Physics 313:

Cosmology

Instructor: Prof. M. Amarian

Old Dominion University

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Topics to be Covered

1. The Expansion of the Universe
2. The Cosmic Microwave Background
3. The Early Universe
4. Inflation

1.The Expansion of the Universe

- The visible universe seems the same in all directions
- There is no reason to assume that we are in any special position in the universe
- The universe is isotropic and homogeneous
- This principle allows to choose the specific coordinate system so that the metric takes simple form of Robertson-Walker metric.

Spacetime geometry

Geometry of three-dimensional homogeneous and isotropic universe is encoded in a metric in the line element

$$ds^2 = g_{ij}dx^i dx^j$$

The coordinate transformations that leave this invariant are simply ordinary three-dimensional rotations and translations

Spacetime geometry (continued)

Another possibility is a **spherical surface** in four-dimesional Euclidean space with some radius a, with line element

$$ds^2 = dx^2 + dz^2$$

$$z^2 + x^2 = a^2$$

The only other possibility (can be proved) is a **hyperspherical surface** in four-dimensional pseudo-Euclidean space with line element

$$ds^2 = dx^2 - dz^2$$

$$z^2 - x^2 = a^2$$

Spacetime geometry (continued)

Rescaling coordinates

$$x' \equiv ax$$

$$z' \equiv az$$

dropping primes

$$ds^2 = a^2[dx^2 \pm dz^2]$$

differentiating

$$z^2 \pm x^2 = 1$$

gives $\Rightarrow$

$$zdz = \mp x \cdot dx$$

so $\Rightarrow$

$$ds^2 = a^2[dx^2 \pm \frac{(x \cdot dx)^2}{1 \mp x^2}]$$

Spacetime geometry (continued)

We can write line element as

$$ds^2 = a^2[dx^2 \pm K\frac{(x\cdot dx)^2}{1-Kx^2}]$$

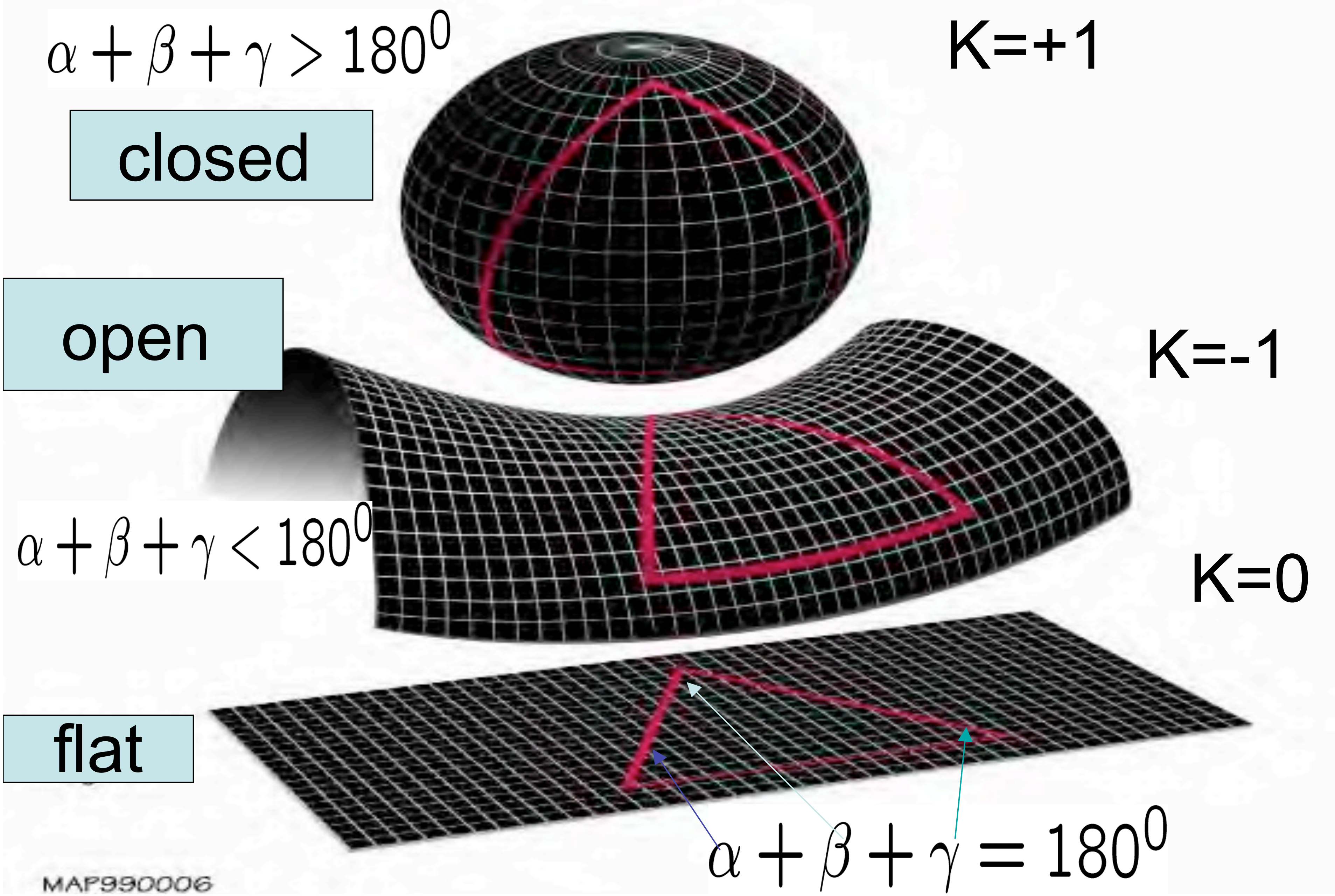
K=+1(spherical),K=-1(hyperspherical), K=0(Euclidean)

Extending this geometry to spacetime with Robertson-Walker scale factor a(t) gives

