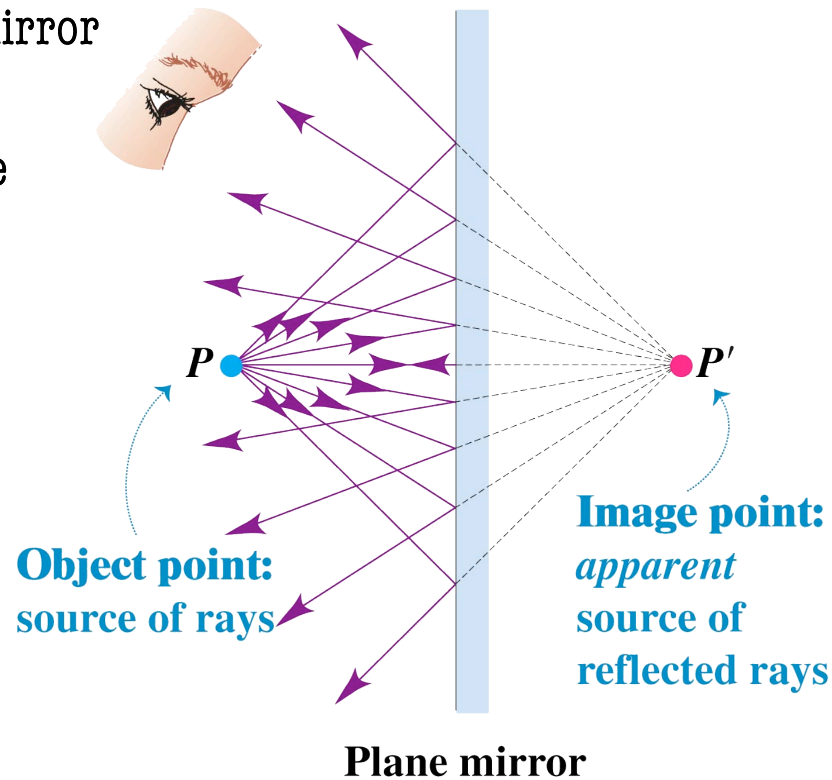
- this is the study of systems where the *ray model* of light works well
- in chapter 26 we'll find that this model isn't complete when we consider *diffraction*
- but it does a good job of describing a lot of situations, so let's take a look at it

geometric optics of a plane mirror

- we've already learnt a rule that applies when light reflects from a smooth surface
  - *angle of incidence = angle of reflection*

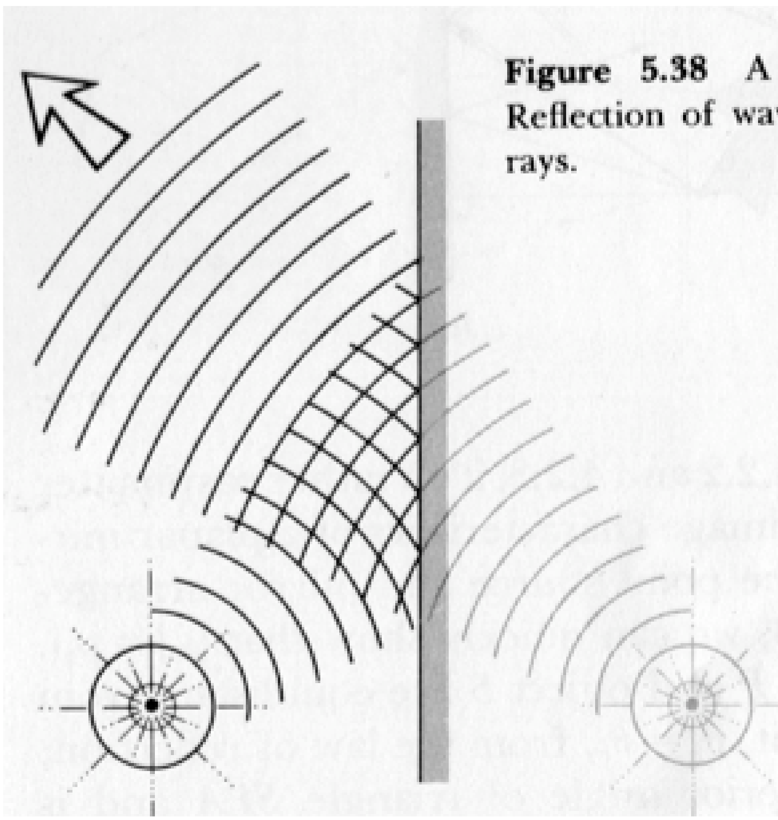


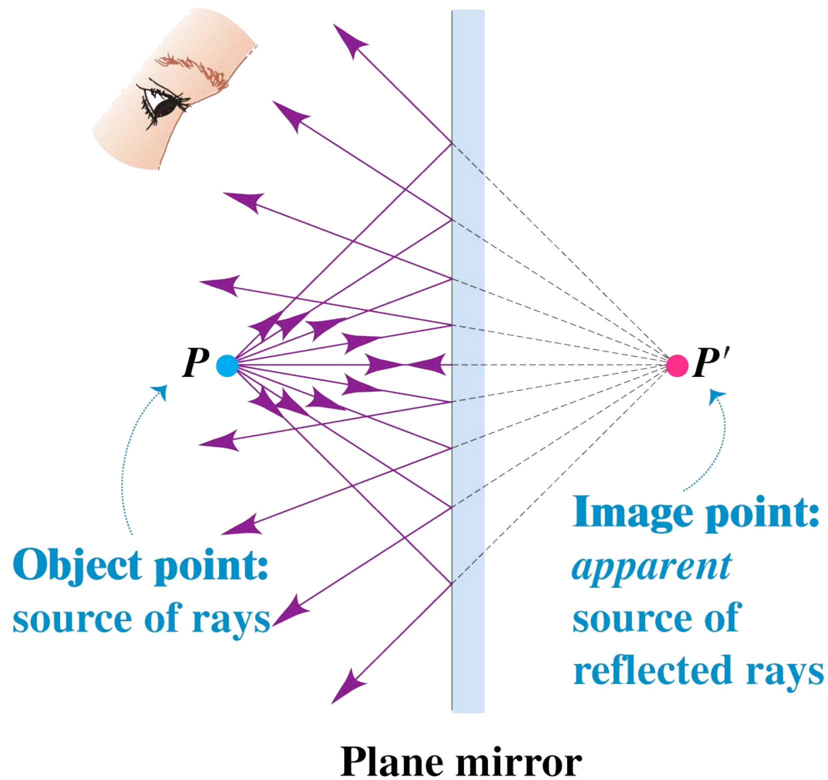
- consider an object which emits spherical wavefronts, or in other words, rays in every direction *(this is basically anything illuminated)*
- place the object in front of a plane mirror
- we can draw rays diverging from the object
- they are reflected at the mirror
- tracking the reflected rays back they *appear* to be diverging from a point behind the mirror
  - *we call this the IMAGE*



geometric optics of a plane mirror

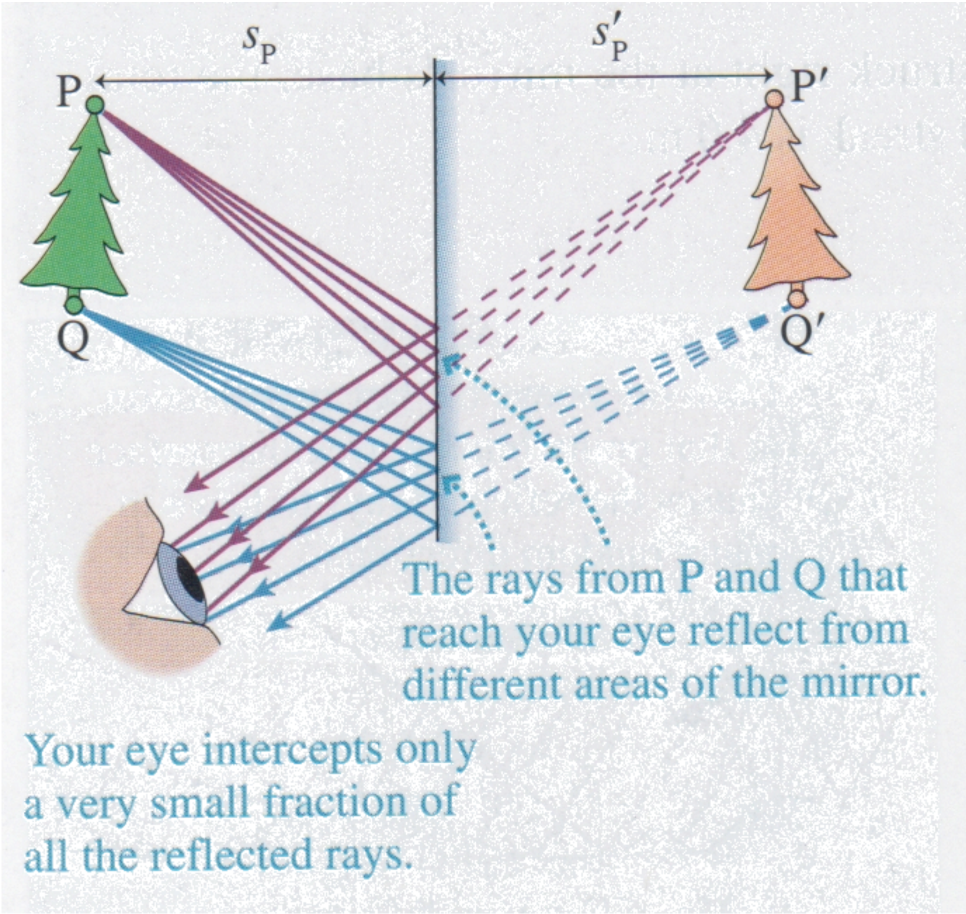
rays simpler to deal with than wavefronts



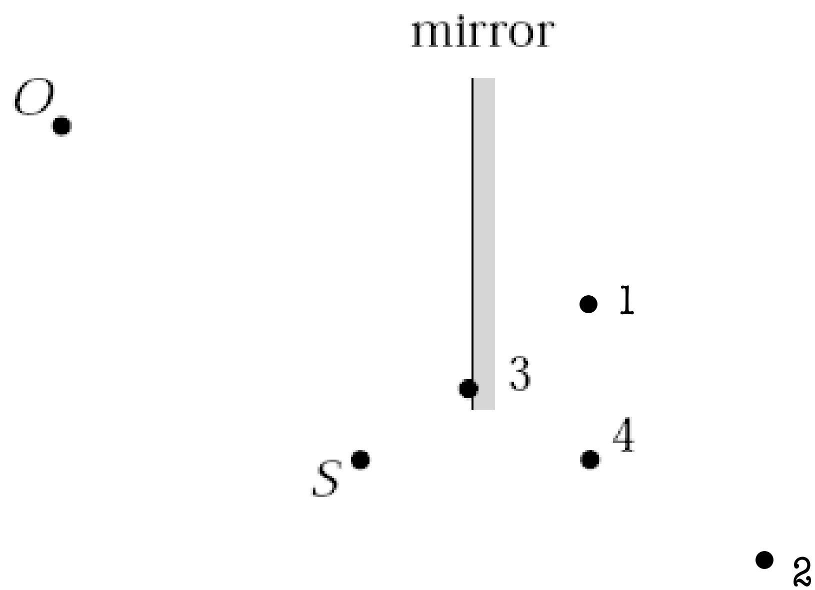


geometric optics of a plane mirror

image formation for an extended object : consider rays from different parts of the object:



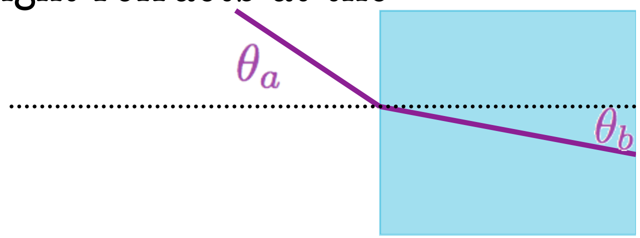
An observer O, facing a mirror, observes a light source S. Where does O perceive the mirror image of S to be located?



A. 1
B. 2
C. 3
D. 4

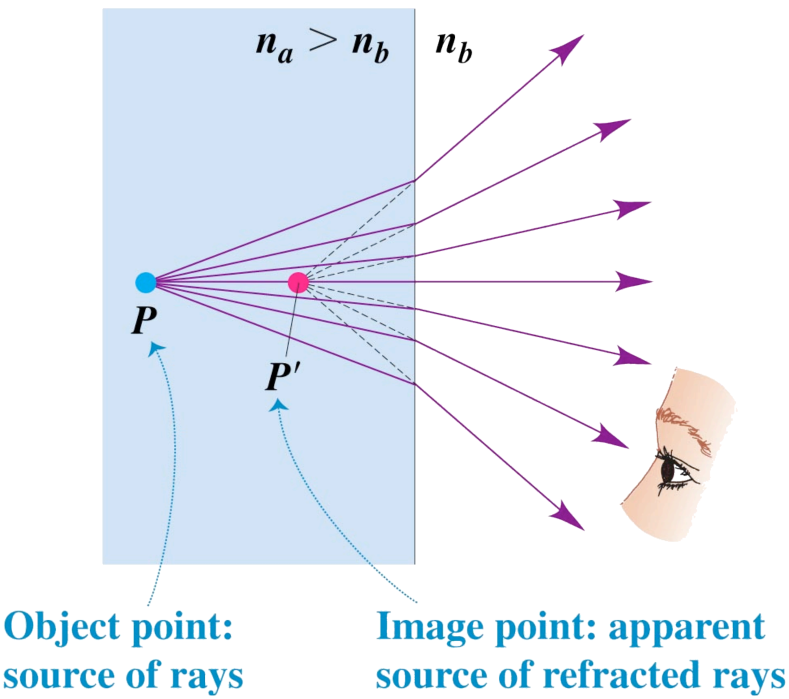
geometric optics of a refracting surface

- we've already learnt a rule that applies when light refracts at the interface between two media
  - *Snell's Law* $n_a \sin\theta_a = n_b \sin\theta_b$

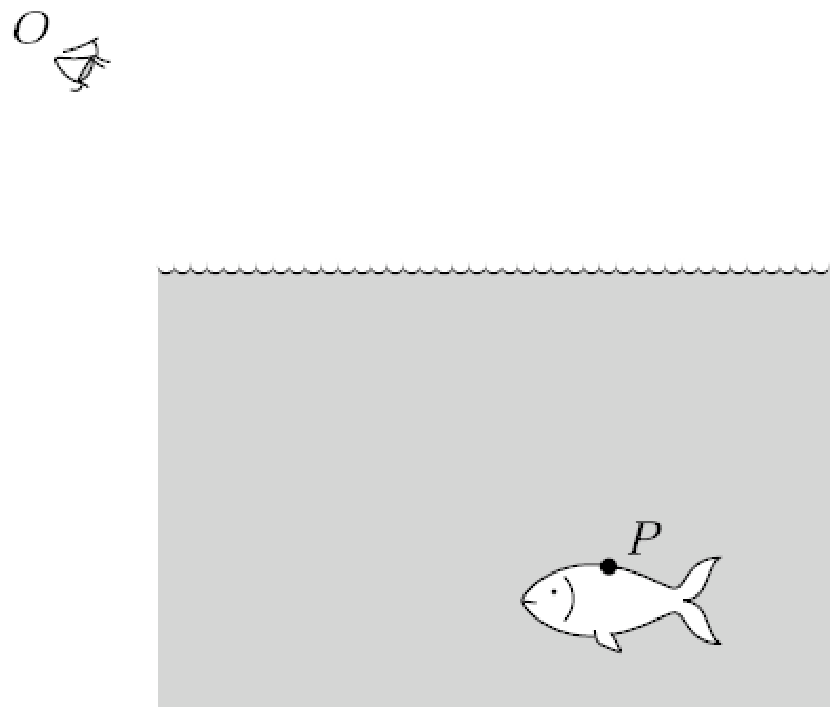


- consider an object which emits spherical wavefronts, or in other words, rays in every direction
- place the object in medium *a*
- we can draw rays diverging from the object
- they are refracted at the interface
- tracking the reflected rays back they *appear* to be diverging from a different point in medium *a*
  - *we call this the IMAGE*

When $n_a > n_b$, P' is closer to the surface than P; for $n_a < n_b$, the reverse is true.

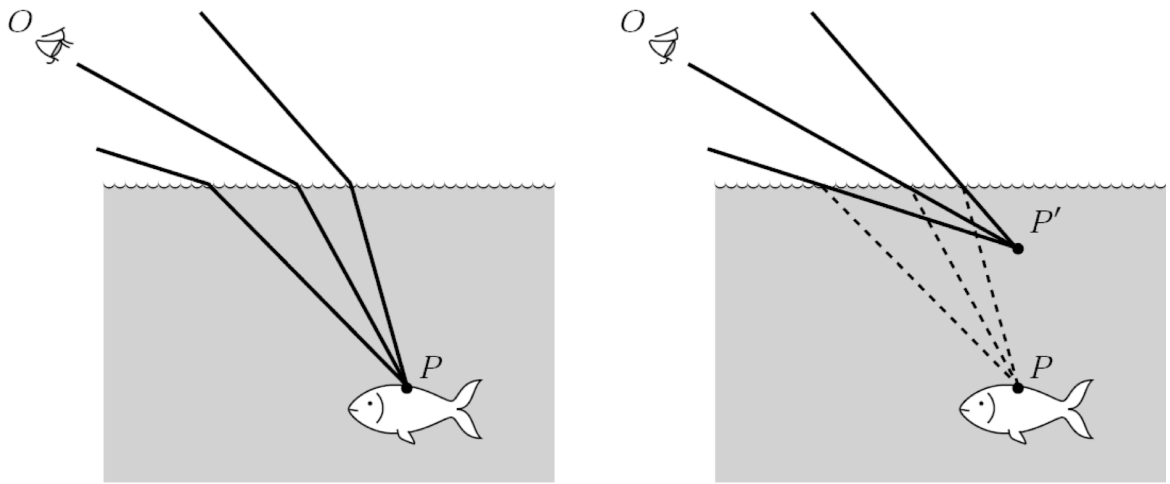


A fish swims below the surface of the water at P. An observer at O sees the fish at

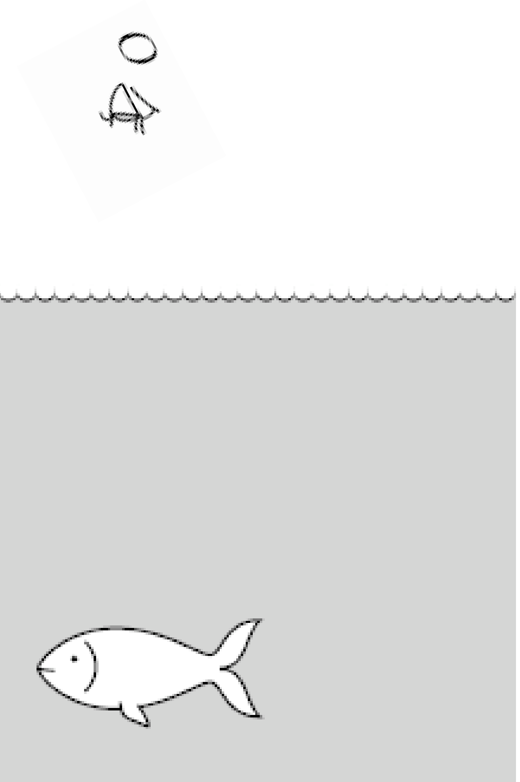


A. a greater depth than it really is.
B. the same depth.
C. a smaller depth than it really is.

Explanation



A fish swims below the surface of the water. Suppose an observer is looking at the fish from point O' straight above the fish. The observer sees the fish at



