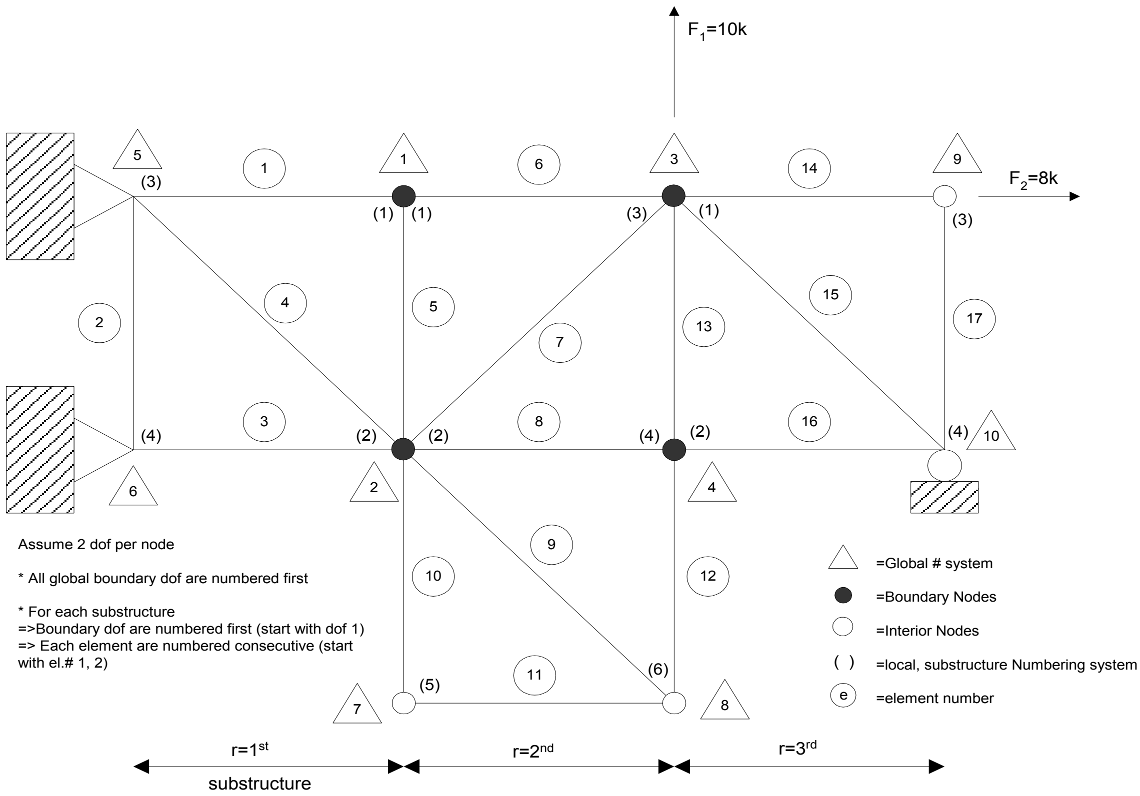
Substructuring Formulations
(Linear, Statics)



Assume 1 substructure only

[K] {z} = {F}

$$\begin{pmatrix} K_{BB} & K_{BI} \\ K_{IB} & K_{II} \end{pmatrix} \cdot \begin{pmatrix} Z_B \\ Z_I \end{pmatrix} = \begin{pmatrix} F_B \\ F_I \end{pmatrix} \qquad \begin{matrix} -\ (1) \\ -\ (2) \end{matrix}$$

Solve Z_I from Eq(2) ➔ $Z_I = [K_{II}]^{-1}(F_I - K_{IB}Z_B)$ - (3)

Substitute Eq. (3) into Eq. (1) ➔

$$K_{BB} \cdot Z_B + K_{BI} \cdot Z_I = F_B$$

$$K_{BB} \cdot Z_B + K_{BI} \cdot \left[(K_{II})^{-1} \cdot (F_I - K_{IB} \cdot Z_B)\right] = F_B$$

$$\left[K_{BB} - K_{BI}(K_{II})^{-1} K_{IB}\right] \cdot Z_B = \left[F_B - K_{BI}(K_{II})^{-1} F_I\right] \qquad -(4)$$

$$\text{or} \qquad \overline{K_B} \cdot Z_B = \overline{F_B} \qquad -(5)$$

Generalized to "NSU" substructures ➔ r=1, 2, …, NSU

Equation (3) becomes:

$$Z_I^{(r)} = \left[K_{II}^{(r)}\right]^{-1}\left(F_I^{(r)} - K_{IB}^{(r)} Z_B^{(r)}\right) \qquad - (6)$$

Equation (5) becomes:

$$\overline{K_B}^{(r)} = K_{BB}^{(r)} - K_{BI}^{(r)} K_{II}^{(r)-1} K_{IB}^{(r)} \qquad - (7)$$

$$\overline{F_B}^{(r)} = F_B^{(r)} - K_{BI}^{(r)} K_{II}^{(r)-1} F_I^{(r)} \qquad - (8)$$