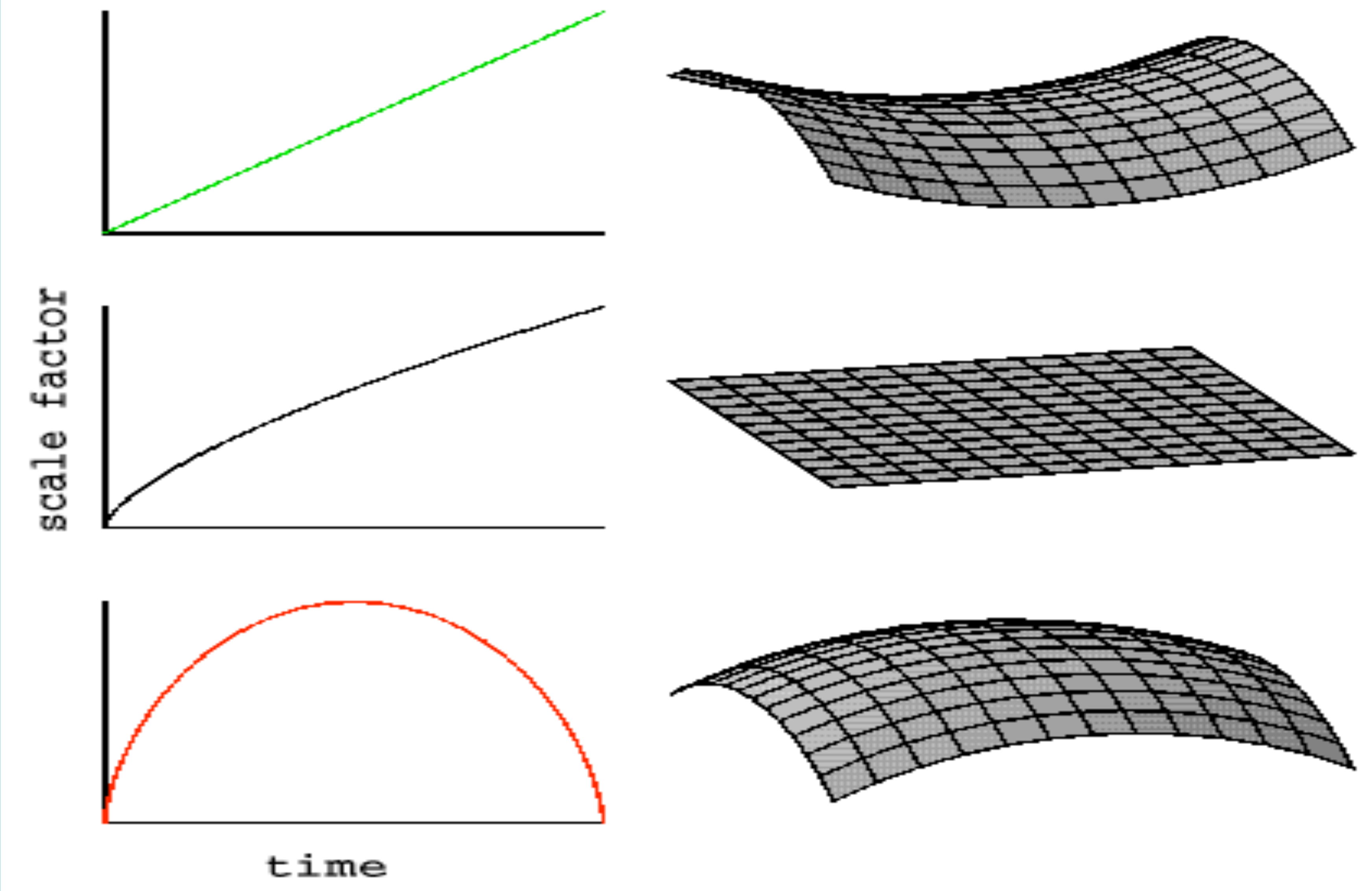
$$d\tau^2 \equiv -g_{\mu\nu}(x)dx^\mu dx^\nu$$

$$= dt^2 - a^2(t)[dx^2 + K\frac{(x\cdot dx)^2}{1-Kx^2}]$$

Which Universe do we live in?



Expansion or contraction?



Spacetime geometry and dynamics

Homogeneity and isotropy of spacetime along with energy conservation lead to equation

$$\frac{d\rho}{dt} + \frac{3\dot{a}}{a}(p + \rho) = 0$$

Where ρ is density, p is a pressure

for $p=w\rho$ solution gives

$$\rho \propto a^{-3-3w}$$

Three Extreme cases:

$$\frac{d\rho}{dt} + \frac{3\dot{a}}{a}(p + \rho) = 0$$

1. Cold Matter (e.g. dust) : p=0

$$\rho \propto a^{-3}$$

2. Hot matter(e.g. radiation): $p=\rho/3$

$$\rho \propto a^{-4}$$

3. Vacuum Energy: $p=-\rho$

$\rho=\text{const}$

Cosmological Constant or the **Vacuum Energy**

The Cosmological Redshift

The general arguments give **no indication** whether the scale factor in the RW metric is

$a(t)>0$, $a(t)<0$ or $a(t)=const$?

This information comes to us from the **observation of a shift** in a frequencies of spectral lines from distant galaxies as compared with their values observed in terrestrial laboratories.

How to calculate this frequency shifts?

