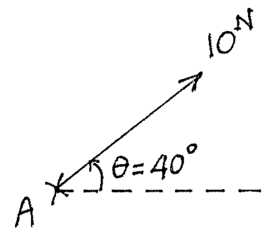
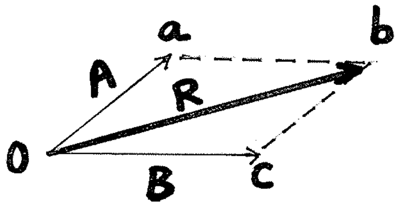
Chapter 2 : Statics of Particles

<u>Force</u>



Magnitude, Direction = Vector
(and Location force applied)

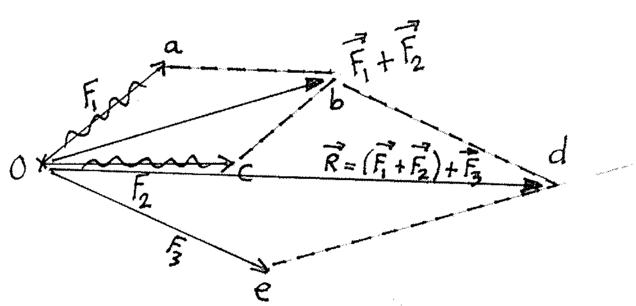
Adding 2 Forces (or Vectors): Parallelogram Law



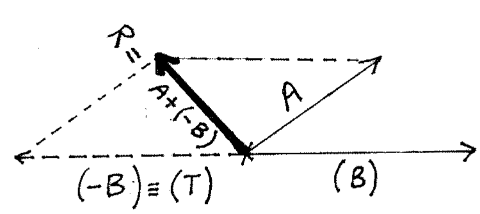
$\vec{R} = \vec{A} + \vec{B}$

oabc = parallelogram

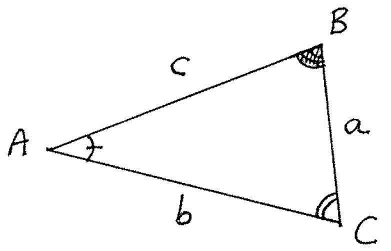
Adding 3 (or more) Vectors : by adding 2 vectors at a time !



Subtracting 2 Vectors : $\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = \vec{A} + (\vec{T})$



High School Reviewed



$$\frac{\sin(\hat{A})}{a} = \frac{\sin(\hat{B})}{b} = \frac{\sin(\hat{C})}{c}$$

$$c^2 = a^2 + b^2 - 2ab\cos\underbrace{(a,b)}_{\hat{C}}$$

what if $\hat{C} = 90°$?

$$\cos(\hat{A}+\hat{B}) = \cos(\hat{A})\cos(\hat{B}) - \sin(\hat{A})\sin(\hat{B})$$

Thus: $\cos(2\alpha) = \cos(\alpha+\alpha) = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha$

$$= 2\cos^2\alpha - 1$$

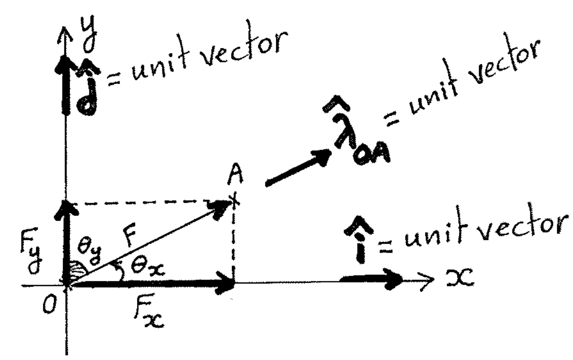
$$\sin(\hat{A}+\hat{B}) = \sin(\hat{A})\cos(\hat{B}) + \sin(\hat{B})\cos(\hat{A})$$