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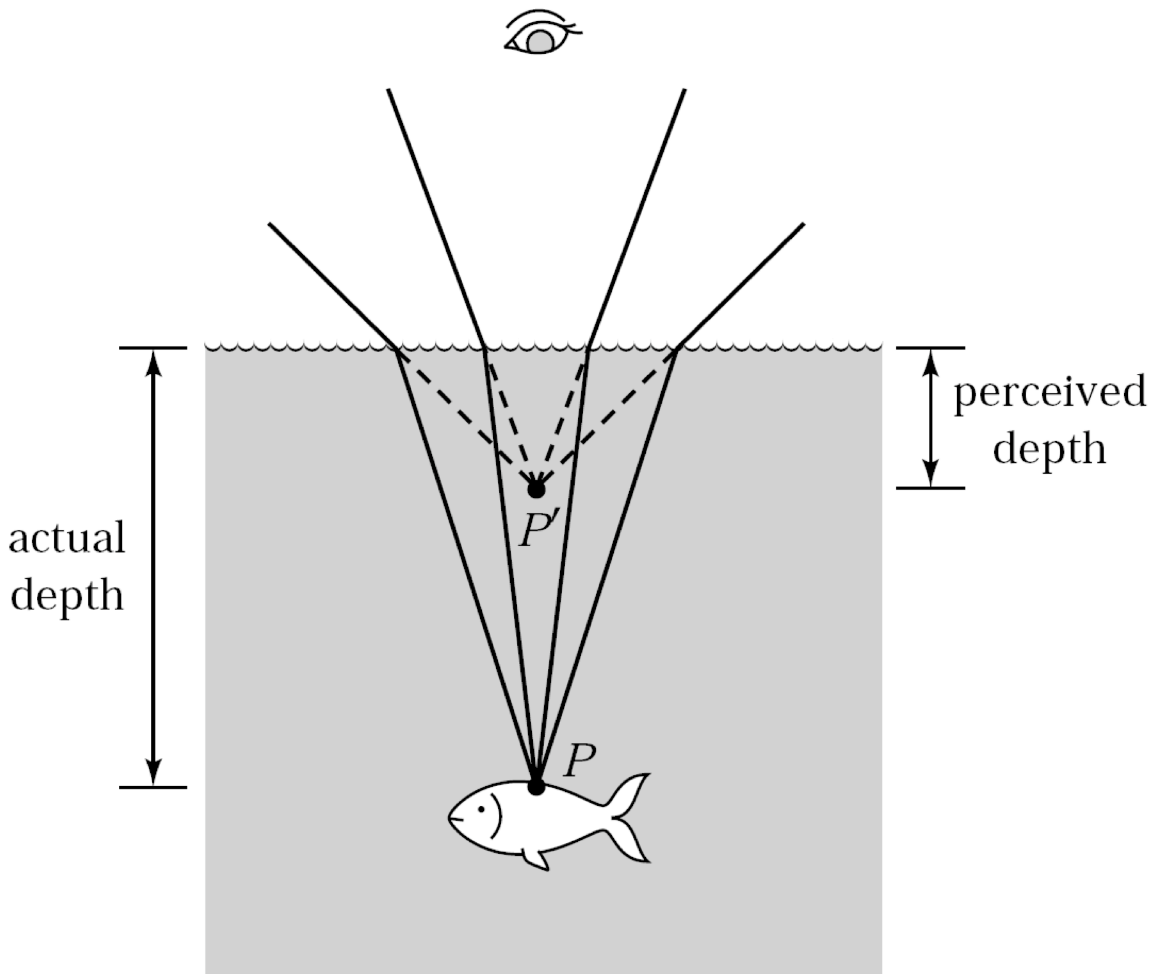
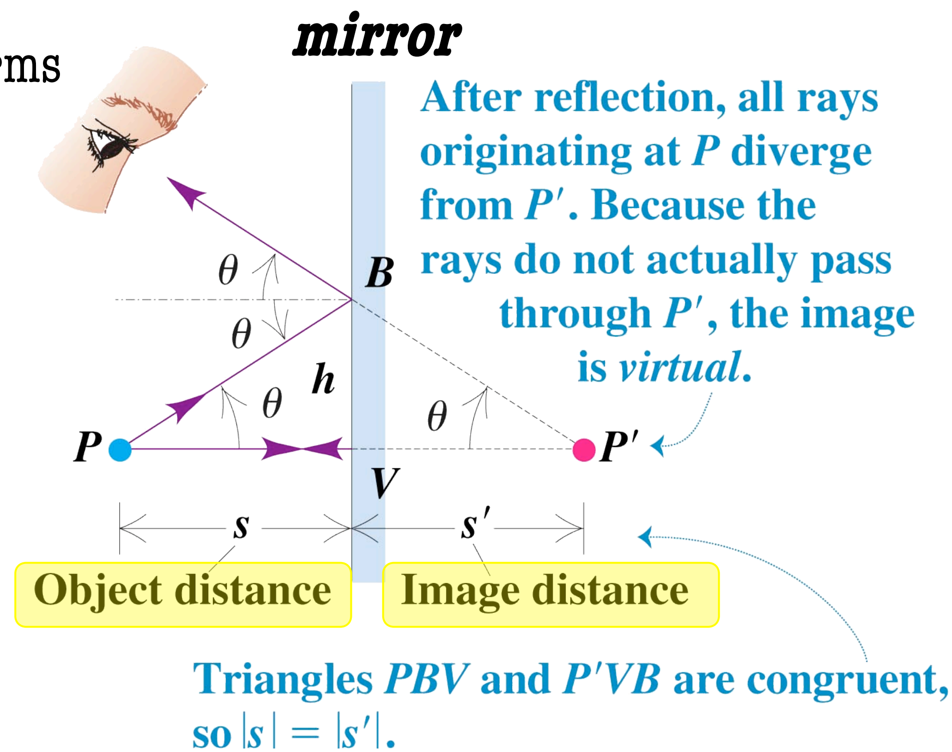


quantitative geometric optics

- we can do better than just these qualitative statements and diagrams
- we can derive formulae that tell us exactly where the image will be and how magnified it is relative to the object
  - *all we need are the rules we already know: law of reflection and Snell's law*

- first we should define some terms
  - object distance
  - image distance

- we call this image a *virtual image* - the rays appear to come from *P'* but if we put a screen at *P'* we wouldn't see an image

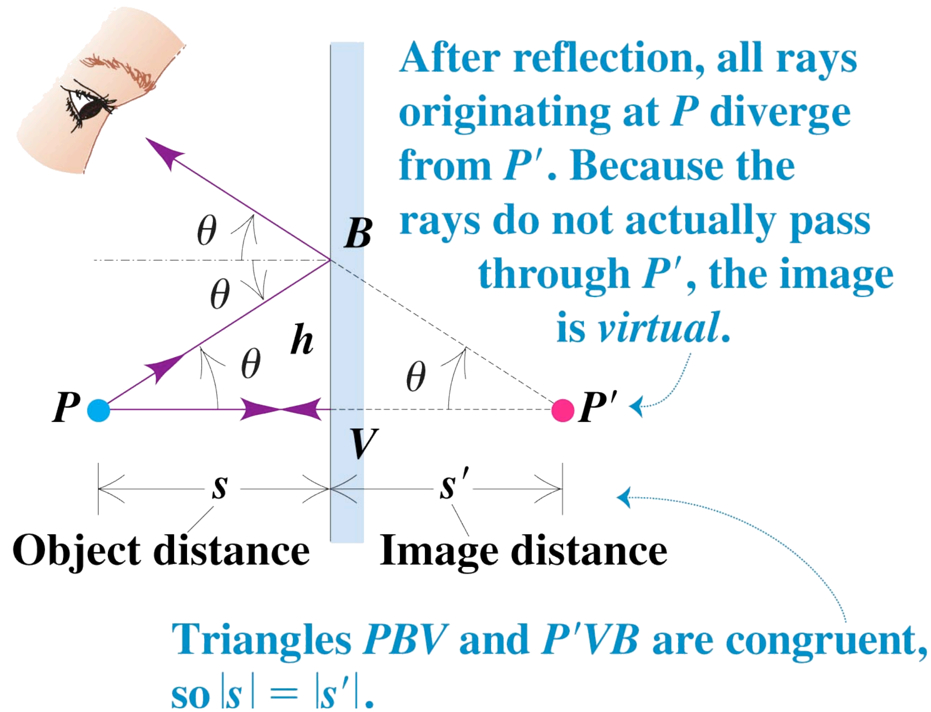


quantitative geometric optics

- some simple trig on these triangles indicates that

$$|s| = |s'|$$

- we'll now define some *sign rules* that look tricky now, but will be very helpful later on when we deal with more complicated systems

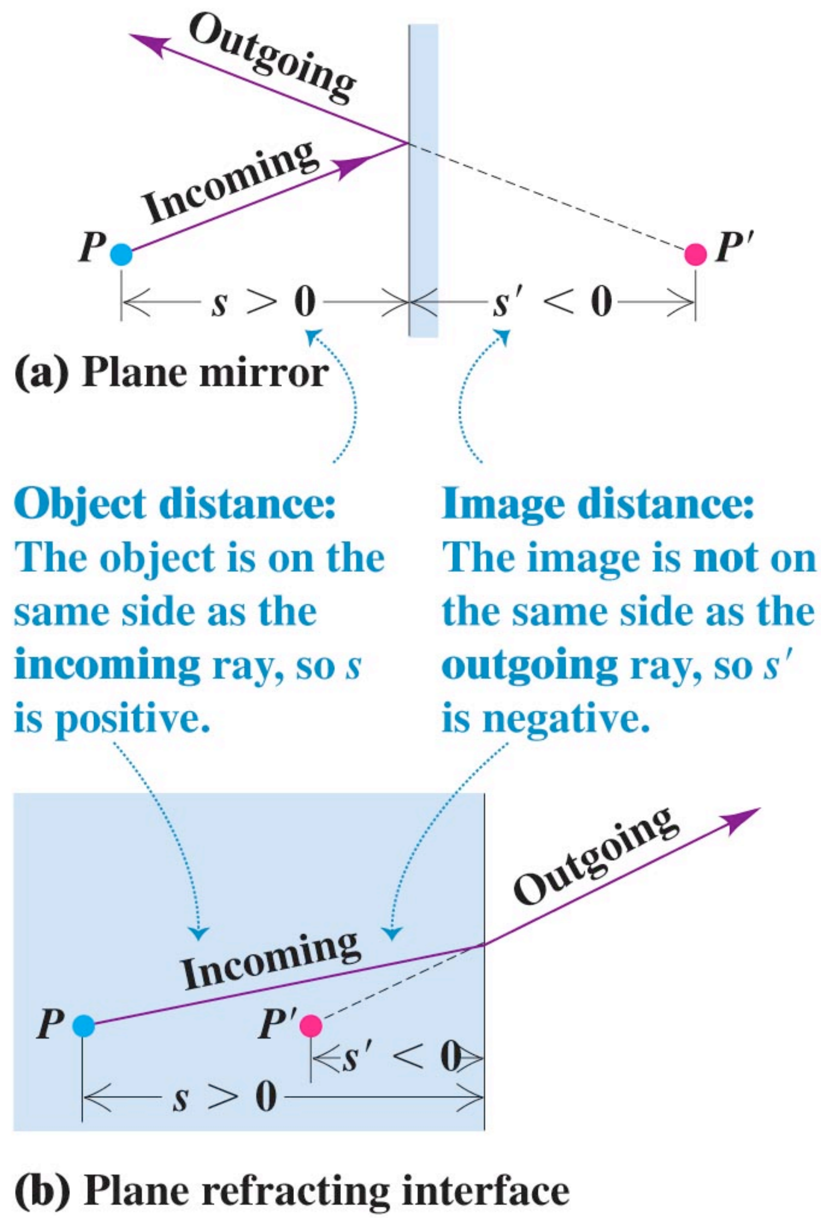


quantitative geometric optics

we'll now define some *sign rules* that look tricky now, but will be very helpful later on when we deal with more complicated systems

object distance: positive when the object is on the same side of the reflecting or refracting surface as the incoming rays (otherwise negative)

image distance: positive when the image is on the same side of the reflecting or refracting surface as the outgoing rays (otherwise negative)



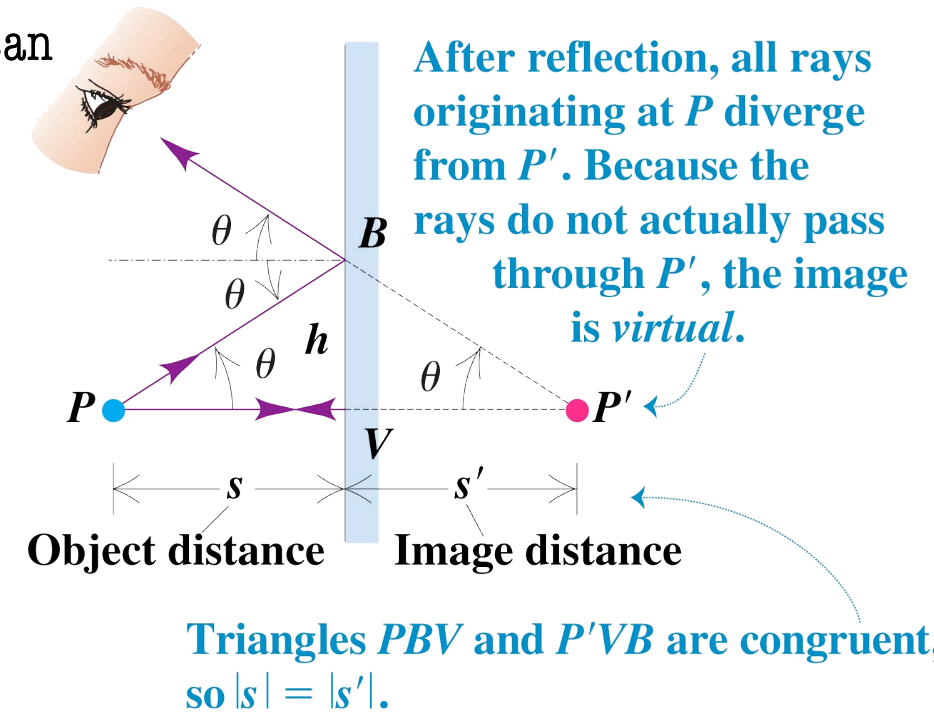
quantitative geometric optics

returning to the plane mirror we can add signs to our equation:

$$|s| = |s'|$$

$$s = -s'$$

“image is as far behind the mirror as the object is in front & is virtual”



quantitative geometric optics

what about objects that have a size?

potentially, reflection or refraction could magnify or diminish the image relative to the object

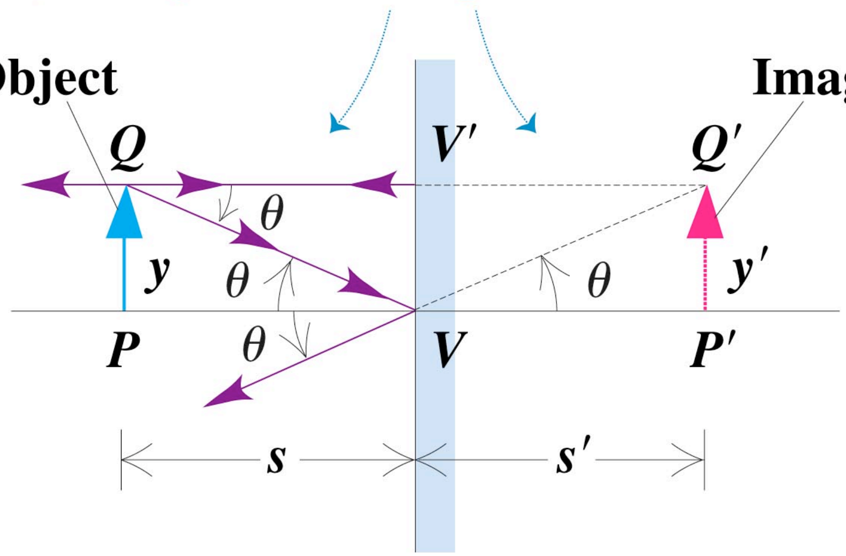
for a plane mirror

$$y = y'$$

more generally we define linear magnification by

$$m = \frac{y'}{y}$$

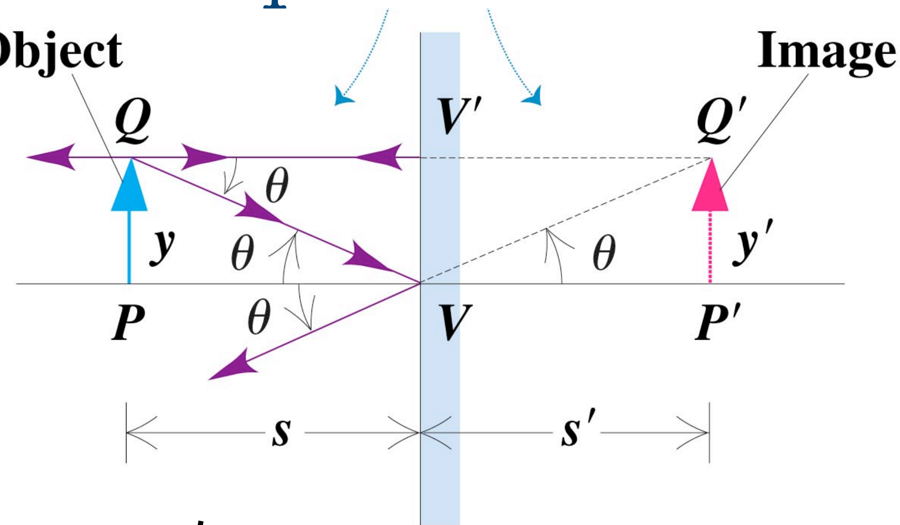
For a plane mirror, *PQV* and *P′Q′V* are congruent, so $y = y'$ and the lateral magnification is 1 (the object and image are the same size).

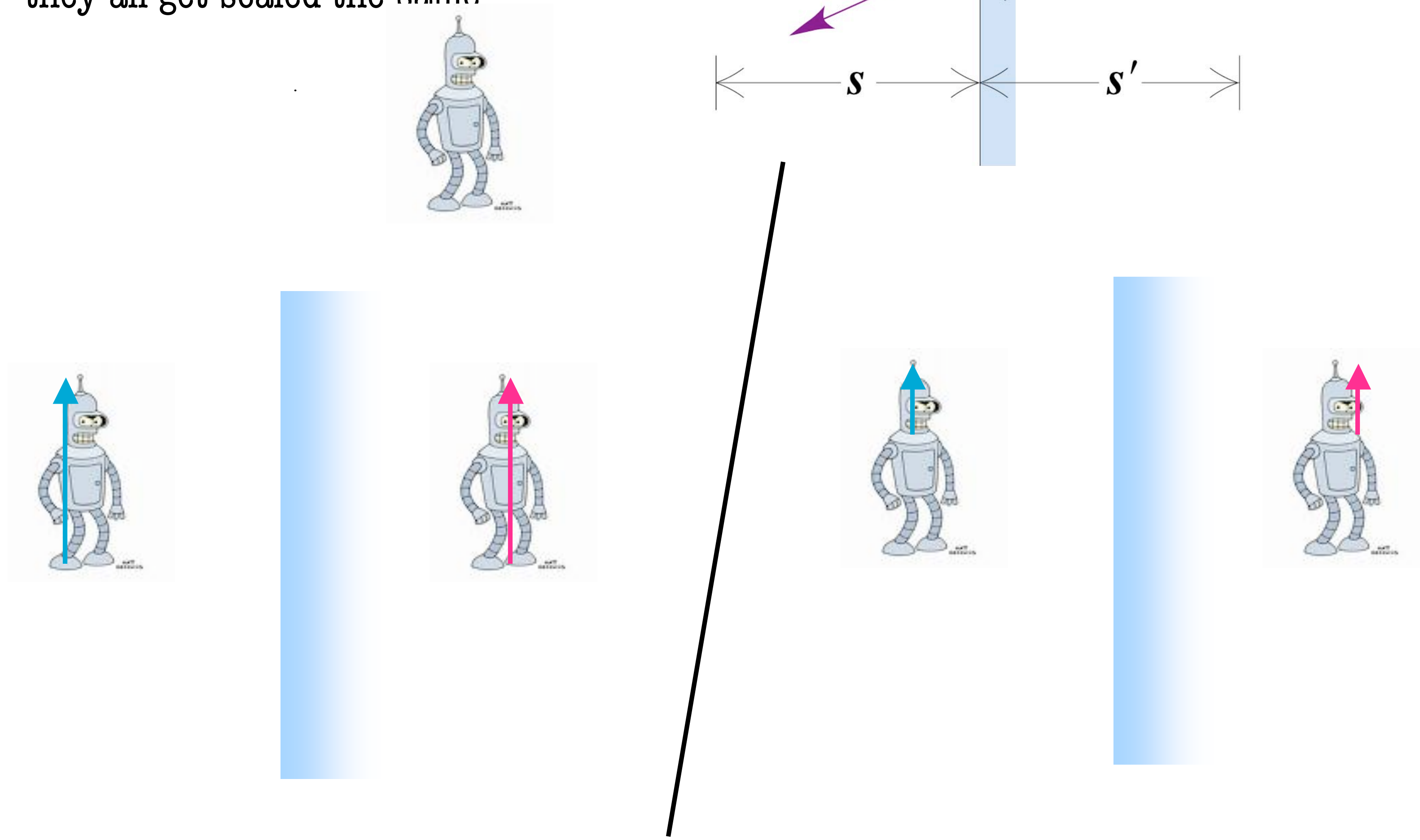


the arrows represent any two points in the object and the same two points in the image - they all get scaled the same

quantitative geometric optics

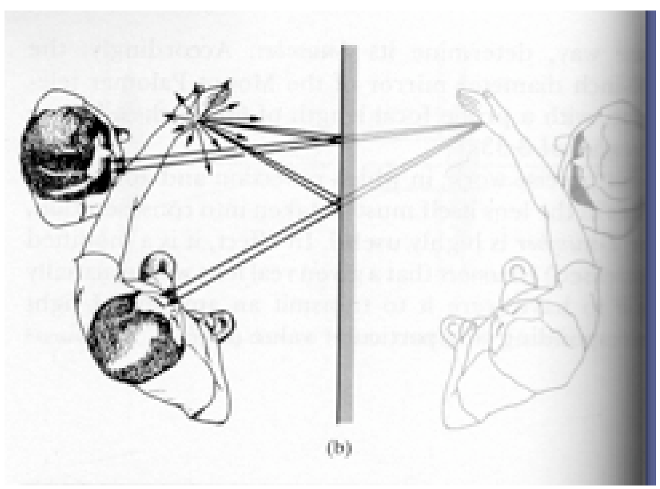
the arrows represent any two points in the object and the same two points in the image - they all get scaled the same