Redshift as Doppler effect

Proper distance to a comoving object is

$$d(r,t) = a(t)\frac{dr}{\sqrt{1-Kr^2}}$$

For a light ray coming toward the orgin

$$\int_{t_1}^{t_o} \frac{cdt}{a(t)} = \int_{o}^{r_1} \frac{dr}{\sqrt{1-Kr^2}}$$

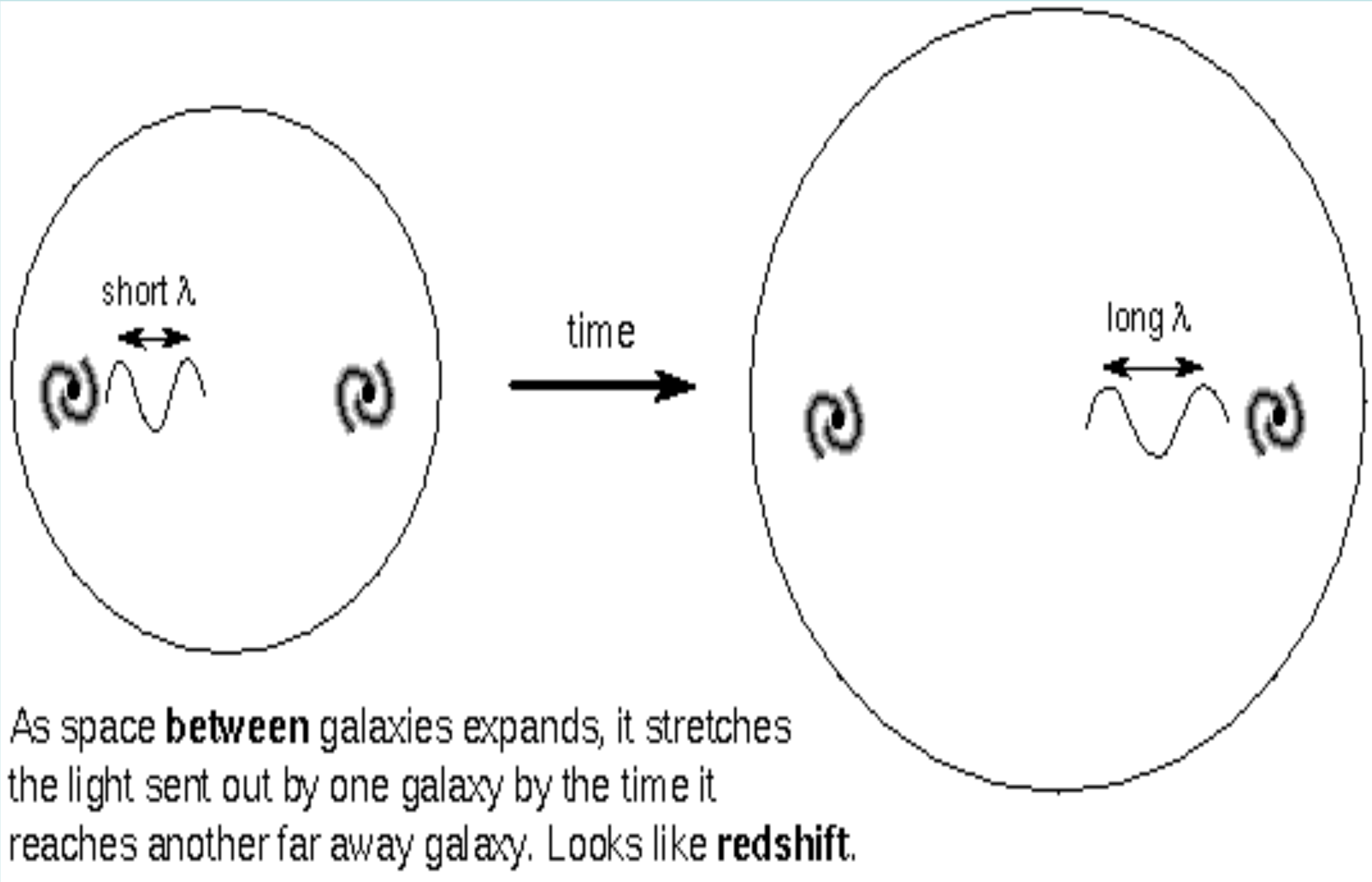
taking the differential we arrive to:

$$\delta \int_{t_1}^{t_o} \frac{cdt}{a(t)} = 0 \Rightarrow \frac{\delta t_1}{a(t_1)} = \frac{\delta t_o}{a(t_o)} \Rightarrow \frac{\nu_0}{\nu_1} = \frac{a(t_1)}{a(t_0)}$$

or alternatively:

$$1 + z = \frac{a(t_0)}{a(t_1)}$$

Redshift


short λ
time
long λ
As space **between** galaxies expands, it stretches the light sent out by one galaxy by the time it reaches another far away galaxy. Looks like **redshift**.

The Hubble constant

For nearby sources, we may expand a(t) in a power series

$$a(t) \simeq a(t_o)[1 + (t - t_o)H_0 + ...]$$

where H_0 is the Hubble constant

$$H_0 \equiv \dot{a}(t_o)/a(t_o)$$

And fractional increase in wavelength

$$z = H_0(ct_o - ct_1) = H_0 d$$

↓ Proper distance

Dynamics of Expansion

Einstein field equation from General Relativity is:

$$R_{\mu\nu} = -8\pi G S_{\mu\nu}$$

$R_{\mu\nu}$ -Ricci tensor, descibes space-time curvature

$$S_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\lambda}_{\lambda}$$

$T_{\mu\nu}$ - Energy-momentum tensor

$g_{\mu\nu}$ - metric tensor, encodes dynamics and physical effects of gravitation

Friedmann Equation

Using FRW metric Einstein eq. become

$$-\frac{2K}{a^2} - \frac{2\dot{a}}{a^2} - \frac{\ddot{a}}{a} = -4\pi(\rho - p) \qquad (1)$$

$$\frac{3\ddot{a}}{a} = -4\pi G(3p + \rho) \qquad (2)$$

From (1) and (2)

$$\Rightarrow \quad \dot{a}^2 + K = \frac{8\pi G\rho a^2}{3} \qquad (3)$$

(Friedmann equation)

Critical density and the Flatness problem

We may define critical density for any value of Hubble constant $H_0 \equiv \dot{a}(t)/a(t_0)$

$$\rho_{0,crit.} \equiv \frac{3H_0^2}{8\pi G} = 1.878 \cdot 10^{-29} h^2 g/cm^3$$

h is Hubble const. in units $100 km \cdot s^{-1} \cdot Mpc^{-1}$

1. K=+1 $\rho_0 > \rho_{0,crit}$
2. K=0 $\rho_0 = \rho_{0,crit}$
3. K=-1 $\rho_0 < \rho_{0,crit}$

At sufficiently early times K~0
then from Friedmann equation

$$\dot{a}^2 + K = \frac{8\pi G \rho a^2}{3}$$

becomes $\Rightarrow$ $\frac{\dot{a}^2}{a^2} \rightarrow \frac{8\pi G \rho}{3}$

from here $\Rightarrow$ $\rho = \rho_{0,crit} = \frac{3H^2}{8\pi G}$

at present $\rho_0 \simeq \rho_{0,crit}$

How is it not very different after billions of years?
This is called called flatness problem.