Thus: $\sin(2\alpha) = \sin(\alpha+\alpha) = 2\sin(\alpha)\cos(\alpha)$

Resolving a Given Force Into 2 Components, Unit Vectors

$$\vec{F} = \vec{F_x} + \vec{F_y} = (F_x)\,\hat{i} + (F_y)\,\hat{j}$$

Also, $\vec{F} = (F)\,\hat{\lambda}_{OA}$



where: $F_x = F\cos(\theta_x)$

$F_y = F\cos(\theta_y)$

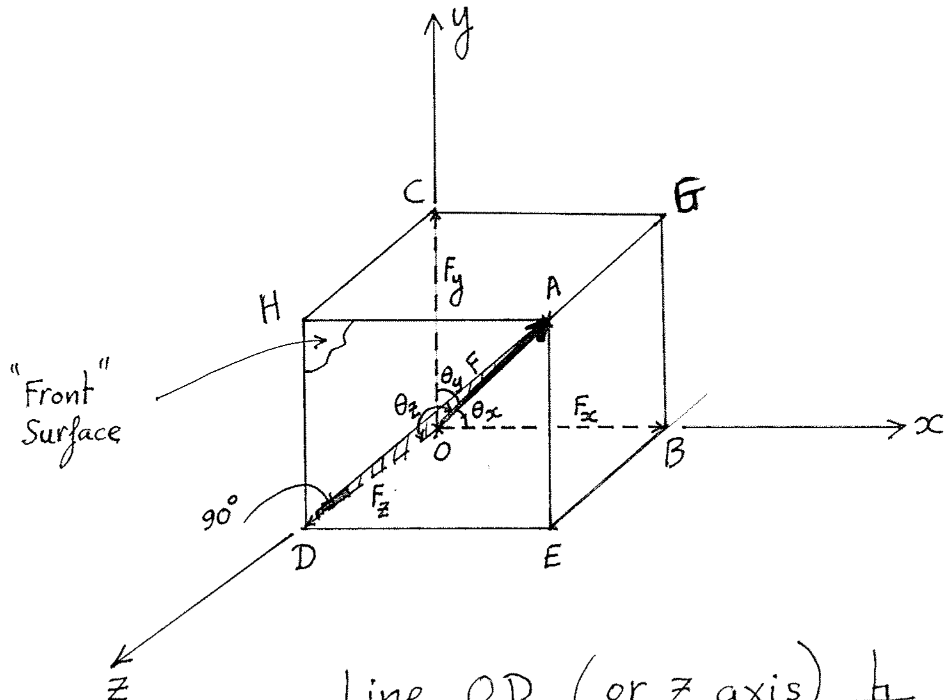
$F = \sqrt{F_x^2 + F_y^2}$

In 3-D cases: $\vec{F} = (F_x)\,\hat{i} + (F_y)\,\hat{j} + \mathbf{(F_z)\,\hat{k}}$

$$\mathbf{F_z = F\cos(\theta_z)} \longrightarrow \cos(\theta_z) = \frac{F_z}{F}$$