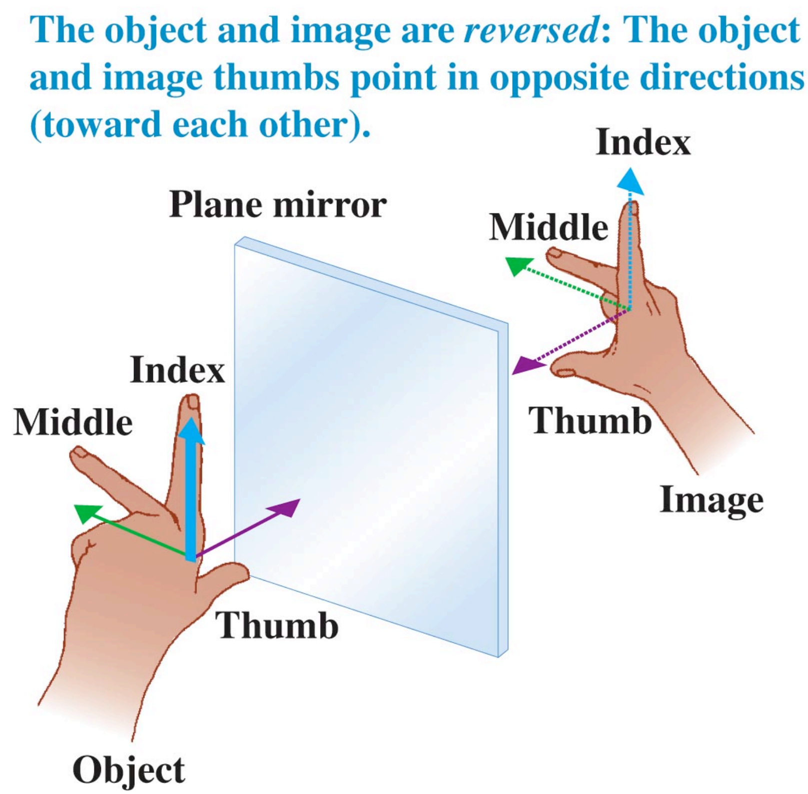




reversal of images

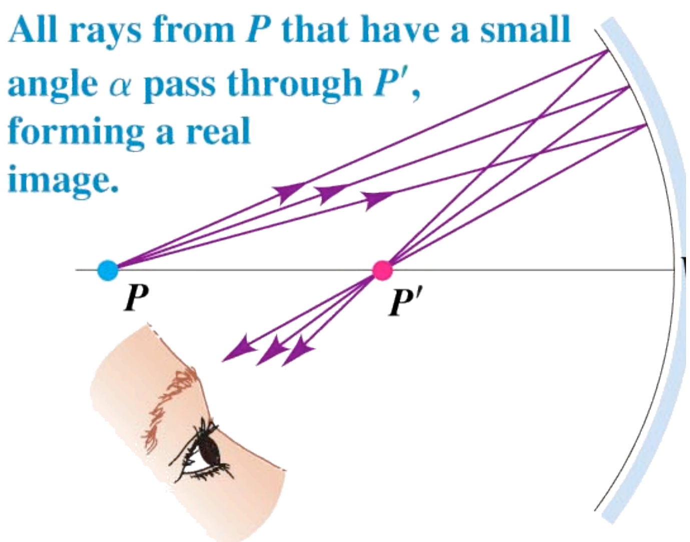
careful though, although the image in a plane mirror is the same size as the object it is not identical to it since it is *reversed*



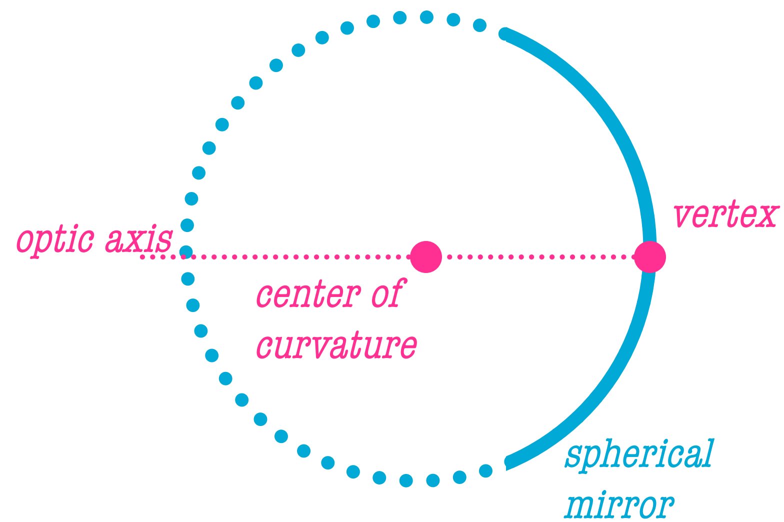


reflection at a spherical surface

- we're going to do something a little unusual here
  - first develop equations
  - then learn a simple graphical method
- you'll see though that this way round here is better



(b) The paraxial approximation, which for rays with small α.

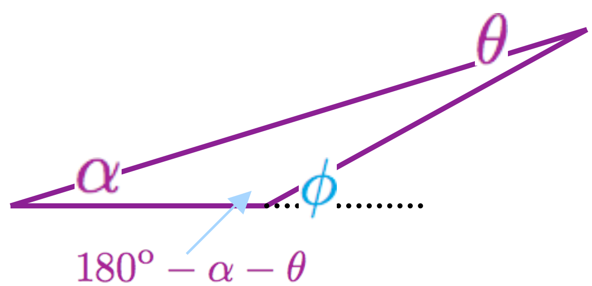


P' is a ***real*** image - **if we placed a screen here we'd see the image**

reflection at a spherical surface

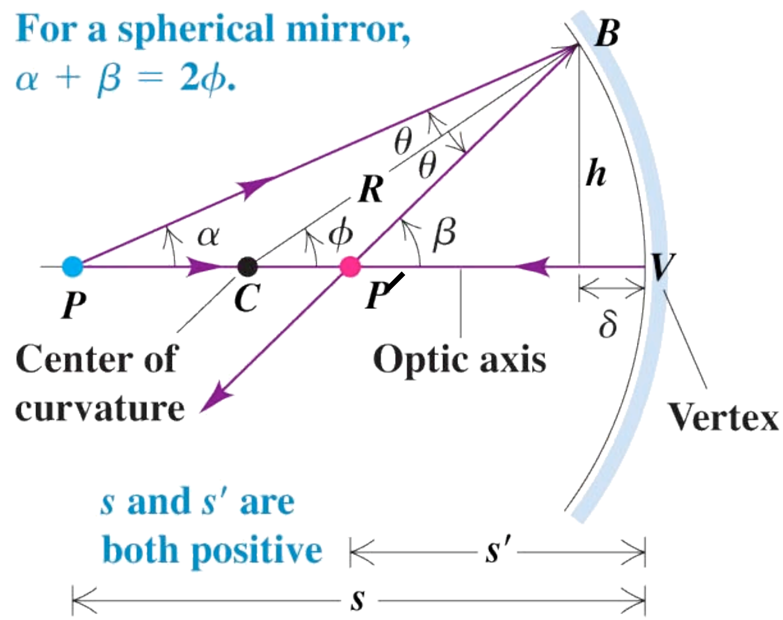
- just drawing a couple of cleverly chosen rays we can derive an equation linking the object distance and the image distance

$$\phi = \alpha + \theta$$



$$\beta = \phi + \theta$$

$$\alpha + \beta = 2\phi$$



(a) Construction for finding the position *P* of an image formed by a concave spherical mirror.

reflection at a spherical surface

- just drawing a couple of cleverly chosen rays we can derive an equation linking the object distance and the image distance

$$\alpha + \beta = 2\phi$$

$$\tan\alpha = \frac{h}{s-\delta}$$

$$\tan\beta = \frac{h}{s'-\delta}$$

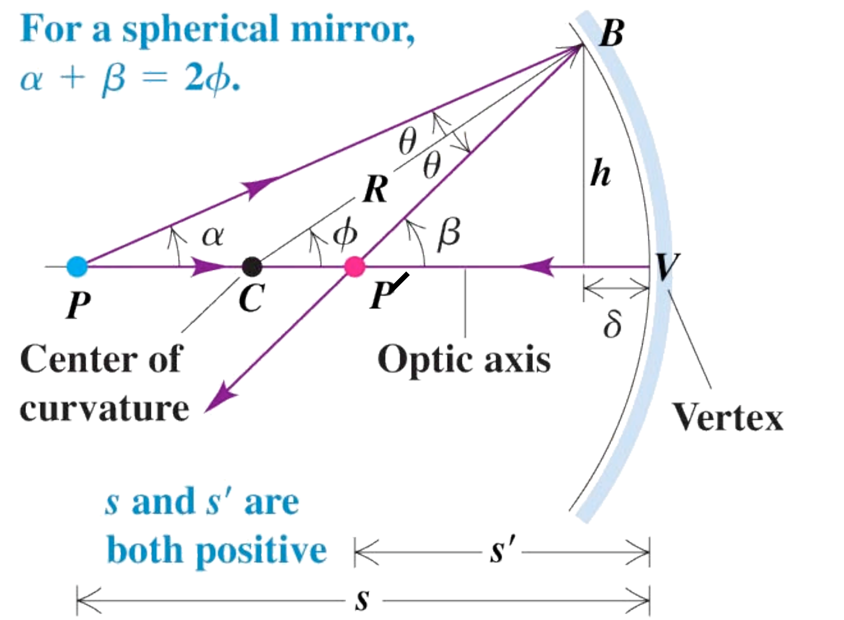
$$\tan\phi = \frac{h}{R-\delta}$$

"paraxial" approximation

*(basically, **small** angles)*

$$\tan\phi \approx \phi$$

also follows that δ is small



(a) Construction for finding the position P' of an image formed by a concave spherical mirror.

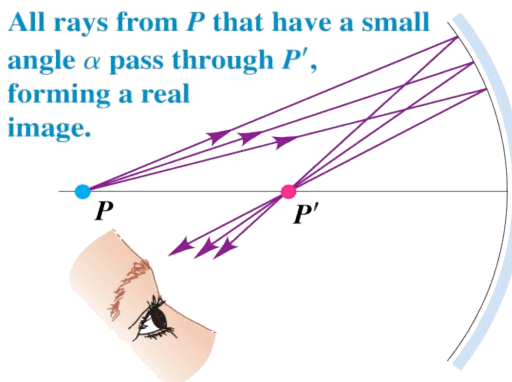
$$\phi \approx \frac{h}{R-\delta} \qquad \alpha \approx \frac{h}{s-\delta} \qquad \beta \approx \frac{h}{s'-\delta}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

reflection at a spherical surface

- notice that in the *"paraxial" approximation* the angle α does not appear, so **all** rays that make a small angle with the optic axis will intersect at *P'*

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$



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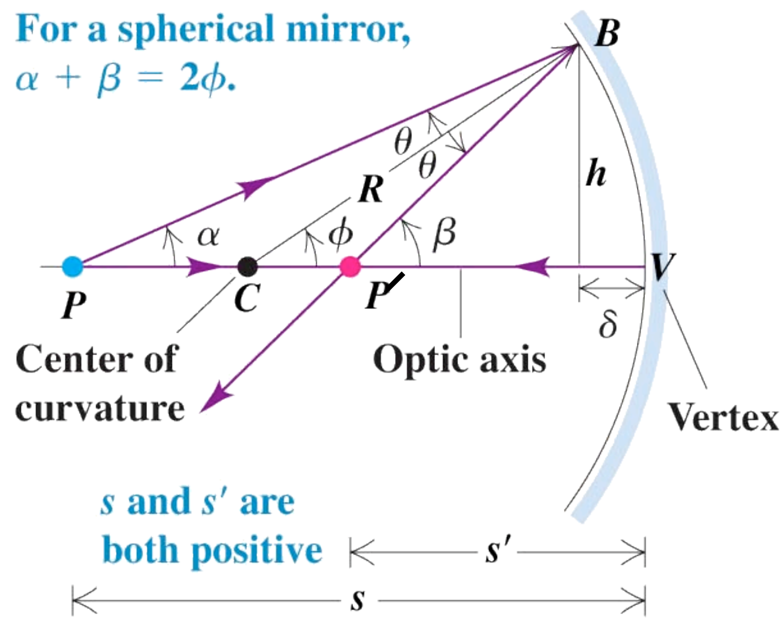
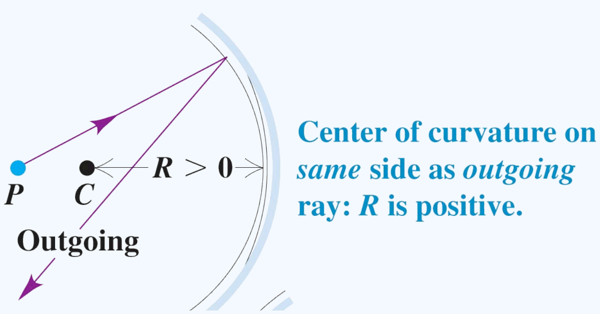
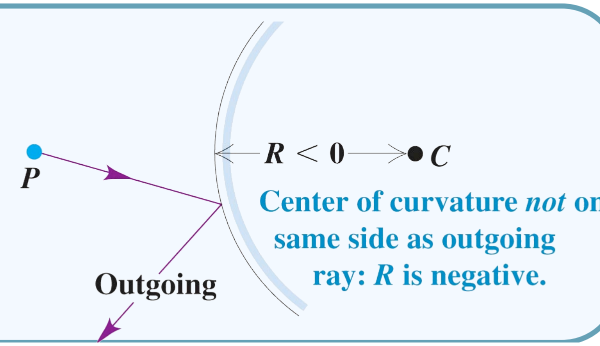


(a) Construction for finding the position P' of an image formed by a concave spherical mirror.

sign rule for center of curvature:





spherical mirrors in the paraxial approx

- from now on we'll always assume the angles are small enough that the equation

  $$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

  holds

- we can get a feel for the equation by looking at some special cases:
  - plane mirror $R \longrightarrow \infty$ $\quad s = -s'$

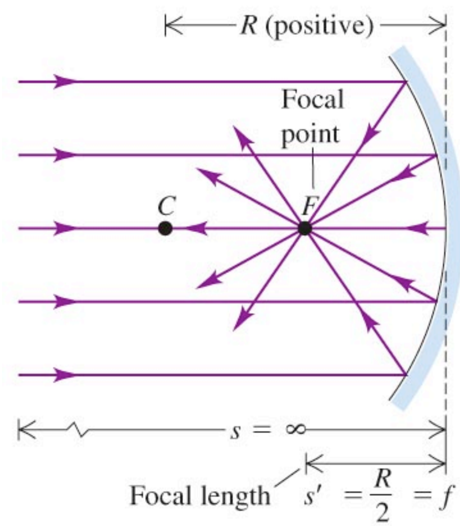
- what if the object point P is a long way away: $s \longrightarrow \infty$

- the incoming rays will all be parallel to the optic axis

$$s' = \frac{R}{2}$$

all rays focus to the same point!

focal point



(a) All parallel rays incident on a spherical mirror reflect through the focal point.

spherical mirrors in the paraxial approx

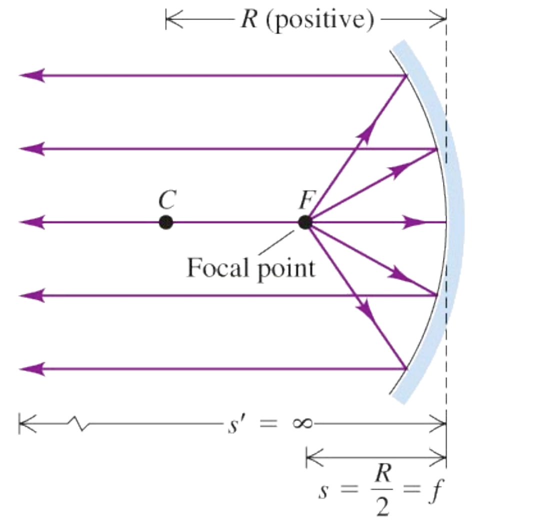
- imagine running this in reverse - put the object at the focal point
  - *then the reflected rays will all be parallel*

$$s = f = \frac{R}{2}$$

$$\frac{1}{s'} + \frac{2}{R} = \frac{2}{R}$$

$$\frac{1}{s'} = 0$$

$$s' \to \infty$$



(b) Rays diverging from the focal point reflect to form parallel outgoing rays.

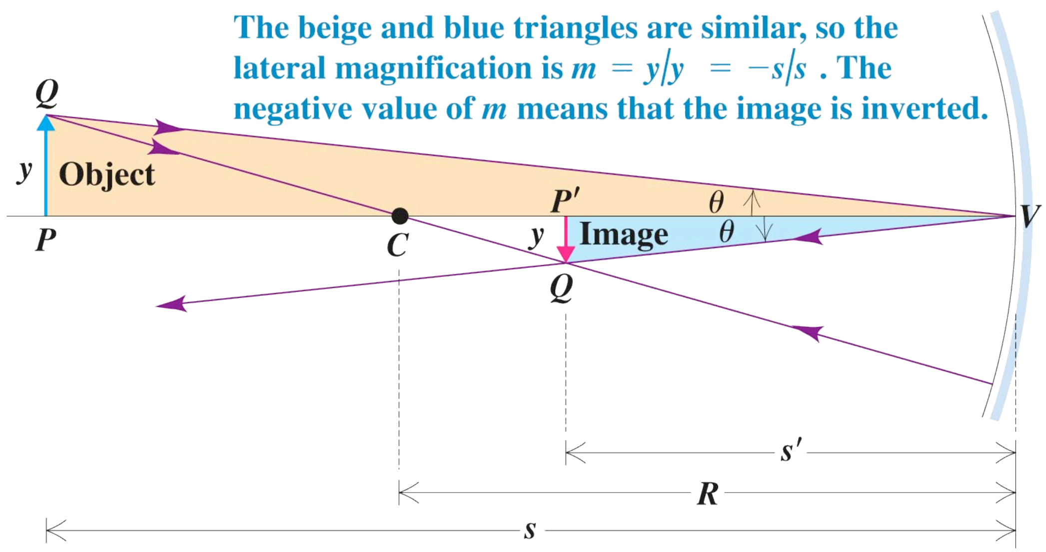
- ***properties of the focal point:***
  - ***any incoming ray parallel to the optic axis is reflected through the focal point***
  - ***any ray passing through the focal point is reflected parallel to the optic axis***

spherical mirrors in the paraxial approx

- we can write the mirror equation another way, featuring the focal length:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

- what about the size of objects?



$$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

negative value of m indicates the inversion of the image

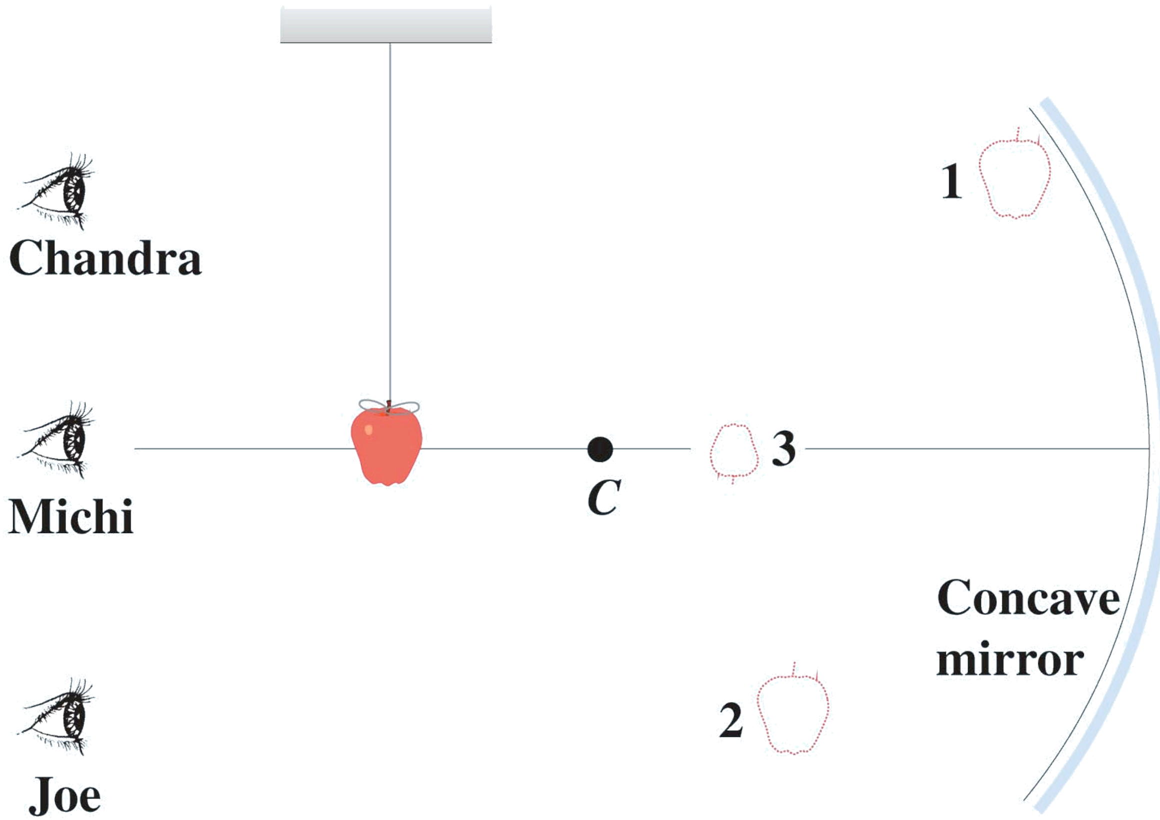
conceptual problem

- do all the observers see the image in the same position?