Deceleration parameter

From equation

$$\frac{3\ddot{a}}{a} = -4\pi G(3p + \rho)$$

we define the deceleration parameter:

$$q_0 \equiv -\frac{\ddot{a}(t_0)a(t_0)}{\dot{a}(t_0)} \quad \Rightarrow$$

$$q_0 = \frac{4\pi G(\rho_0 + 3p_0)}{3H_0^2} = \frac{\rho_0 + 3p_0}{2\rho_{0,crit}}$$

$$q_0 = \frac{4\pi G(\rho_0 + 3p_0)}{3H_0^2} = \frac{\rho_0 + 3p_0}{2\rho_{0,crit}}$$

1. $p_0 \ll \rho_0$ (Non-relativistic matter)

then $\Longrightarrow$ $q_0 = 1/2$ (K=0)

2. $p_0 = \rho_0/3$ (Relativistic matter)

$q_0 = 1$ (K=0)

3. $p_0 = -\rho_0$ (Vacuum energy)

$q_0 = -1$ (K=0)

Solution of Friedmann equation for K=0

1.Non-relativistic (cold) matter

$$\rho = \rho_0 (\frac{a}{a_0})^{-3} \rightarrow a(t) \sim t^{2/3}$$

$$q_0 = 1/2$$

Age of universe

$$t_0 = \frac{2}{3H_0} = 6.52 \cdot 10^9 h^{-1} yr.$$

(Einstein-de Sitter Model)

2.Relativistic (hot) matter

$$\rho = \rho_0 (\frac{a}{a_0})^{-4}$$

$$q_0 = 1$$

$$t_0 = \frac{1}{2H_0}$$

3.Vacuum energy

$$\rho_v = const$$

$$q_0 = -1$$

$$H = \sqrt{\frac{8\pi G \rho_v}{3}}$$

$$t = \infty$$

(De Sitter Model)

Lecture 2

Energy density for arbitrary K

Denote fractions of critical energy density

$$\Omega_\Lambda, \Omega_M, \Omega_R$$

Then for arbitrary K the density is:

$$\rho = \frac{3H_0^2}{8\pi G}[\Omega_\Lambda + \Omega_M(\frac{a_0}{a})^3 + \Omega_R(\frac{a_0}{a})^4]$$

From Friedmann equation we get:

$$\Omega_\Lambda + \Omega_M + \Omega_R + \Omega_K = 1$$

$$\Omega_K \equiv -\frac{K}{a_0^2 H_0^2}$$

Einstein eq. and Cosmological constant

If we express energy-momentum tensor as the sum of vacuum and matter terms then the Einstein equations take the form :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\lambda}_{\lambda} = -8\pi G T^{M}_{\mu\nu} + 8\pi G \rho_V g_{\mu\nu}$$

Or modifying it to read

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = -8\pi G T^{M}_{\mu\nu}$$

where

$$\boxed{\Lambda = 8\pi G \rho_V}$$

is cosmological constant

Why Einstein added Λ ?

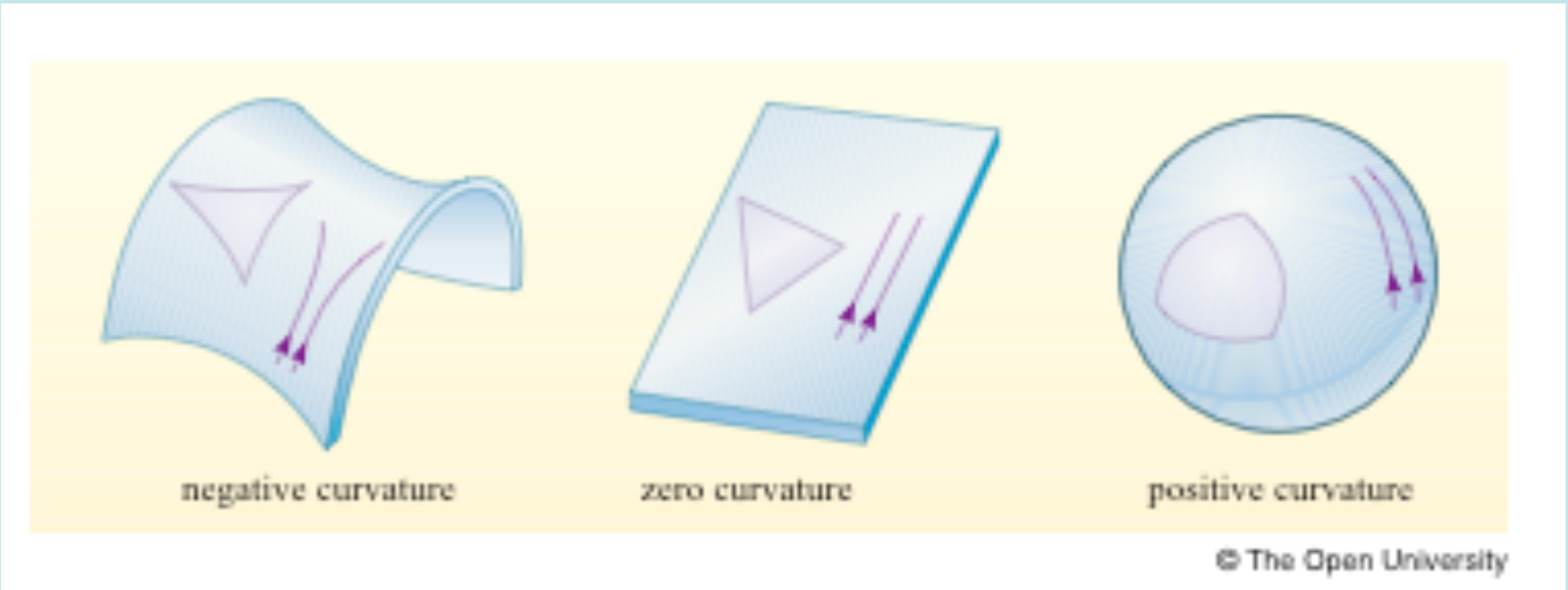
Einstein wanted to have Static Universe.
It is possible only when :