Then: $\overline{K}_B = \sum_{r=1}^{NSU} \overline{K}_B^{(r)}$ - (9)

$$\overline{F}_B = \sum_{r=1}^{NSU} \overline{F}_B^{(r)} \qquad - (10)$$

Now solve: $\left[\overline{K}_B\right]\overrightarrow{Z_B} = \overrightarrow{F_B}$ - (11)

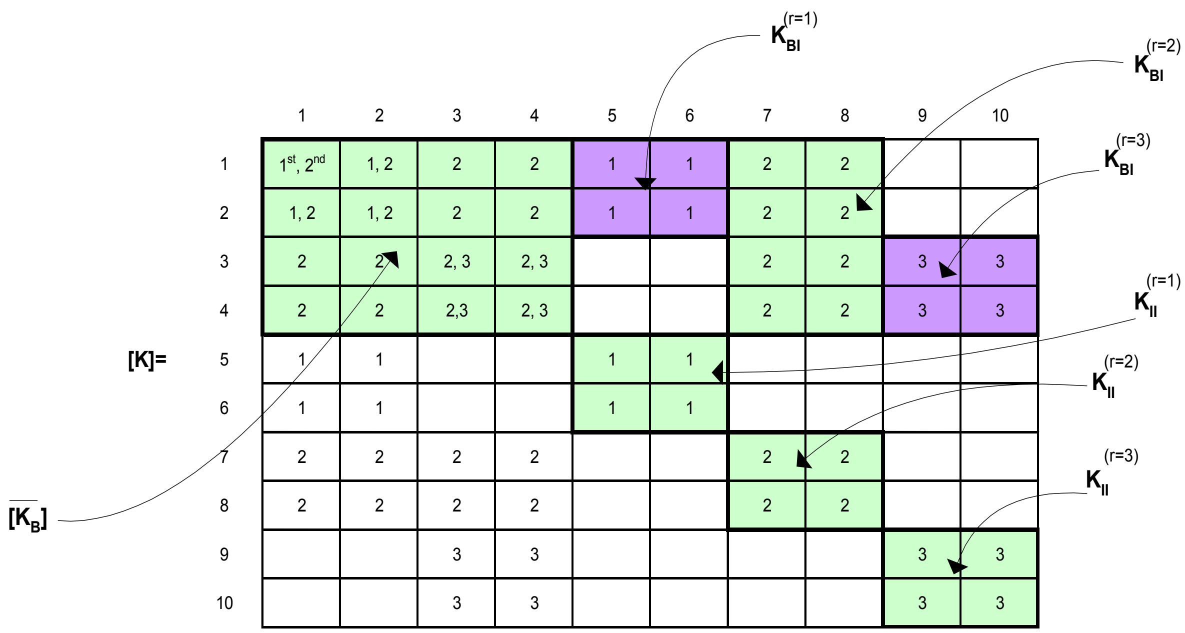
Nearly dense, sym. (or unsym), Non-singular

Notes:

(a) $\left[\overline{K_B}^{(r)}\right] \cdot \overrightarrow{Z_B}^{(r)} = \overline{F_B}^{(r)}$

(b) $\overrightarrow{Z_B}^{(r)}$ is a subset of $\overrightarrow{Z_B}$

(c) * assume each node has 1 dof.



$$[K]\cdot\vec{z}=\vec{f}$$

$$[K]_{orig.}=\begin{bmatrix}112 & 7 & . & . & . & 2\\ & 110 & 5 & 4 & 3 & F\\ & & 88 & F & F & 1\\ & & & 66 & F & F\\ & & & & 44 & F\\ & & & & & 11\end{bmatrix}$$

Use MMD, or ND...

switching rows & columns (pivoting)

$$\text{IPERM}\begin{pmatrix}1\\2\\3\\4\\5\\6\end{pmatrix}=\begin{pmatrix}6\\5\\4\\3\\2\\1\end{pmatrix}$$

NEW OLD

$$[K]_{new}=\begin{bmatrix}11 & . & . & 1 & . & 2\\ & 44 & . & . & 3 & .\\ & & 66 & . & 4 & .\\ & & & 88 & 5 & F\\ & & & & 110 & 7\\ & & & & & 112\end{bmatrix}$$

Given

Non-Singular

$$[A]=\begin{bmatrix}\varepsilon & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0\\ \frac{1}{\varepsilon} & 1\end{bmatrix}\begin{bmatrix}\varepsilon & 0\\ 0 & -\frac{1}{\varepsilon}\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{\varepsilon} & 1\end{bmatrix}^{T}$$

switching rows/ columns will add "Fills"!

☹

$$\lambda = Evalues = \frac{\varepsilon \pm \sqrt{\varepsilon^{2}+4}}{2} \Rightarrow \lambda_1 = \ominus \ \& \ \lambda_2 = \oplus$$