A. yes, at position 3

B. no, Chandra at 2, Michi at 3, Joe at 1

C. no, Chandra at 1, Michi at 3, Joe at 2



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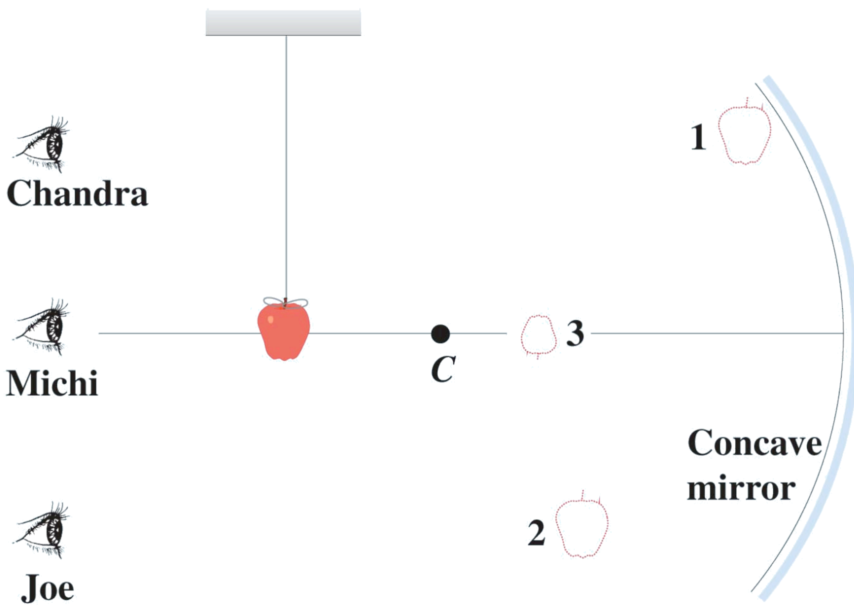
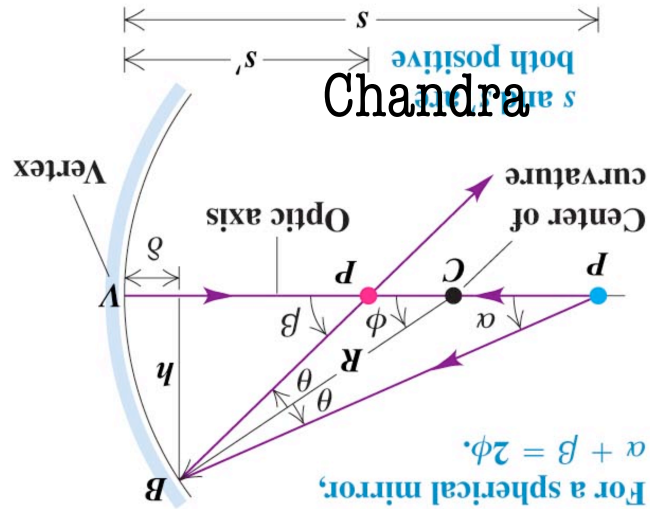
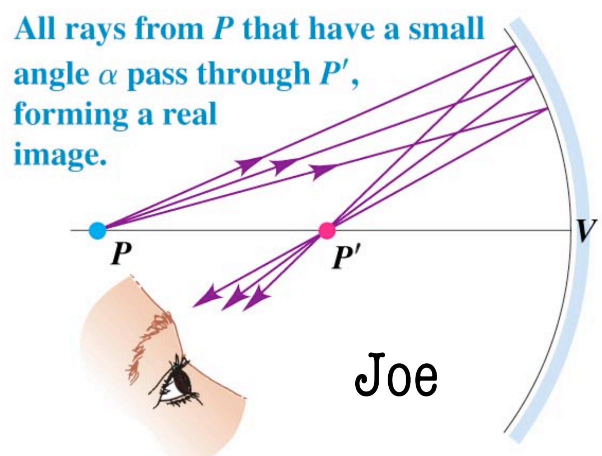


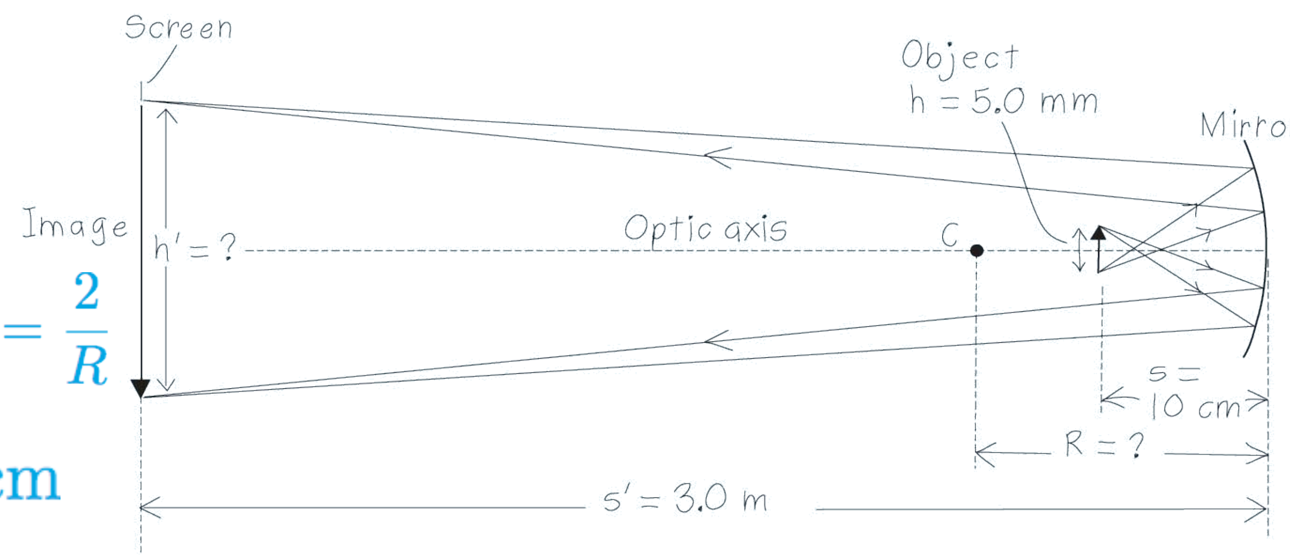




yes,
position 3

image from a concave mirror

- a lamp is placed 10 cm in front of a concave spherical mirror that forms an image of the filament on a screen placed 3.0 m from the mirror. Is this image real or virtual? What is the radius of curvature of the mirror? If the lamp filament is 5.0 mm high, how tall is its image? What is the lateral magnification?



$$\frac{1}{10\,\mathrm{cm}} + \frac{1}{300\,\mathrm{cm}} = \frac{2}{R}$$

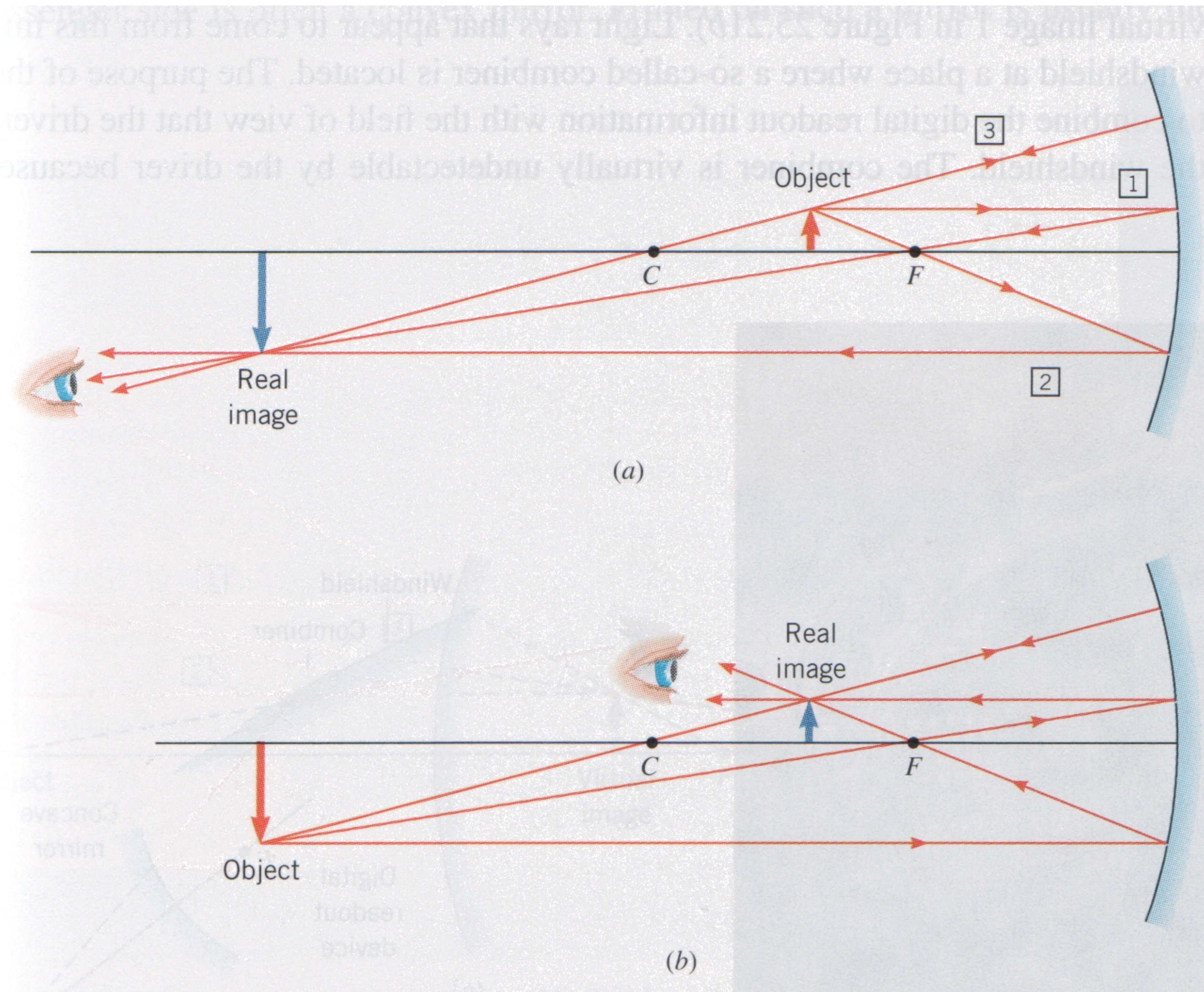
$$R = 19.4\,\mathrm{cm}$$

$$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

$$y' = -\frac{s'}{s}y = -\frac{300}{10}\,5\,\mathrm{mm} = -150\,\mathrm{mm}$$

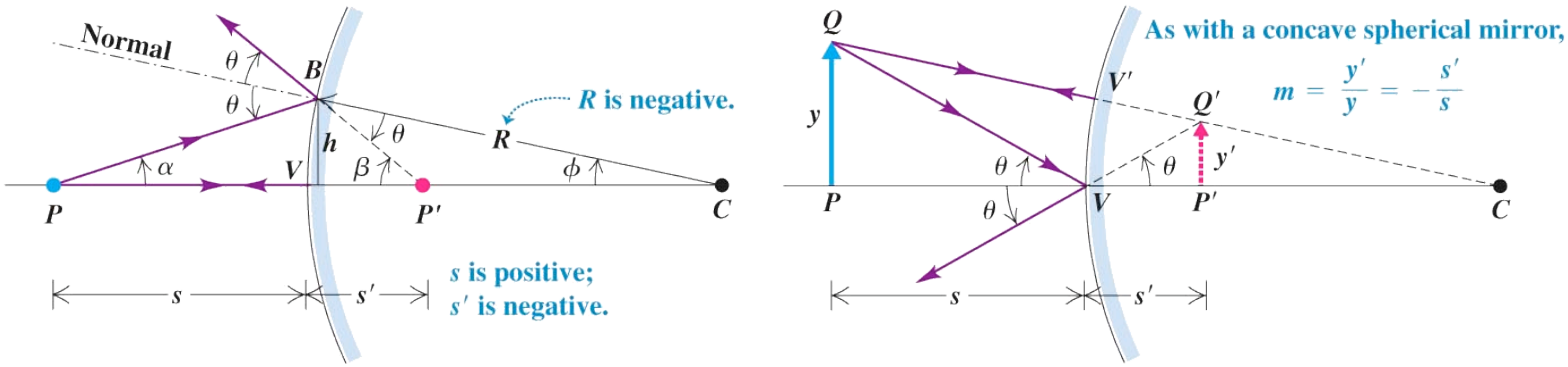
$$m = -30$$

reversibility in ray optics



convex spherical mirrors

- the mirrors we've looked at have been *concave* - we can also have *convex*



(a) Construction for finding the position of an image formed by a convex mirror.

(b) Construction for finding the magnification of an image formed by a convex mirror.

- geometric constructions are similar and lead to the same equations:

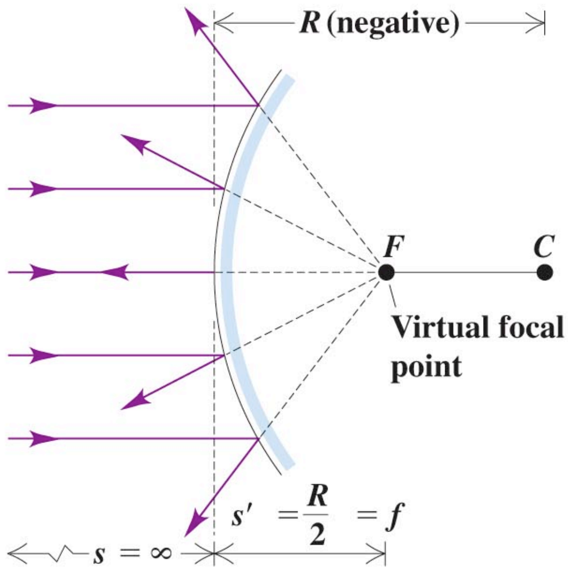
$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

$$m \equiv \frac{y'}{y} = -\frac{s'}{s}$$

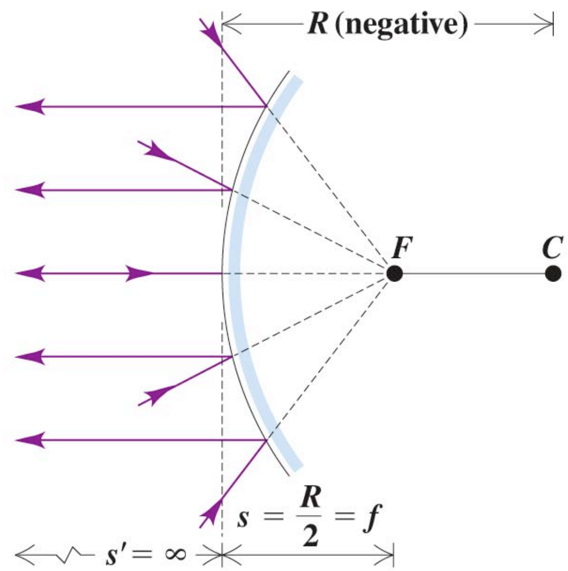
- to account for the convexity with outgoing wave to the right, R must be negative and hence s' is negative
  - *a virtual image*

focal point of convex spherical mirrors

- what happens if we have an object at infinity and hence parallel rays - they should go through a *focal point*



(a) Paraxial rays incident on a convex spherical mirror diverge from a virtual focal point.



(b) Rays aimed at the virtual focal point are parallel to the axis after reflection.

- actually have a *virtual focal point* in this case - treat the focal length f as being negative (just as R is negative here)

reading quiz

- the focal length of a diverging thin lens is
  - A. always positive
  - B. sometimes positive, sometimes negative
  - C. always negative

- a ray, initially parallel to the optic axis, approaching a converging thin lens will pass through
  - A. the focal point on the far side of the lens
  - B. the center of the lens
  - C. the vertex of the lens

graphical methods for mirrors

- the equations are very useful, but the sign conventions can be tricky, so a graphical method makes an excellent cross-check
- we've learnt about some special rays that we can call *principal rays*
  - *a ray parallel to the optic axis is reflected through the focal point of the mirror*
  - *a ray through the focal point is reflected parallel to the optic axis*
  - *a ray along the radius passing through the center of curvature is reflected back along the same line*
  - *a ray reflecting at the vertex is reflected forming an equal angle to its original direction*
- usually drawing any two of these rays describes the image position and size - drawing more checks our answer
  - *no tricky sign conventions here*

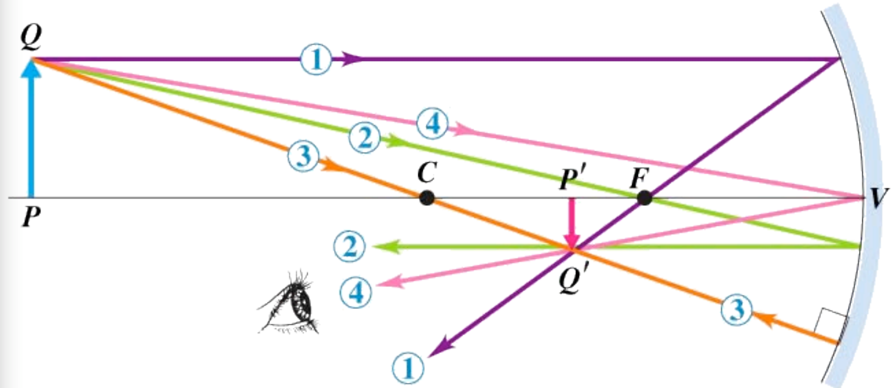
graphical methods for mirrors

① Ray parallel to axis reflects through focal point.

② Ray through focal point reflects parallel to axis.

③ Ray through center of curvature intersects the surface normally and reflects along its original path.

④ Ray to vertex reflects symmetrically around optic axis.



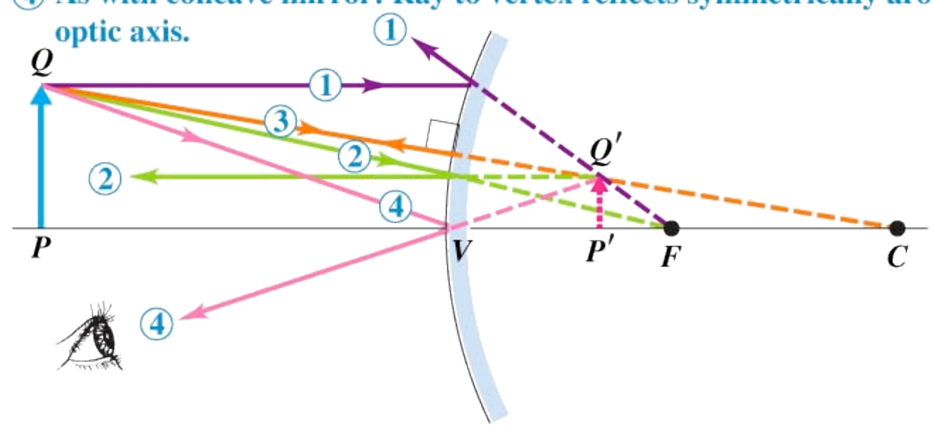
(a) Principal rays for concave mirror

① Reflected parallel ray appears to come from focal point.

② Ray toward focal point reflects parallel to axis.