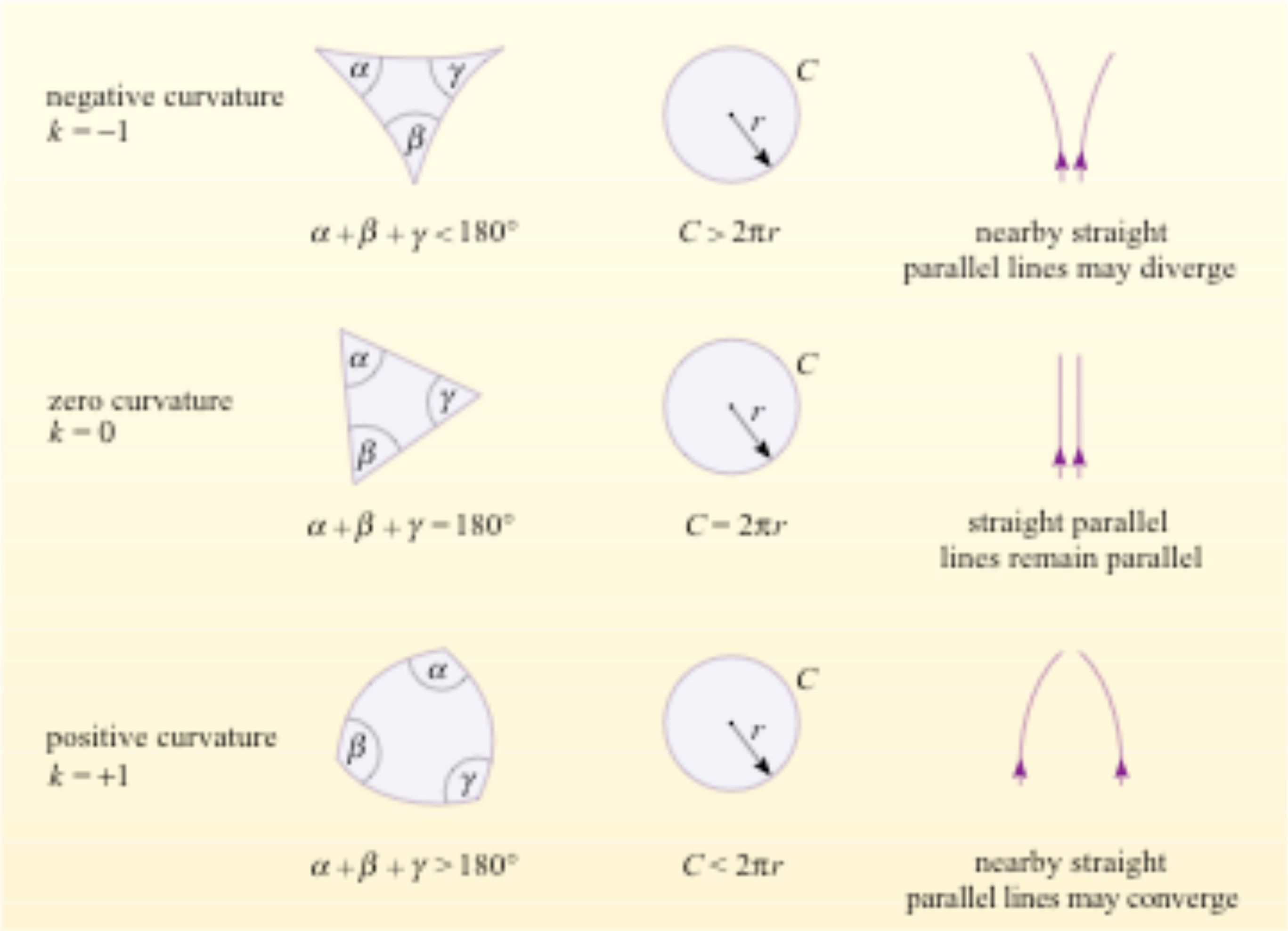
$3p + \rho = 0$ and $K = 8\pi G \rho a^2/3$

