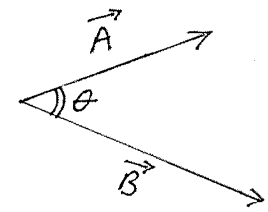
Chapter 3: Rigid Bodies, Equivalent Force Systems

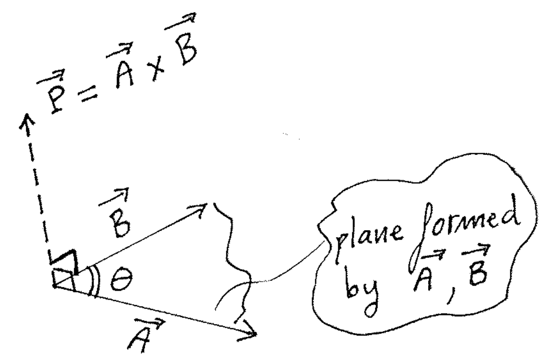
High School Reviewed



(A) Dot Product of 2 Vectors

$$\vec{A} \cdot \vec{B} = \begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix} \cdot \begin{Bmatrix} B_x \\ B_y \\ B_z \end{Bmatrix} = A_x B_x + A_y B_y + A_z B_z = \text{scalar} \equiv S$$

$$\vec{A} \cdot \vec{B} \overset{\text{or}}{=} |A| * |B| * \cos\underbrace{(\vec{A}, \vec{B})}_{\theta} \equiv S$$



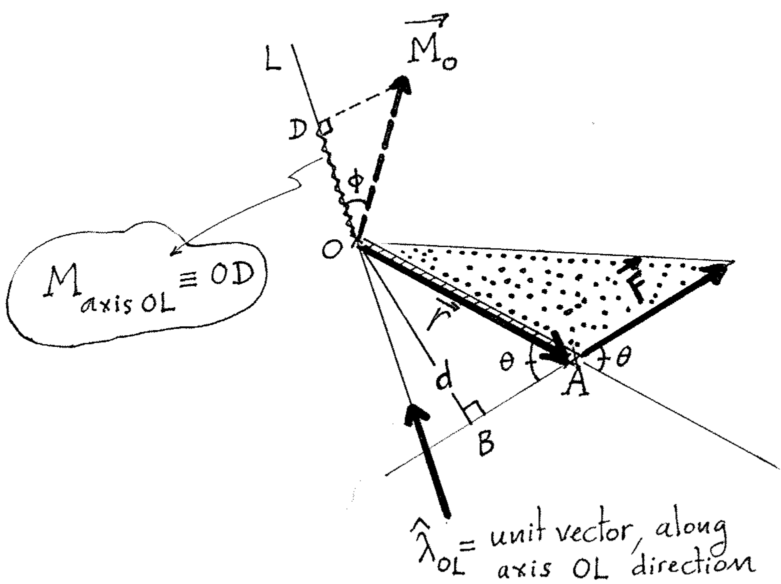
(B) Cross Product of 2 Vectors

$$\vec{A} \times \vec{B} = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \left(A_y B_z \hat{i} + \hat{j} A_z B_x + \hat{k} A_x B_y\right) - \left(B_x A_y \hat{k} + B_y A_z \hat{i} + B_z A_x \hat{j}\right) \equiv \vec{P}$$

$$\vec{A} \times \vec{B} = \left(A_y B_z - A_z B_y \, , \, A_z B_x - A_x B_z \, , \, A_x B_y - A_y B_x\right) \equiv \vec{P}$$

$$|\vec{A} \times \vec{B}| \overset{\text{or}}{=} |A| * |B| * \sin\underbrace{(\vec{A}, \vec{B})}_{\theta}$$

C1 Moment About Point "O" Due to Force Applied at Point "A"



$$\vec{M}_o = \vec{r} \times \vec{F}$$

$\vec{r}$: Position vector, from point "o" (where you wish to compute moment) to point "A" (where force applied)

Hence:

$$\vec{M}_o = \vec{OA} \times \vec{F}$$

$$\vec{M}_o = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ OA_x & OA_y & OA_z \\ F_x & F_y & F_z \end{vmatrix}$$

Also:

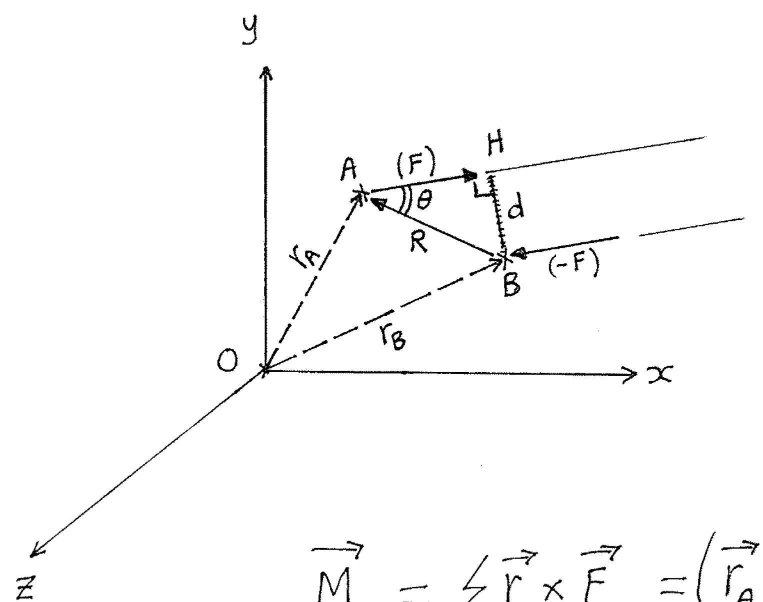
$$|\vec{M}_o| = |r| * |F| * \sin(\underbrace{r, F}_{\theta}) = |F| * d$$

(d = perpendicular distance between point "o" and force $\vec{F}$)

$M_{axis\, OL} \equiv$ projection of vector $\vec{M}_o$ onto axis OL

$$M_{axis\, OL} = \underbrace{\vec{M}_o \cdot \hat{\lambda}_{OL}}_{\text{scalar}} = |M_o| * |1| * \cos(\underbrace{M_o, OL}_{\equiv \phi}) = |M_o| \cos(\phi) \equiv OD$$

C_2 Moment of a Couple



$$\vec{M}_o = \sum \vec{r} \times \vec{F} = \left(\vec{r}_A \times \vec{F}\right) + \left(\vec{r}_B \times -\vec{F}\right)$$

$$\vec{M}_o = \left(\vec{r}_A - \vec{r}_B\right) \times \vec{F}$$

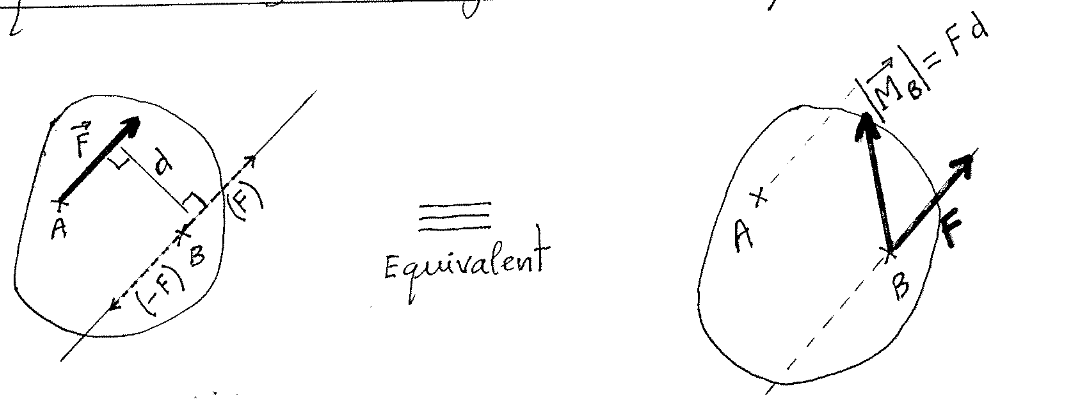
$$\vec{M}_o = \left(\vec{R}\right) \times \vec{F} \quad ; \text{ where } \vec{r}_A - \vec{r}_B \equiv \vec{R}$$

Hence: $\left|\vec{M}_o\right| = |\vec{R}| * |\vec{F}| * \sin\underbrace{(\vec{R}, \vec{F})}_{\equiv \theta}$

$$|M_o| = |F| * (d)$$

perpendicular distance between $(\vec{F})$ & $(-\vec{F})$

(D) Equivalent Systems of Forces and/or Moments



System I
(Force $\vec{F}$ applied @ A)

System II
(Force $\vec{F}$ & $\vec{M}_B$ applied @ B)

Proof's Hint: Applied $(+\vec{F})$ and $(-\vec{F})$ @ B in System I !

Two (2) systems of force(s) and/or moment(s) are said to be EQUIVALENT iff:

* $\left(\sum \vec{F}\right)_I = \left(\sum \vec{F}\right)_{II}$

* and $\left(\sum \vec{M}_{@\,\text{any point}}\right)_I = \left(\sum \vec{M}_{@\,\text{same point}}\right)_{II}$