③ As with concave mirror: Ray radial to center of curvature intersects the surface normally and reflects along its original path.

④ As with concave mirror: Ray to vertex reflects symmetrically around optic axis.



(b) Princpal rays for convex mirror

equations / graphical

- good practice when using the equations is to also draw a quick sketch of principal rays
  - *a concave mirror has a radius of curvature of absolute value 20cm. Find the details of the image in the cases that the object distance is 30cm, 20cm, 10cm, 5cm*

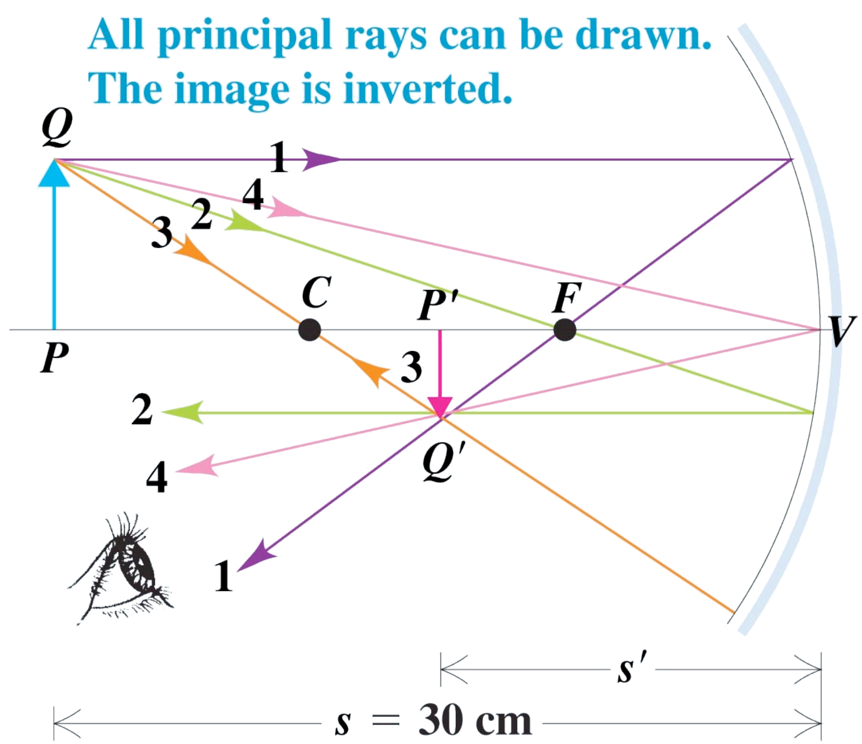
(a) s = 30 cm

$$\frac{1}{30\text{ cm}} + \frac{1}{s'} = \frac{2}{20\text{ cm}}$$

$$s' = 15\text{ cm}$$

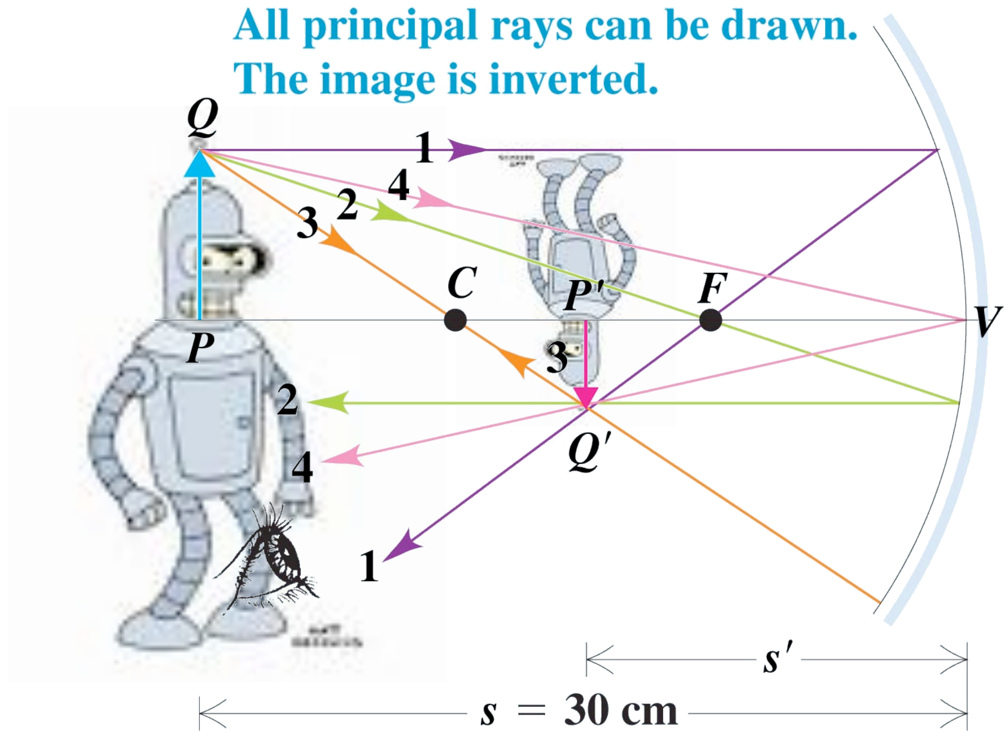
$$m = -\frac{s'}{s} = -\frac{1}{2}$$

real, inverted, reduced image



(a) Construction for *s* = 30 cm

equations / graphical



(a) Construction for *s* = 30 cm

equations / graphical

- good practice when using the equations is to also draw a quick sketch of principal rays
  - *a concave mirror has a radius of curvature of absolute value 20cm. Find the details of the image in the cases that the object distance is 30cm, 20cm, 10cm, 5cm*

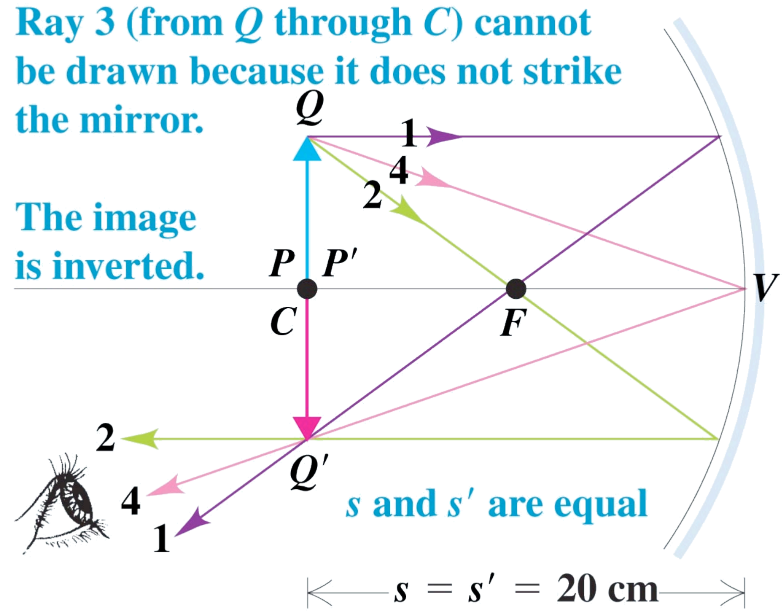
(b) s = 20 cm

$$\frac{1}{20\text{ cm}} + \frac{1}{s'} = \frac{2}{20\text{ cm}}$$

$$s' = 20\text{ cm}$$

$$m = -\frac{s'}{s} = -1$$

real, inverted, unmagnified image



(b) Construction for $s = 20$ cm

equations / graphical

- good practice when using the equations is to also draw a quick sketch of principal rays
  - *a concave mirror has a radius of curvature of absolute value 20cm. Find the details of the image in the cases that the object distance is 30cm, 20cm, 10cm, 5cm*

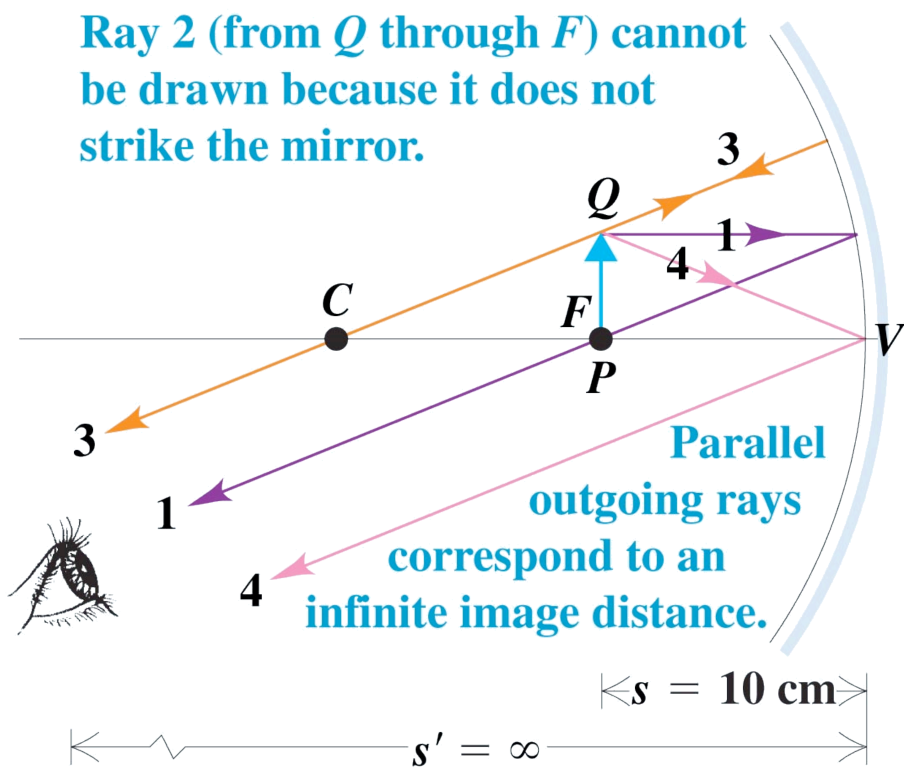
(c) s = 10 cm

$$\frac{1}{10\,\mathrm{cm}} + \frac{1}{s'} = \frac{2}{20\,\mathrm{cm}}$$

$$s' \to \pm\infty$$

$$m = -\frac{s'}{s} = \mp\infty$$

image will just be a blur



(c) Construction for $s = 10$ cm

equations / graphical

- good practice when using the equations is to also draw a quick sketch of principal rays
  - *a concave mirror has a radius of curvature of absolute value 20cm. Find the details of the image in the cases that the object distance is 30cm, 20cm, 10cm, 5cm*

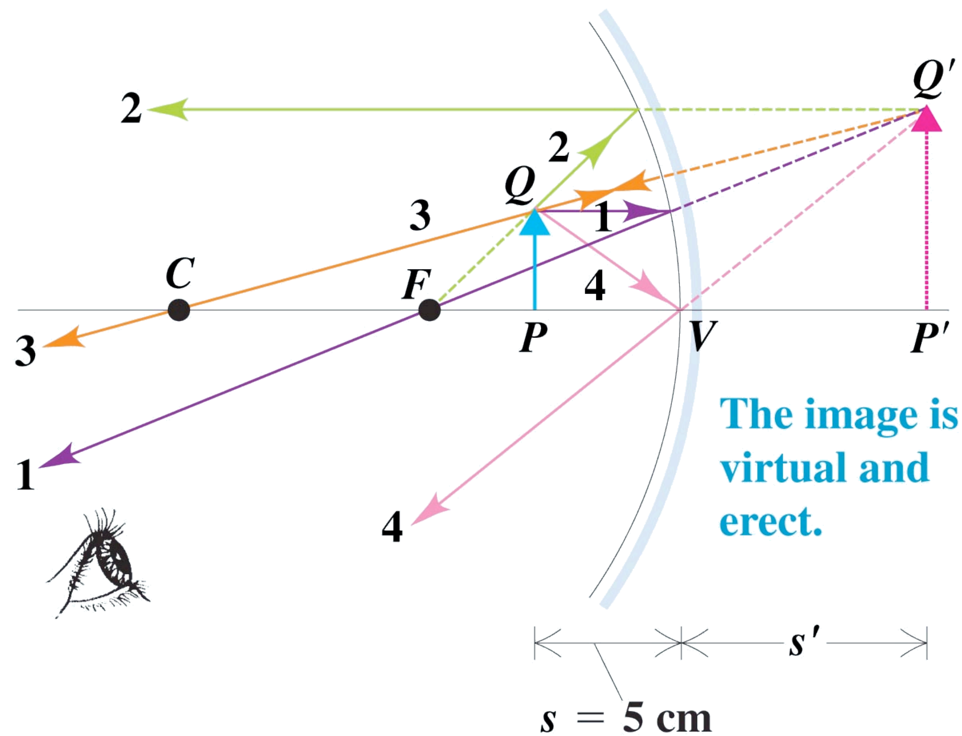
(d) s = 5 cm

$$\frac{1}{5\,\text{cm}} + \frac{1}{s'} = \frac{2}{20\,\text{cm}}$$

$$s' = -10\,\text{cm}$$

$$m = -\frac{s'}{s} = 2$$

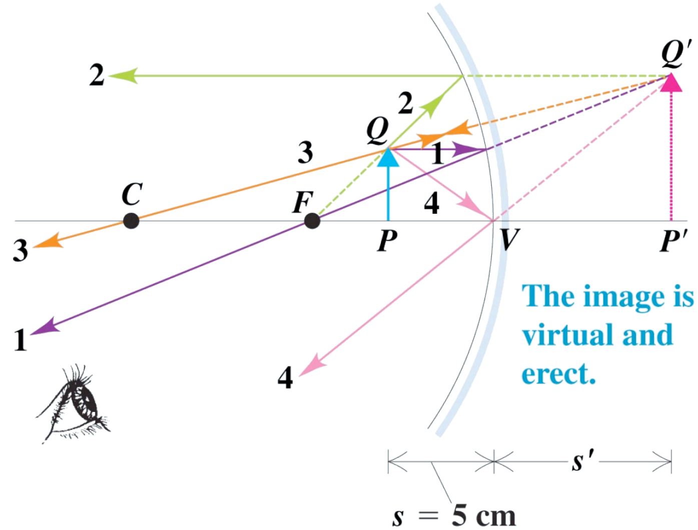
virtual, erect, magnified image



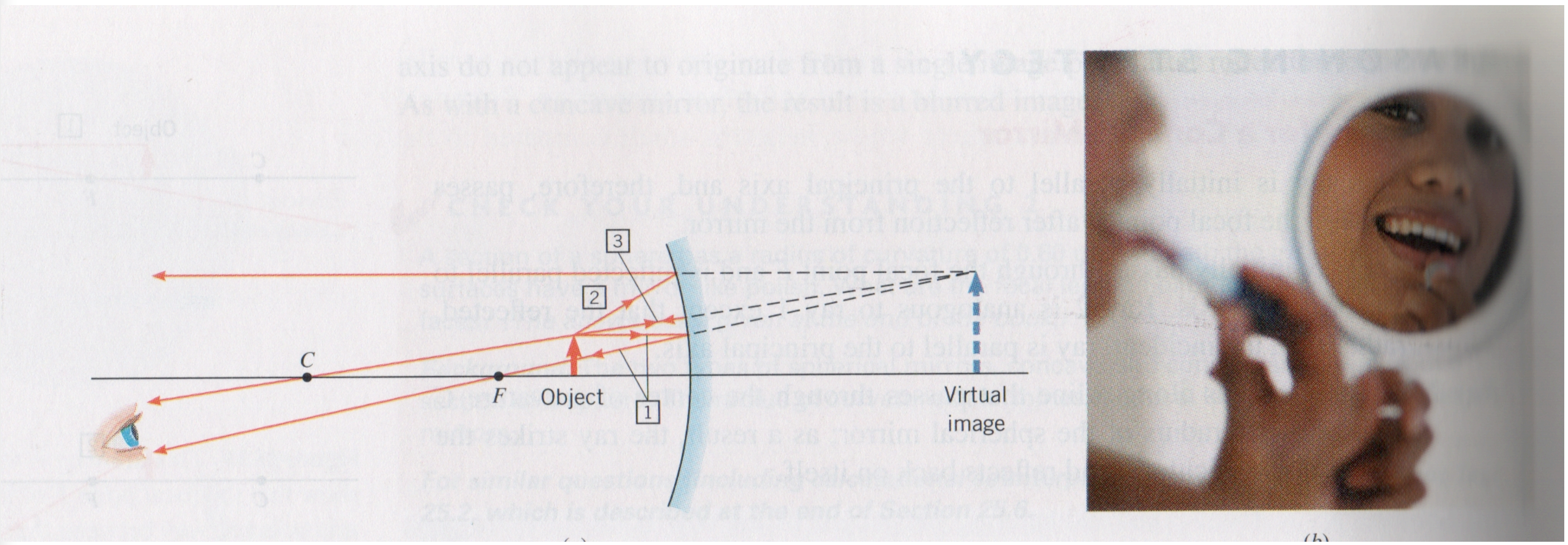
(d) Construction for $s = 5$ cm

equations / graphical

- virtual image



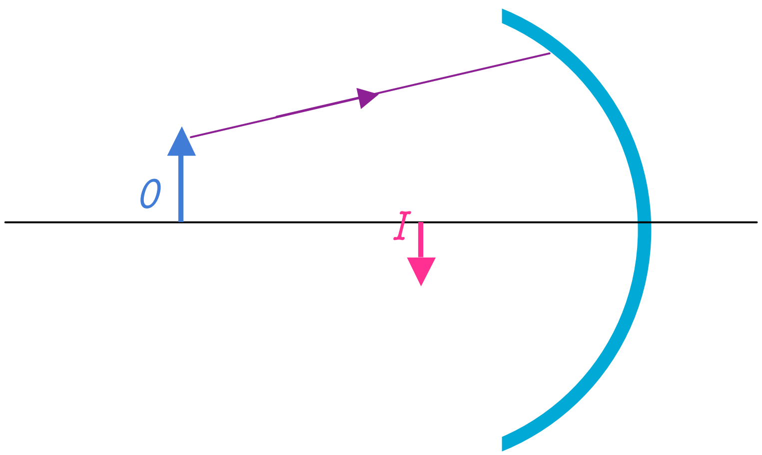
(d) Construction for $s = 5$ cm



other rays

- don't forget that the principal rays just happen to be handy ones
- there are as many rays as you want to draw

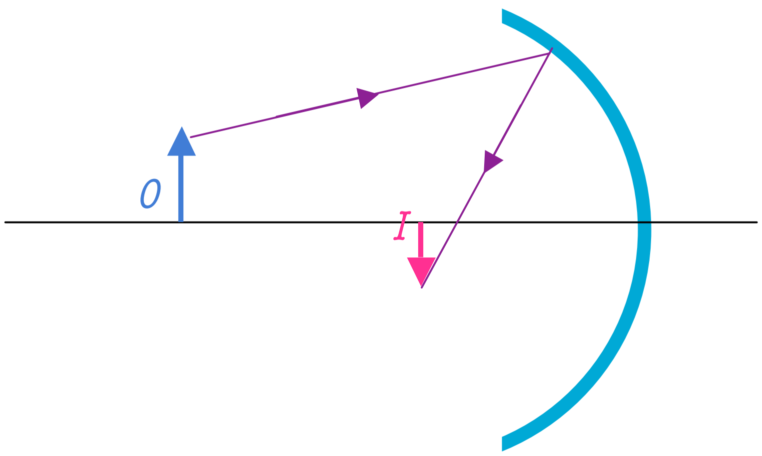
- e.g. we've determined the following image is formed from the object shown



- complete the path of the ray started

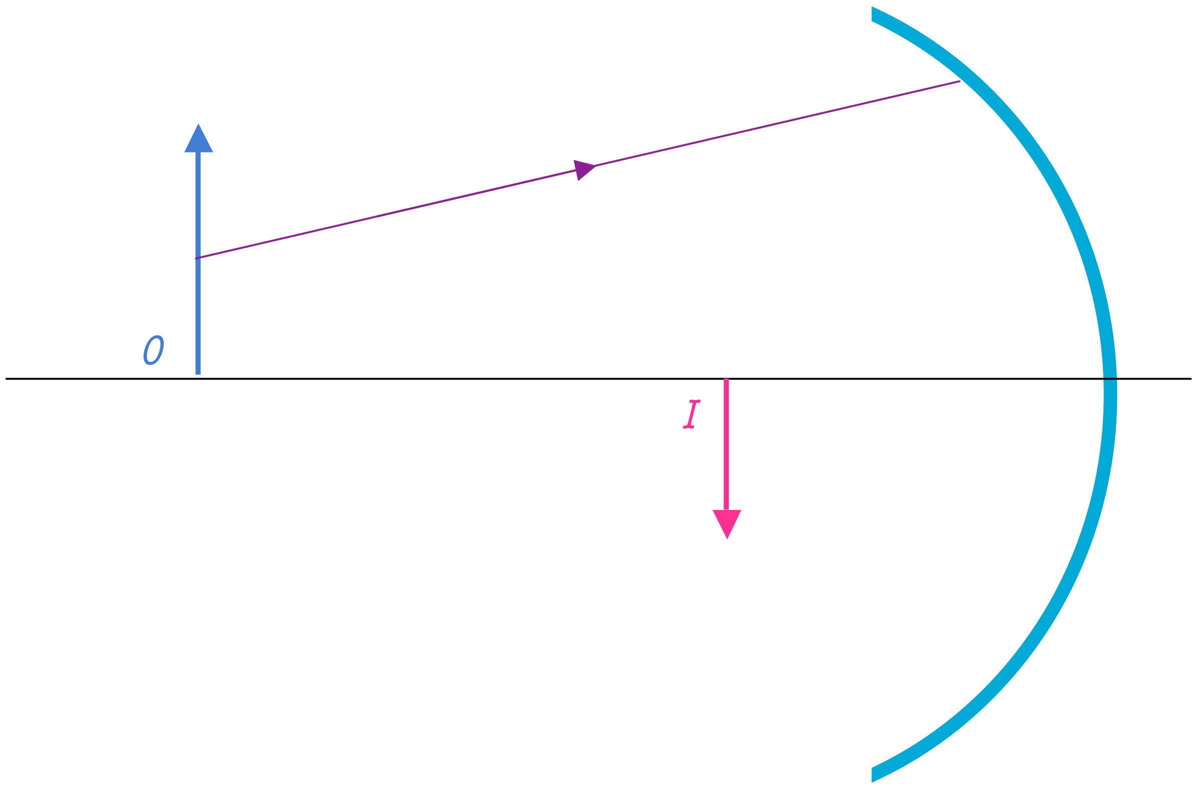
other rays

- even though it's not a principal ray, we know where it has to end up
- on the equivalent point on the image
- or else it isn't an image of the object!