Einstein radius is defined as

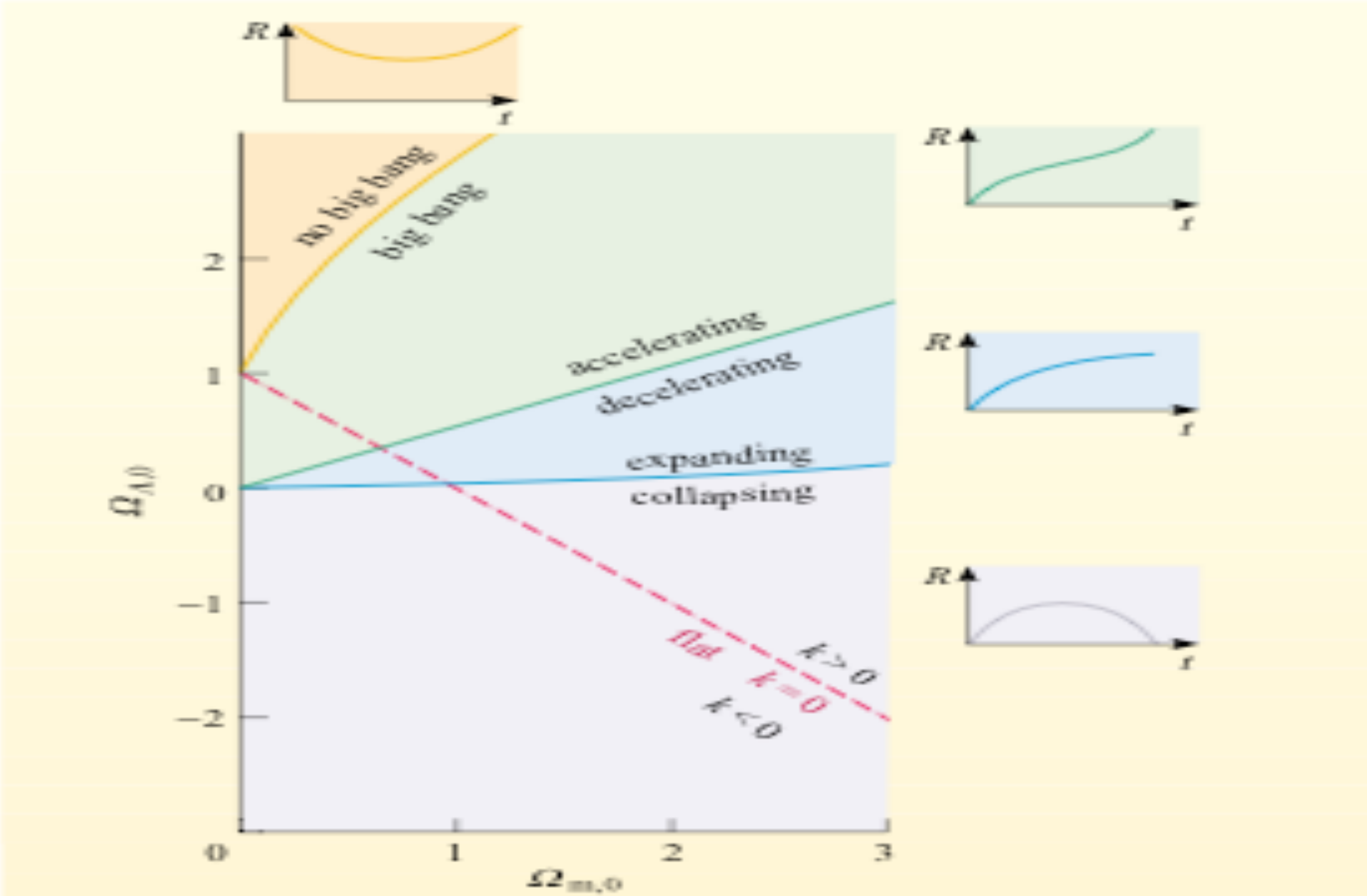
$$a_E = 1/\sqrt{8\pi G \rho_V} = 1/\sqrt{\Lambda}$$