$$F = \sqrt{F_x^2 + F_y^2 + \mathbf{F_z^2}}$$

$$\boxed{\text{Prove that } \cos(\theta_z) = \frac{F_z}{F} \text{ ??}}$$

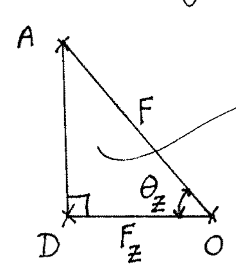


Line OD (or z axis) $\perp$ Front surface

Line AD $\in$ Front Surface

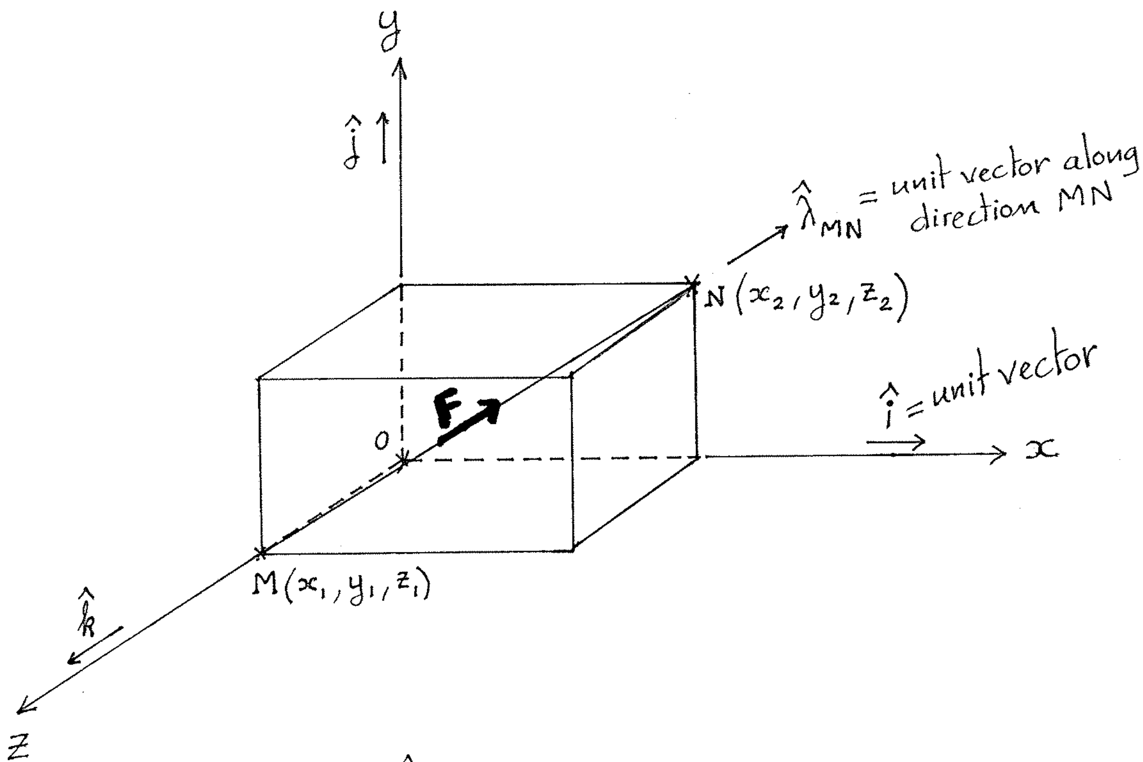
Hence, Line OD $\perp$ Line AD

Thus, triangle OAD has 90° angle @ $\hat{D}$



$$\cos(\theta_z) = \frac{F_z}{F}$$

Force (or Vector) Defined by 2 Points in Space



$$\vec{F} = (F) * \hat{\lambda}_{MN}$$

where: $$\hat{\lambda}_{MN} = \frac{\overrightarrow{MN}}{MN} = \frac{(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

Let: $\Delta x \equiv x_2 - x_1$; $\Delta y \equiv y_2 - y_1$; $\Delta z \equiv z_2 - z_1$

Hence: $$\hat{\lambda}_{MN} = \frac{(\Delta x)\hat{i} + (\Delta y)\hat{j} + (\Delta z)\hat{k}}{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}} = \frac{(\Delta x, \Delta y, \Delta z)}{L_{MN}}$$

$$\vec{F} = (F) * \frac{(\Delta x, \Delta y, \Delta z)}{L_{MN}} \equiv (F_x)\hat{i} + (F_y)\hat{j} + (F_z)\hat{k}$$

Thus, the 3 components of $\vec{F}$ can be identified as:

$$F_x = \frac{F * \Delta x}{L_{MN}} \; ; \; F_y = \frac{F * \Delta y}{L_{MN}} \; ; \; F_z = \frac{F * \Delta z}{L_{MN}}$$

Equilibrium of a Particle

A particle, acted by a system of forces $\vec{F_i}$, is said to be in EQUILIBRIUM iff

$$\sum_{\ell=1}^{N} \vec{F_\ell} = \vec{0} \qquad \text{1 "vector" equation} \qquad (32)$$

For 3-D cases, the above Eq. (32) can be expressed as:

$$\sum_{\ell=1}^{N} \left\{ \left(F_{\ell x}\right)\hat{i} + \left(F_{\ell y}\right)\hat{j} + \left(F_{\ell z}\right)\hat{k} \right\} = (0)\,\hat{i} + (0)\,\hat{j} + (0)\,\hat{k}$$

Comparing both sides of the above "vector equation", the following 3 "scalar equations" can be obtained:

$$\boxed{\begin{aligned} \sum_{\ell=1}^{N} F_{\ell x} &= 0 \\ \sum_{\ell=1}^{N} F_{\ell y} &= 0 \\ \sum_{\ell=1}^{N} F_{\ell z} &= 0 \end{aligned}}$$