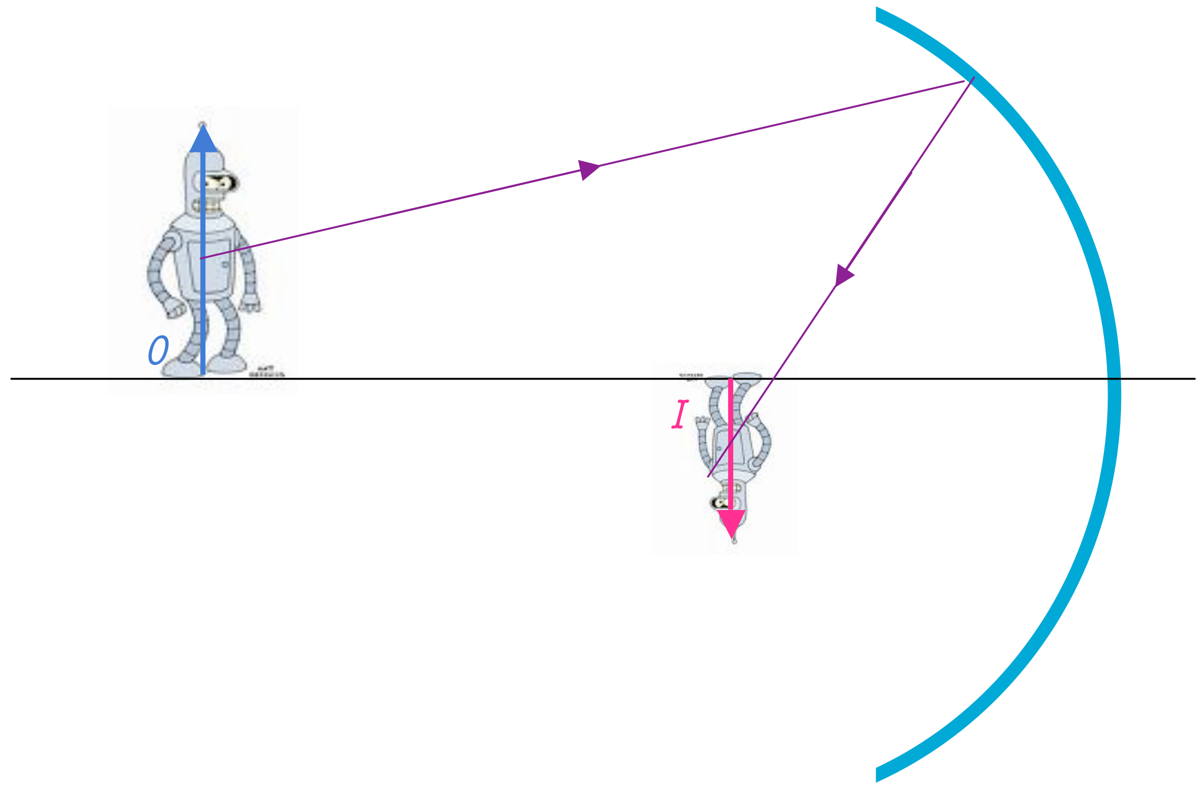
other rays

- where does this ray end up?



other rays

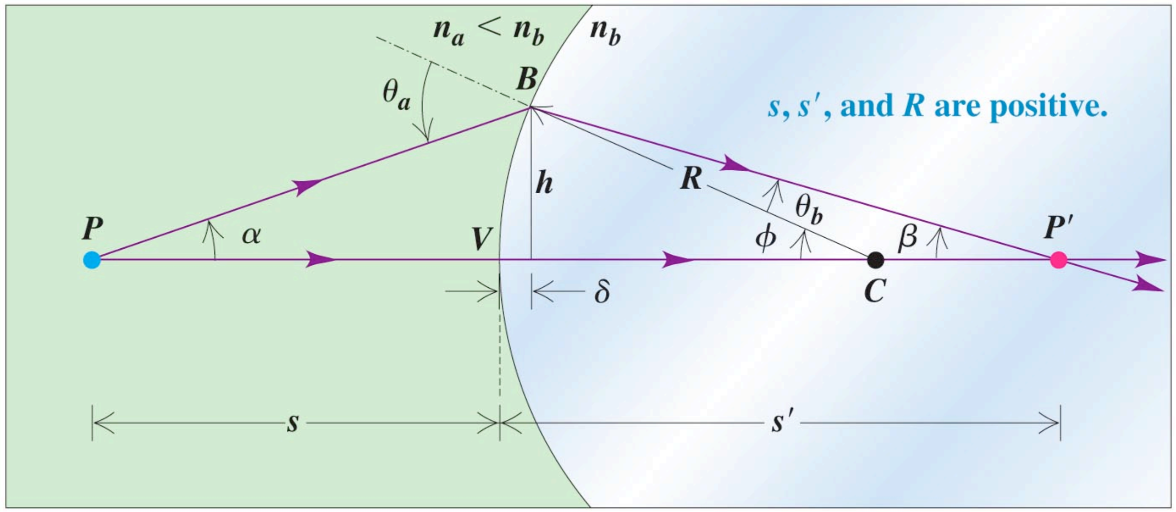
- where does this ray end up?



refraction at a spherical surface

- rather similar to reflection - derive a formula starting from Snell's law

$$n_a \sin\theta_a = n_b \sin\theta_b$$



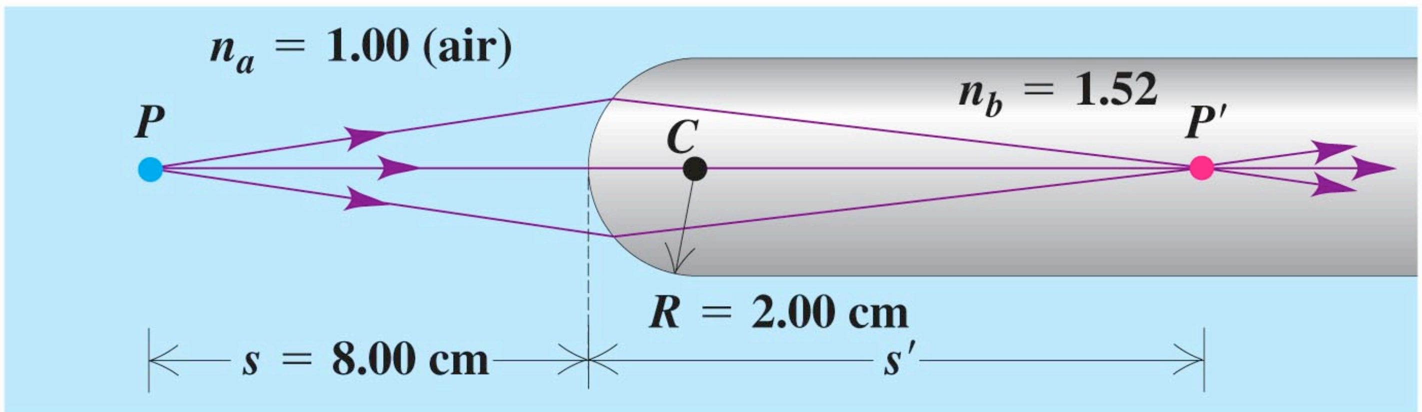
- skipping the details we find $$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

- again this is in the *paraxial approximation* and holds for all small angles α

- lateral magnification $$m \equiv \frac{y'}{y} = -\frac{n_a\, s'}{n_b\, s}$$

glass rod in air

- cylinder of glass (n = 1.52) has one end ground into a hemispherical surface with radius 2.00cm. The rod is surrounded by air.
  - (a) find the image distance of a small object on the axis of the rod and 8.00cm to the left of the vertex
  - (b) find the lateral magnification



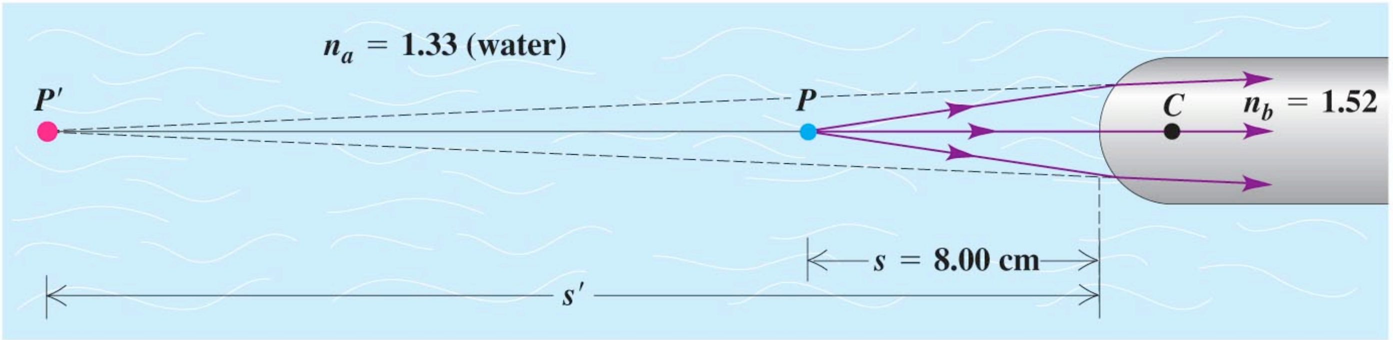
$$\frac{1.00}{8.00\,\text{cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.00}{2.00\,\text{cm}} \qquad s' = 11.3\,\text{cm}$$

$$m = -\frac{1.00 \times 11.3}{1.52 \times 8.0} = -0.929$$

inverted

glass rod in water

- same rod in water (n = 1.33)



$$\frac{1.33}{8.00\,\text{cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.33}{2.00\,\text{cm}}$$

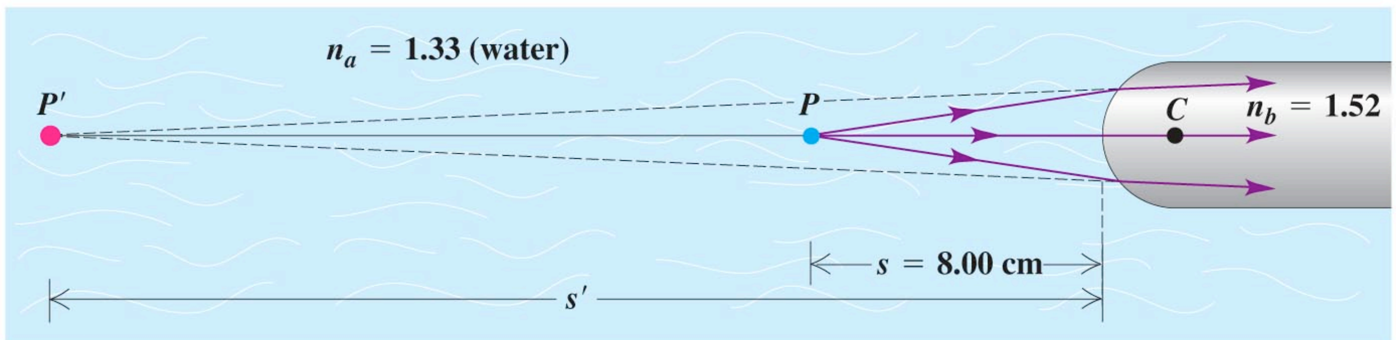
$$s' = -21.3\,\text{cm}$$

$$m = -\frac{1.33 \times (-21.3)}{1.52 \times 8.0} = +2.33$$

upright

reminder of the sign convention for R

- R positive when center of curvature on outgoing ray side
- R negative otherwise



$R > 0$

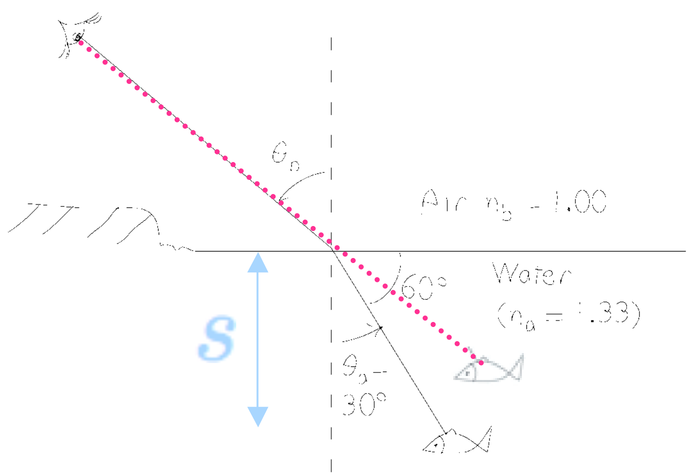
plane refractive surface

- corresponds to $R \to \infty$

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0$$

$$\frac{s'}{s} = -\frac{n_b}{n_a} \qquad m = 1$$

back to the fishpond

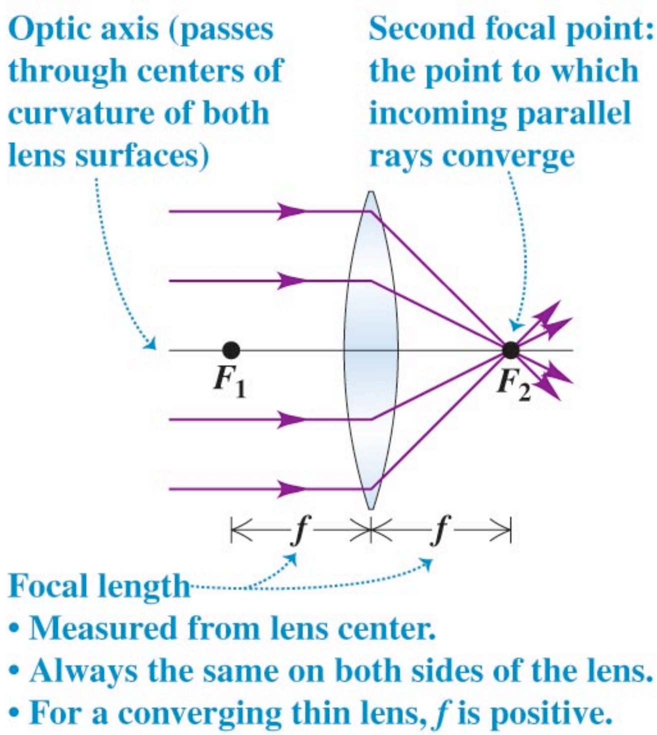


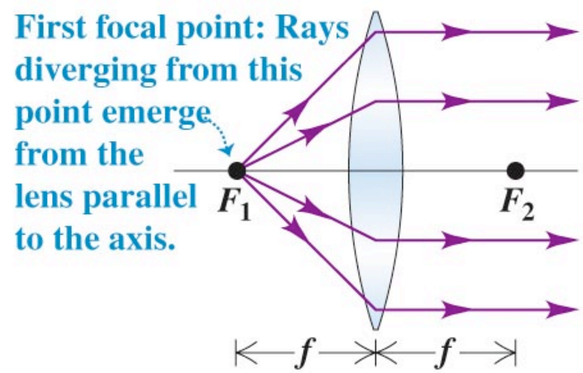
$$|s'| = 0.752s$$

$$\frac{s'}{s} = -0.752$$ *virtual image*

‘thin’ lenses

- these are the classic optical device - used *very* widely (how many of you wear glasses or contacts?)
- we should learn about their properties
  - two spherical surfaces close enough together that we can neglect the distance between them



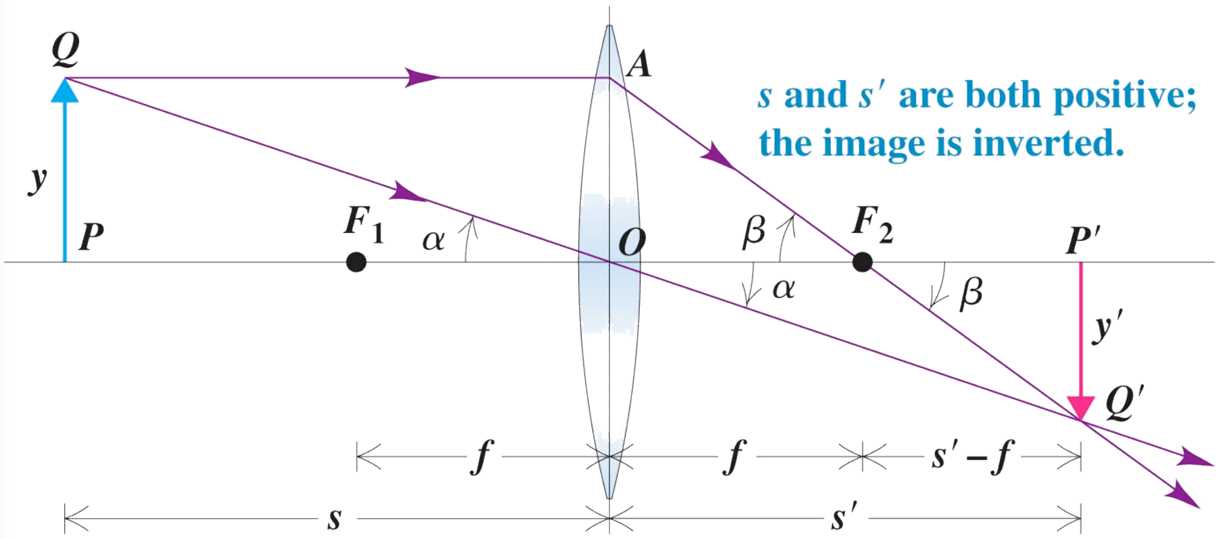


conventional to draw the rays as bending at the midline of the lens

(actually refracted at both surfaces)

image formation by a ‘thin’ *converging* lens

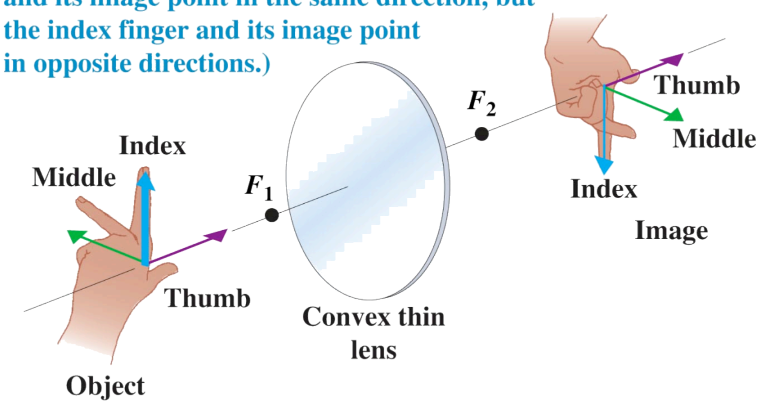
- using just the principal rays through the focal points or the lens center we can find the properties of the image