Cosmological Models

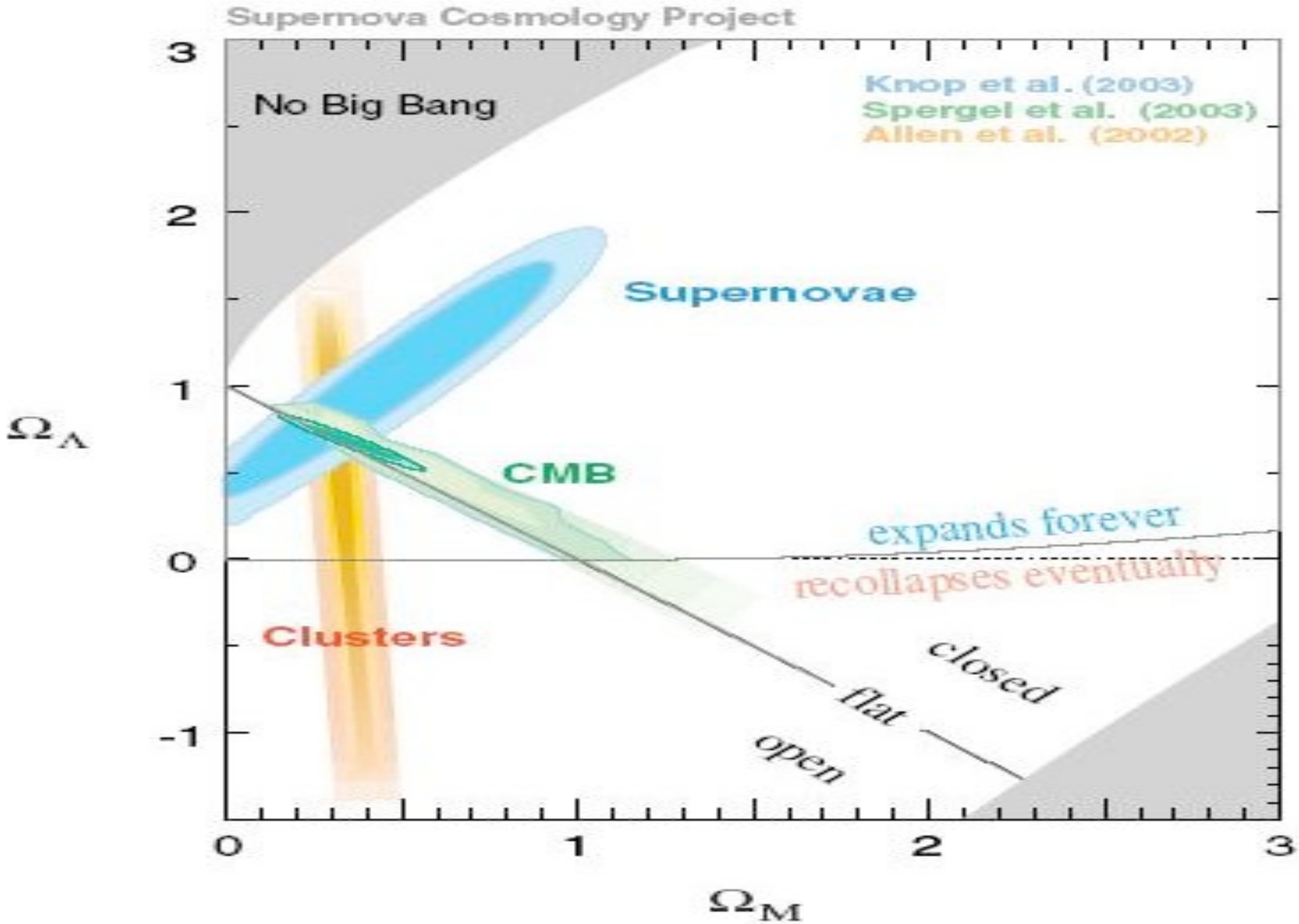



negative curvature
k = −1
α + β + γ < 180°
C > 2πr
nearby straight
parallel lines may diverge
zero curvature
k = 0
α + β + γ = 180°
C = 2πr
straight parallel
lines remain parallel
positive curvature
k = +1
α + β + γ > 180°
C < 2πr
nearby straight
parallel lines may converge

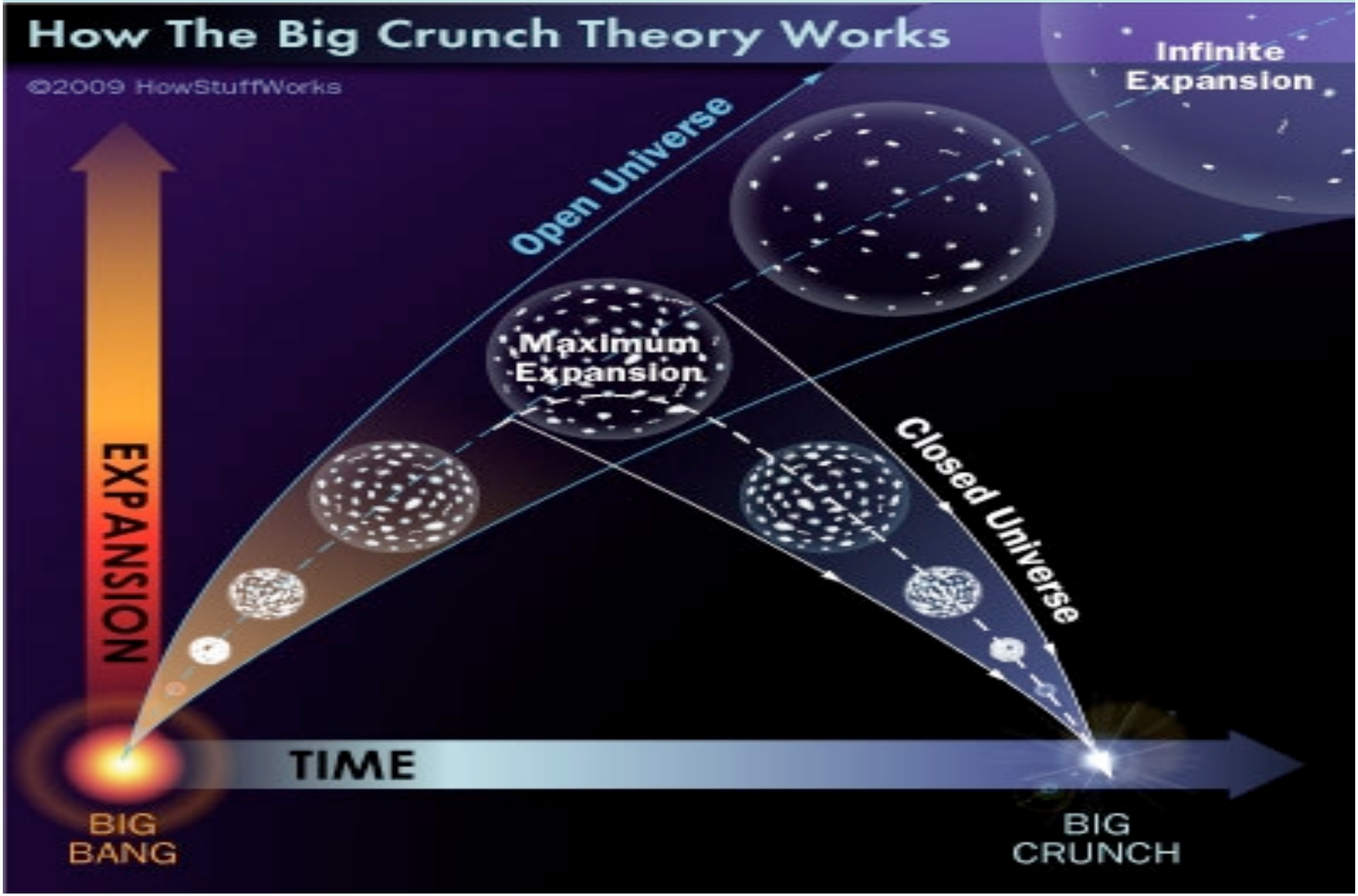
Universe: Flat? Accelerating?Expanding?

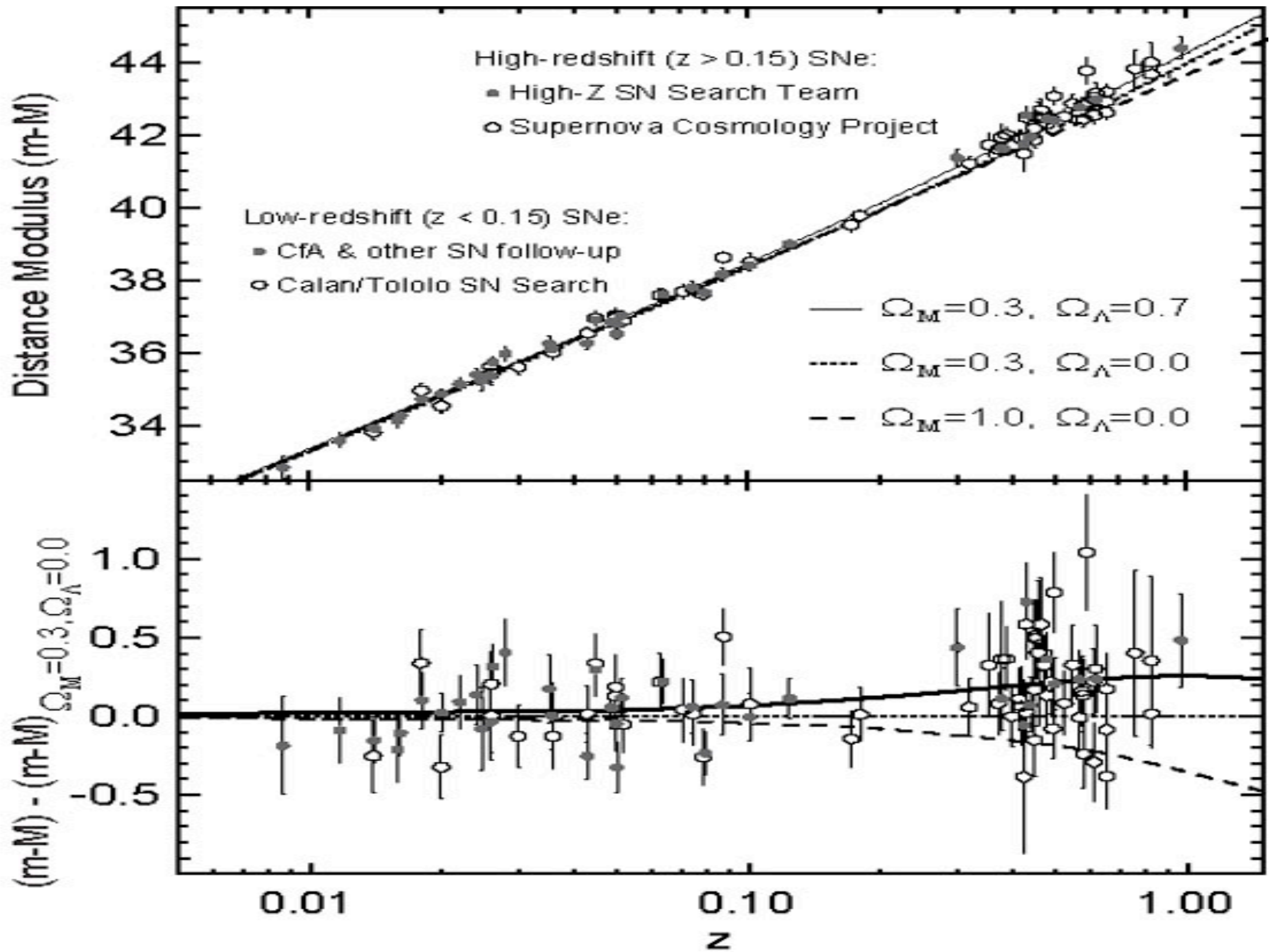


Supernova Cosmology Project



Big Crunch or Ethernal Expansion?





High-redshift (z > 0.15) SNe:
High-Z SN Search Team
Supernova Cosmology Project
Low-redshift (z < 0.15) SNe:
CfA & other SN follow-up
Calan/Tololo SN Search
Ω_M=0.3, Ω_Λ=0.7
Ω_M=0.3, Ω_Λ=0.0
Ω_M=1.0, Ω_Λ=0.0
Distance Modulus (m-M)
44
42
40
38
36
34
$(m-M) - (m-M)_{\Omega_M=0.3,\Omega_\Lambda=0.0}$
1.0
0.5
0.0
-0.5
0.01
0.10
1.00
z

Vacuum energy

Form the distance-redshift of~ 250 Supernovae it isfound that for flat universe

$$\Omega_\Lambda = 0.75^{+0.06}_{-0.07}(stat.) \pm 0.032(syst.)$$

This result indicate $p \simeq -\rho$

Measurement show presense of unknown source of energy in Uverse called

Dark Energy

Cosmic expansion or tired light?

Absorption and scattering of light might invalidate redshift-luminosity theory.

Is it possible that universe is really static and that photons simply suffer a loss of energy and hence a decrease in frequency as they travel to us? (“tired light”)

Answer: No as we $1/(1+z)$ behavior for the rate.

Ages

$$\Omega_M, \Omega_\Lambda, H_0$$

Allow us to estimate the age of universe.

How else can we define it?

Heavy element abundances

Nucleus decay: $$A = A_{init} exp(-\lambda t)$$

$$\Rightarrow \qquad t = \lambda^{-1} ln(A_{init}/A)$$

But we don't know both A_{init}, A

Let us take two nucleus and measure

$$\frac{A_1}{A_2} = (\frac{A_{1init}}{A_{2init}}) exp((\lambda_1 - \lambda_2)t)$$

$$t = \frac{1}{\lambda_2 - \lambda 1}[ln(A_1/A_2) - ln(A_{1init}/A_{2init})]$$

Take two isotopes $^{235}U,^{238}U$

with decay rates

$$^{235}U \rightarrow 0.971 \cdot 10^{-9}/yr$$

$$^{238}U \rightarrow 0.154 \cdot 10^{-9}/yr$$

- Initial A.R. is estimated to be 1.65
- The present A.R. on earth is 0.00723

Then

$$t = \frac{ln(1.65) - ln(0.00723)}{(0.971 - 0.154) \cdot 10^{-9}/yr}$$

$$= 6.6Gyr = 6.6 \cdot 10^{9}$$