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

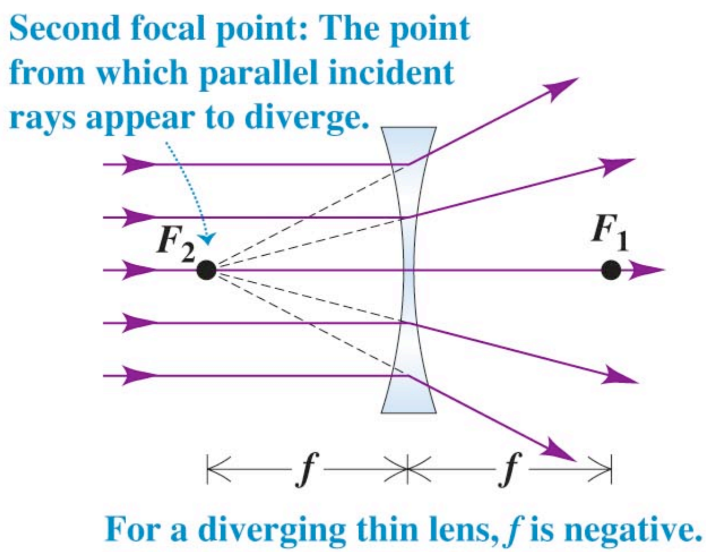
$$m = -\frac{s'}{s}$$

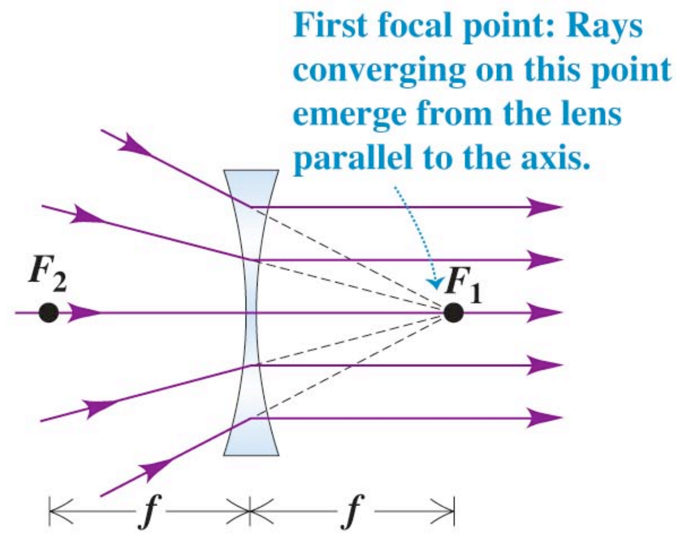
The object and image are inverted, but not reversed. (The thumb and its image point in the same direction, but the index finger and its image point in opposite directions.)



a ‘thin’ *diverging* lens

- this is another common lens type

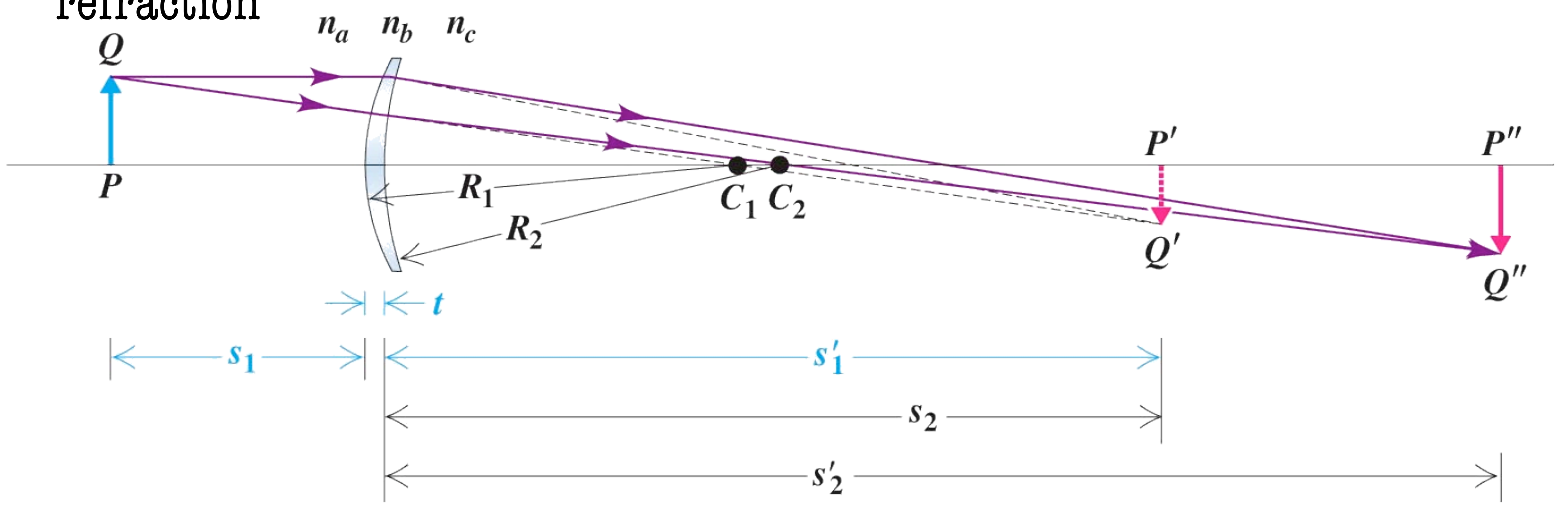


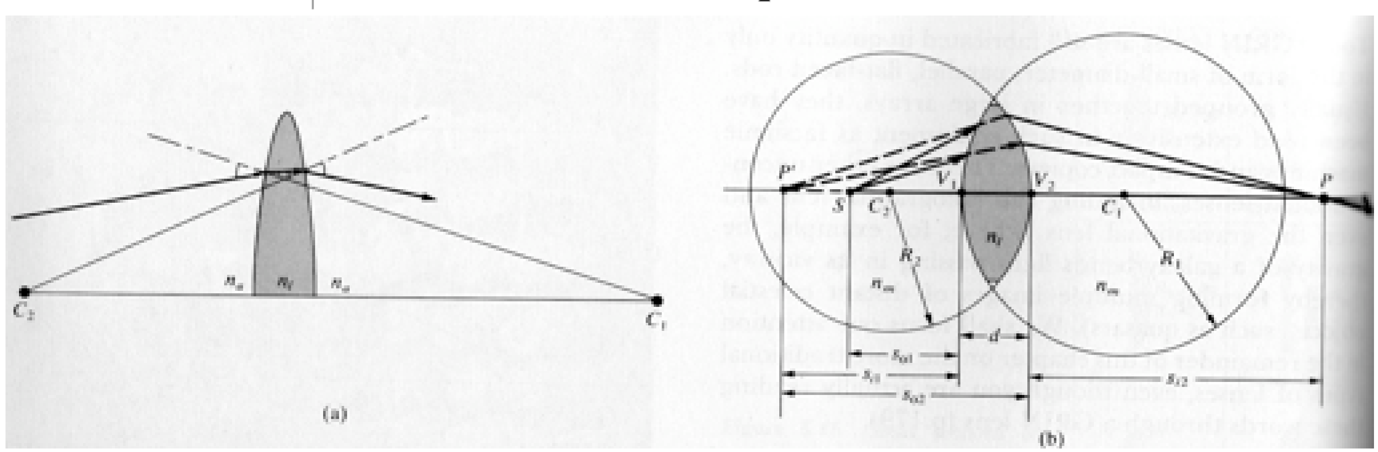


$$f < 0$$

the ‘thin’ lens equation

- using the equations of refraction at spherical surfaces we can derive an equation which will link the focal length to the properties of the lens
- this is our first example of *multiple optics*
- we use the image from the first refraction as an object for the second refraction





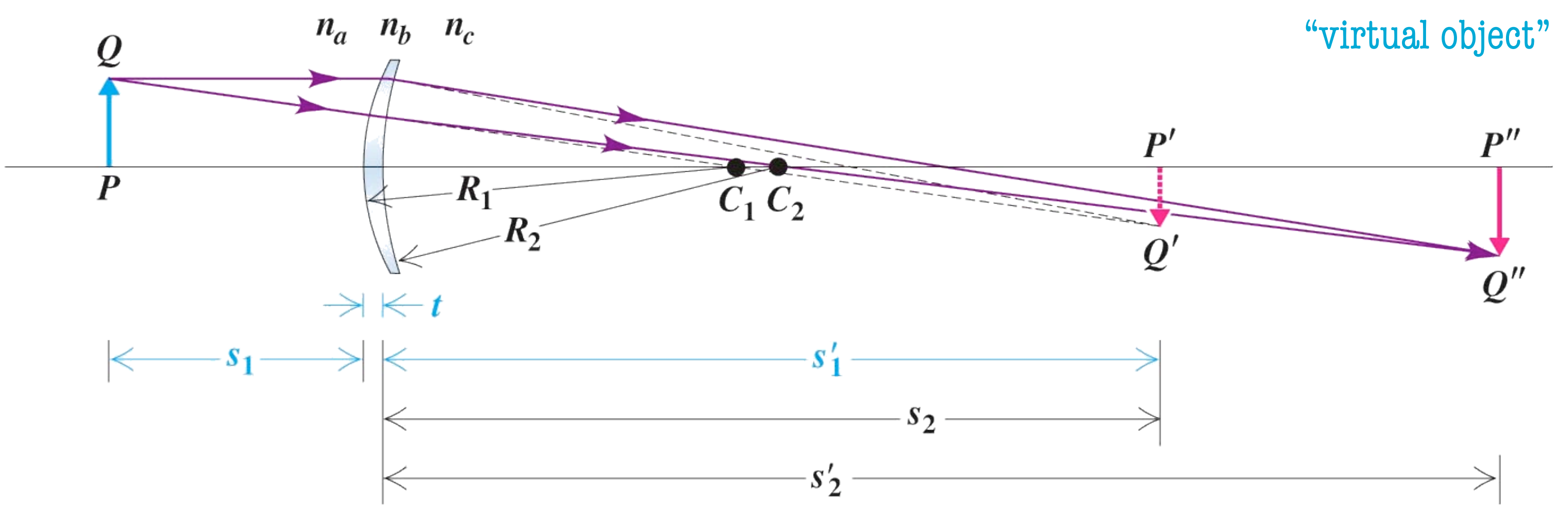
the 'thin' lens equation

- applying the 'refraction at a spherical surface' equation for the a-b surface: $\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}$
- and at the b-c surface: $\frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2}$
- if the lens is surrounded by air then $n_a = n_c = 1$
- for a thin lens, the distance between surfaces can be neglected and then the first image position is the object position for the second refraction:

$$s_2 = -s_1'$$

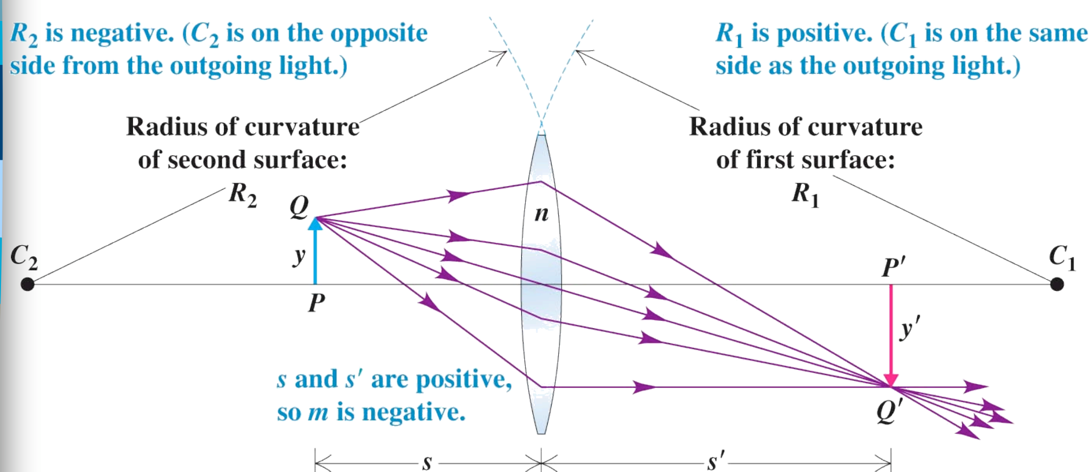
• note that the object distance is negative - the incoming rays are on the opposite side than the object

"virtual object"



the 'thin' lens equation

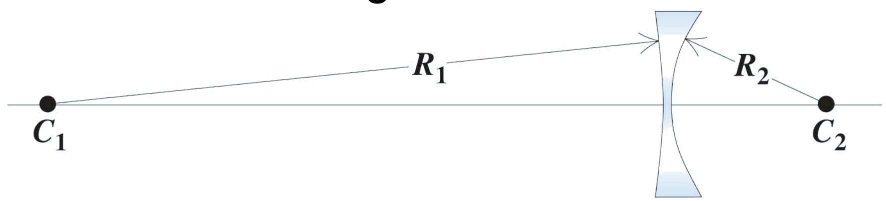
- in this simplified case: $$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1} \qquad -\frac{n}{s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$
- $\frac{n}{s_1'}$ is common to both equations so we can eliminate it $$\frac{1}{s_1} + \frac{1}{s_2'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
- if we consider the lens to be a single optical unit (as before) then $$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

double concave diverging lens

- the two surfaces of the lens shown below have radii of curvature with absolute values of 20cm and 5.0cm respectively. The index of refraction of the lens material is 1.52. What is the focal length of the lens?



  - first surface has center of curvature on opposite side as outgoing light
    then $R_1 = -20\,\text{cm}$
  - second surface has center of curvature on same side as outgoing light
    then $R_2 = 5\,\text{cm}$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.52-1)\left(\frac{1}{-20\,\text{cm}} - \frac{1}{5\,\text{cm}}\right)$$

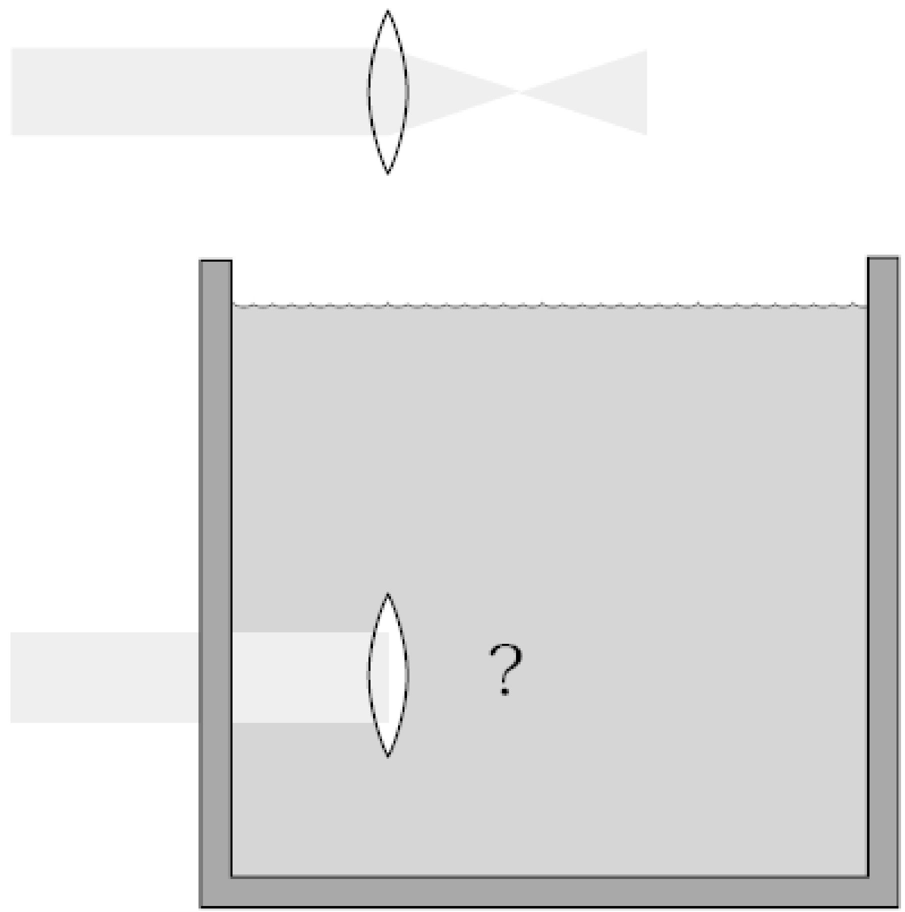
$$f = -7.7\,\text{cm}$$

diverging lenses have negative focal lengths

A parallel beam of light is sent through an aquarium. If a convex glass lens is held in the water, it focuses the beam

A. closer to the lens than
B. at the same position as
C. farther from the lens than

outside the water.

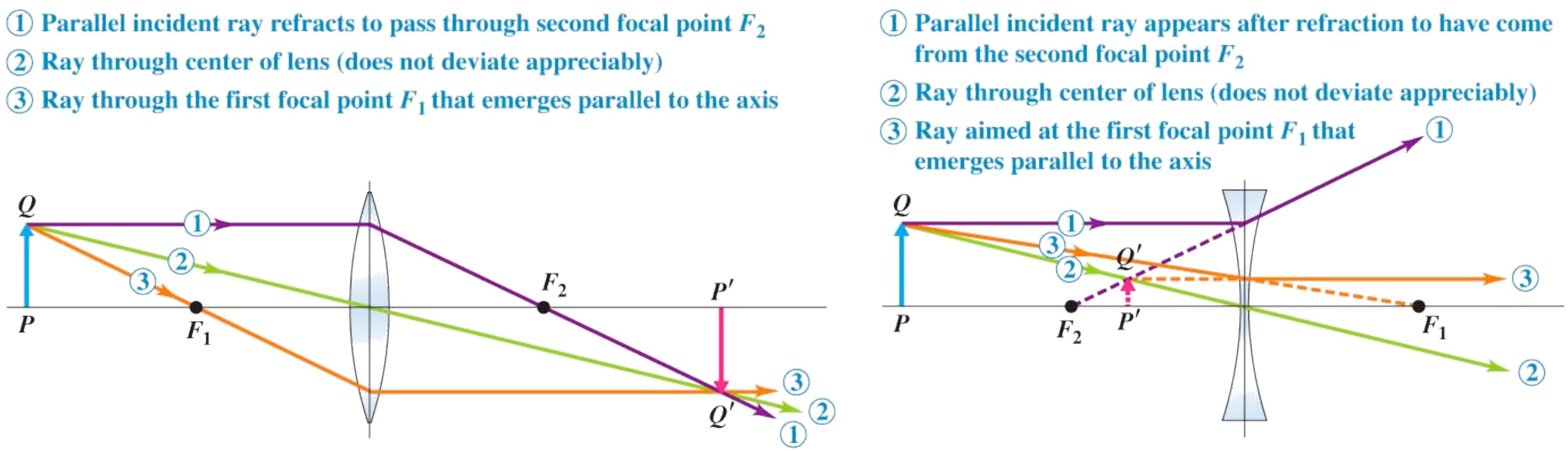


hint: more, less, the same bending between

glass-air, glass-water ?

graphical method

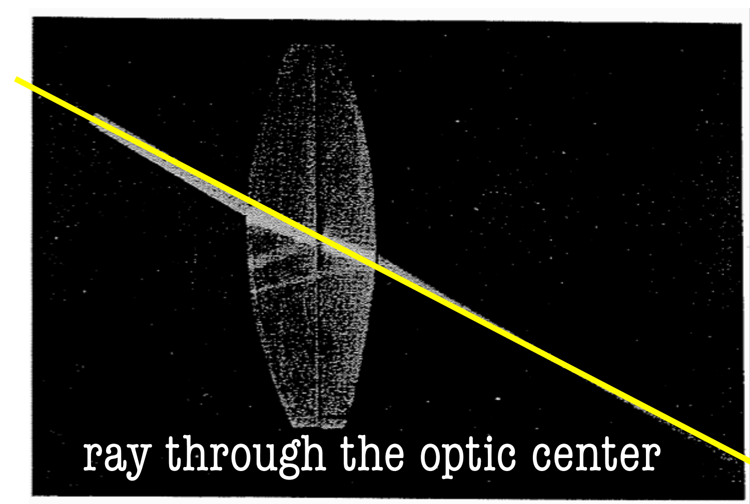
- as with reflection we can develop a graphical method based upon the properties of special or *principal rays*



(a) Converging lens

(b) Diverging lens

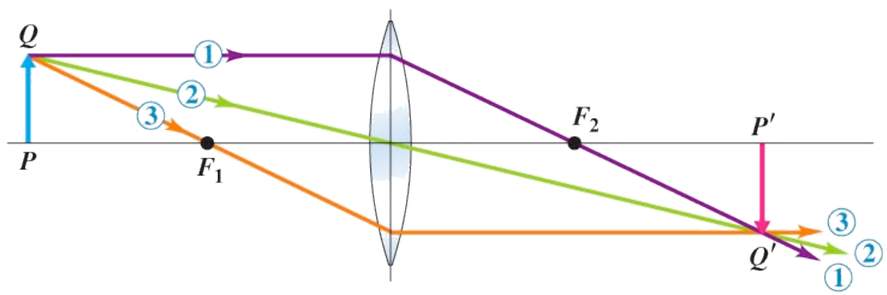
- we draw the ray as if the 'bending' occurs at the midpoint of the lens - this is fine for 'thin' lenses and we'll only ever consider thin lenses



graphical method

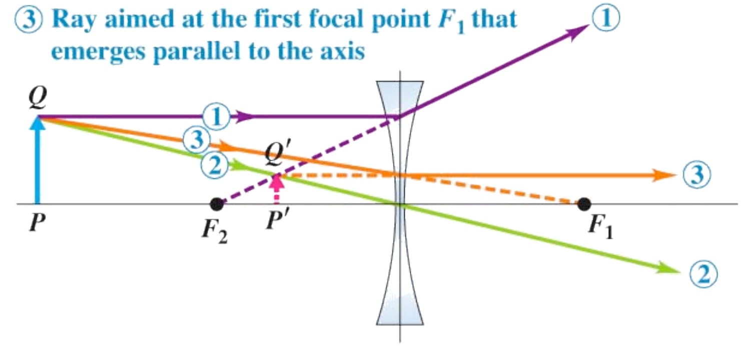
- what does this really look like?

① Parallel incident ray refracts to pass through second focal point F_2
② Ray through center of lens (does not deviate appreciably)
③ Ray through the first focal point F_1 that emerges parallel to the axis

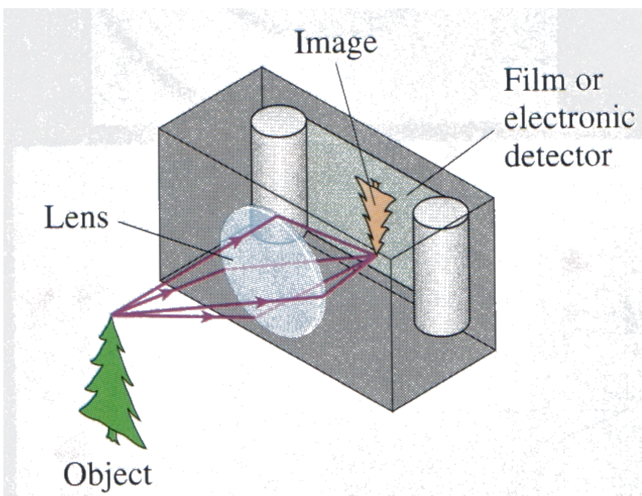


(a) Converging lens

① Parallel incident ray appears after refraction to have come from the second focal point F_2
② Ray through center of lens (does not deviate appreciably)
③ Ray aimed at the first focal point F_1 that emerges parallel to the axis



(b) Diverging lens



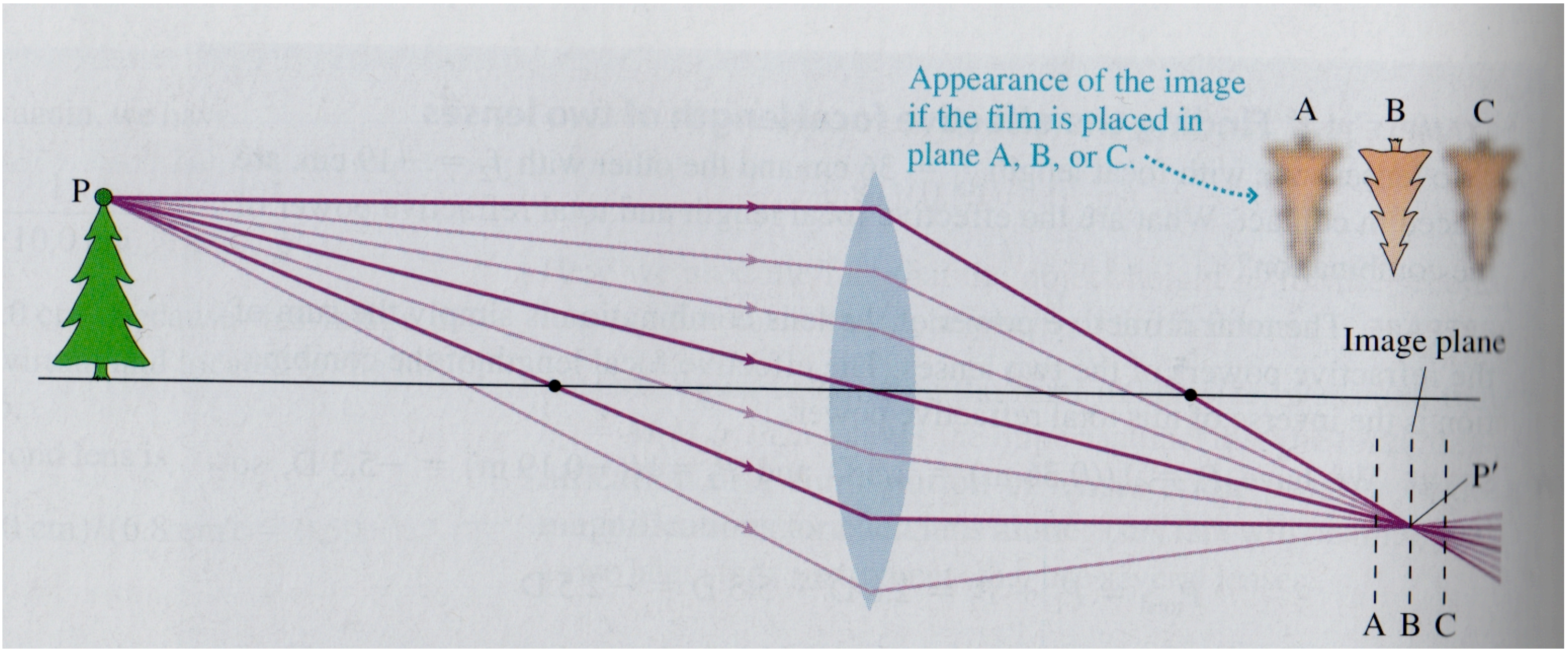
real image from a converging lens,

(reflected to change the direction)

virtual, reduced upright image through a diverging lens

graphical method

- what does it look like if we place a screen or film away from the image point?



- *the image is not focused.*

quiz

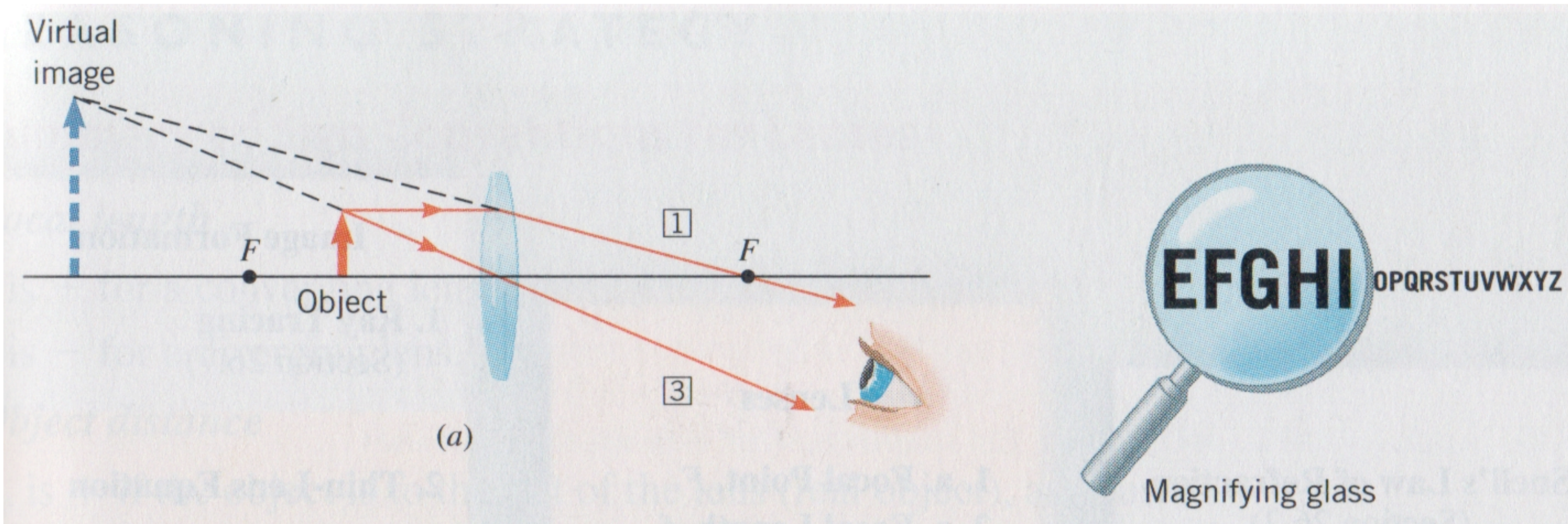
draw a diagram or solve the equation(s)

- an object is placed one and a half focal lengths away from a converging lens, which of the following best describes the image produced?
  - *(A) virtual, inverted, much closer to the lens than the object*
  - *(B) real, inverted, much closer to the lens than the object*
  - *(C) real, inverted, much further from the lens than the object*
  - *(D) virtual, upright, much closer to the lens than the object*

- an object is placed half a focal length away from a converging lens, which of the following best describes the image produced?
  - *(A) virtual, inverted, closer to the lens than the object*
  - *(B) virtual, upright, further from the lens than the object*
  - *(C) real, inverted, further from the lens than the object*
  - *(D) real, upright, closer to the lens than the object*

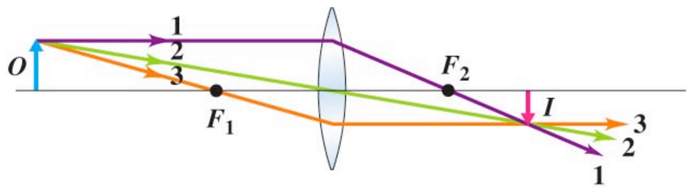
graphical method

- object between converging lens and focal point

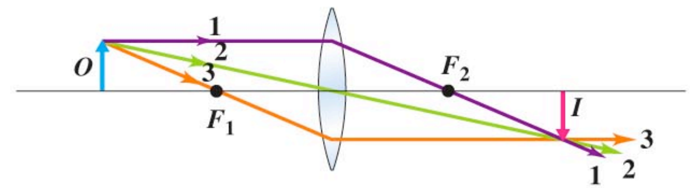


graphical method

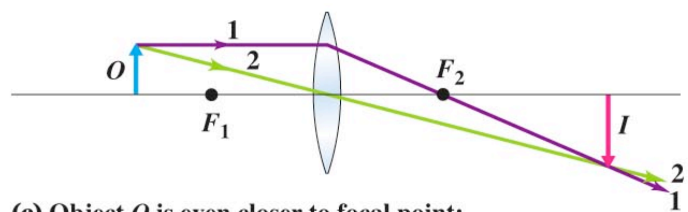
- possibilities with a converging lens



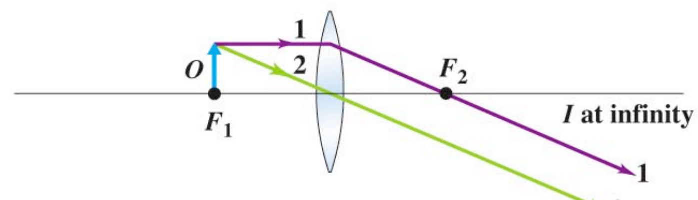
(a) Object O is outside focal point; image I is real.



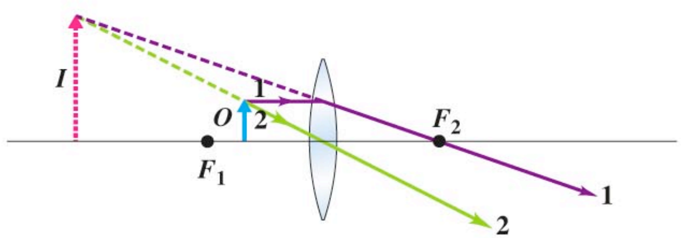
(b) Object O is closer to focal point; image I is real and farther away.



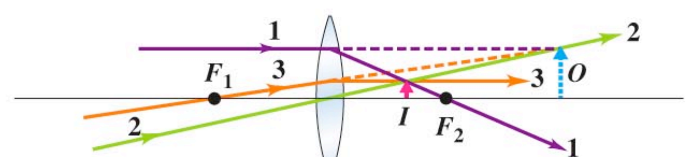
(c) Object O is even closer to focal point; image I is real and even farther away.



(d) Object O is at focal point; image I is at infinity.



(e) Object O is inside focal point; image I is virtual and larger than object.

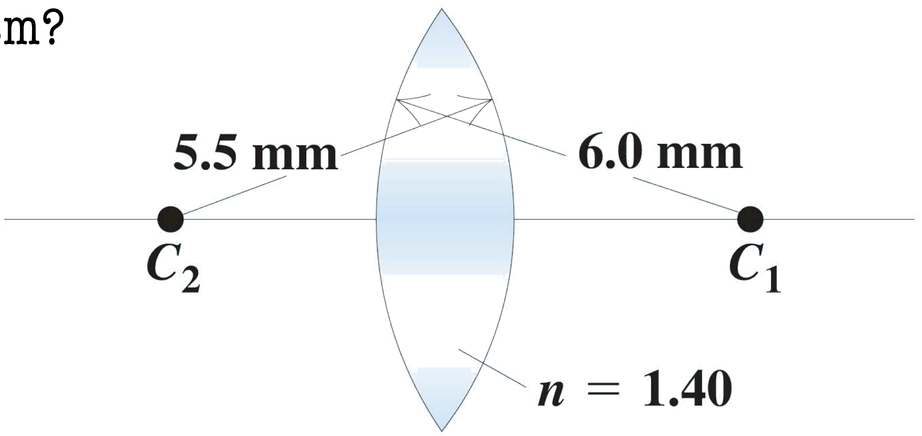


(f) A virtual object O (light rays are *converging* on lens)

multiple optics case - another optical unit is producing a virtual image

the eye's lens

- the eye's lens has a refractive index of about 1.40
- when the lens is as shown in the figure, what is its focal length?
- if you could consider this lens in isolation what would the image distance be for an object distance of 20cm?



$$R_1 = +6.0\,\text{mm}$$
$$R_2 = -5.5\,\text{mm}$$
$$\frac{1}{f} = (1.40 - 1)\left(\frac{1}{6.0\,\text{mm}} - \frac{1}{-5.5\,\text{mm}}\right) \qquad f = 7.2\,\text{mm}$$

$$\frac{1}{200\,\text{mm}} + \frac{1}{s'} = \frac{1}{7.2\text{mm}} \qquad s' = 7.5\,\text{mm}$$

a ‘thin’ lens

- another nice applet
  - demonstrates all the properties we’ve discussed
  - http://phet.colorado.edu/web-pages/index.html

image formed by a diverging lens

- you are given a diverging lens and observe that parallel rays through the lens spread out as though diverging from a point 20cm from the center of the lens
- you want to use this lens to form an erect virtual image that is 1/3 the size of the object
- where should the object be placed?
- draw a principal ray diagram

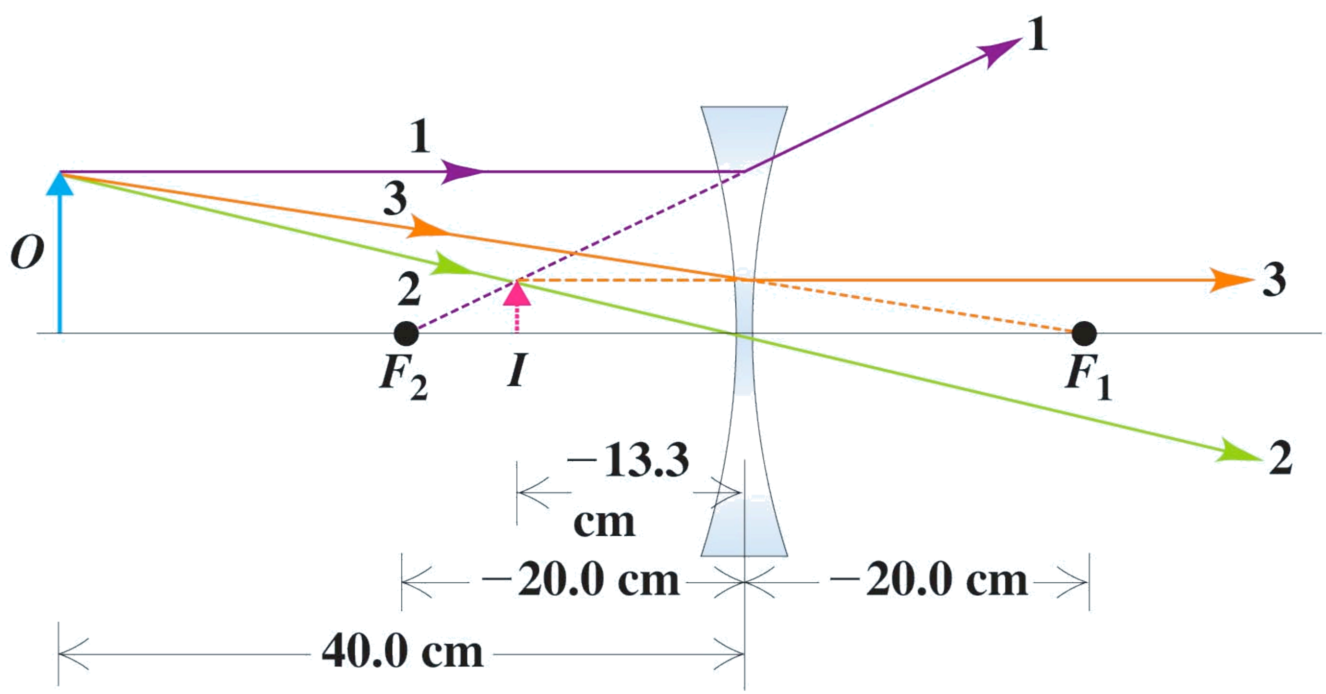
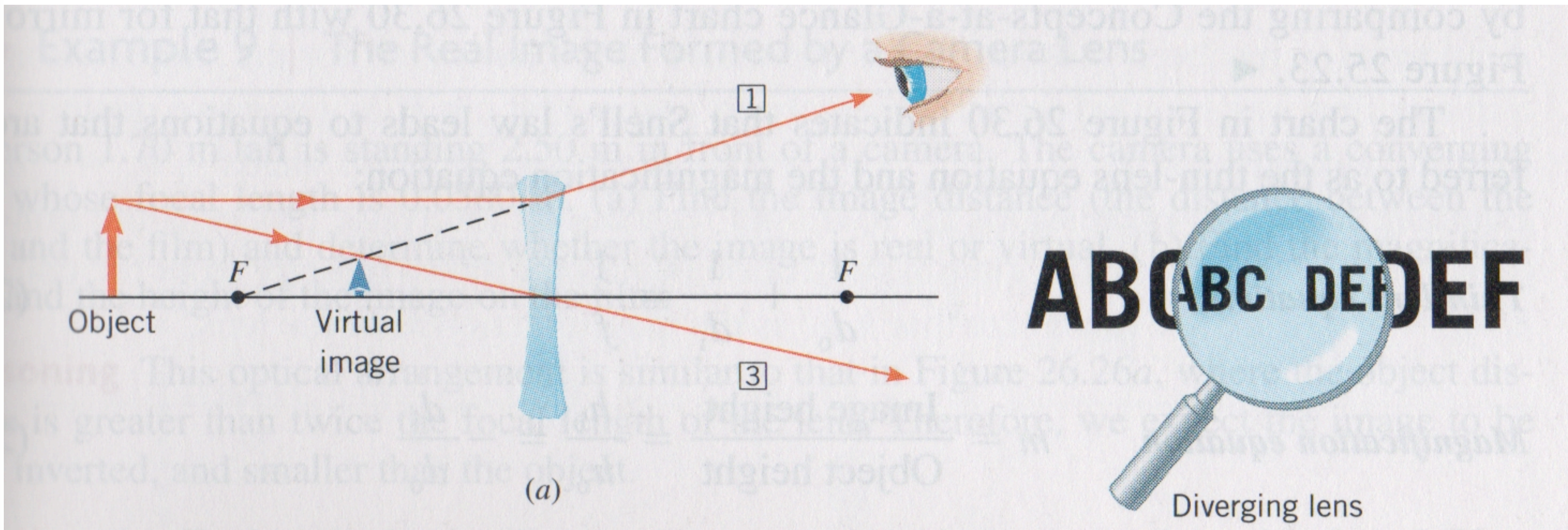


image formed by a diverging lens



quiz

draw a diagram or solve the equation(s)

- an object is placed two focal lengths away from a diverging lens, which of the following best describes the image produced?
  - *(A) virtual, inverted, closer to the lens than the object*
  - *(B) virtual, upright, closer to the lens than the object*
  - *(C) real, inverted, further from the lens than the object*
  - *(D) real, upright, closer to the lens than the object*

- an object is placed half a focal length away from a diverging lens, which of the following best describes the image produced?
  - *(A) real, upright, closer to the lens than the object*
  - *(B) virtual, upright, further from the lens than the object*
  - *(C) real, inverted, further from the lens than the object*
  - *(D) virtual, upright, closer to the lens than the object*