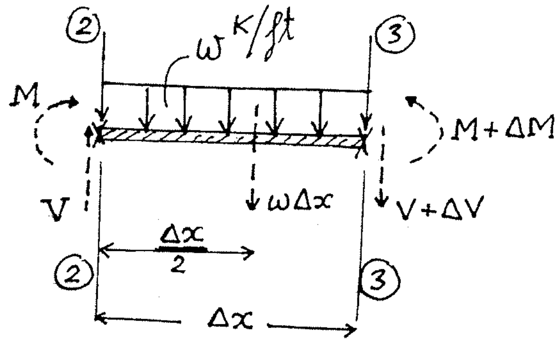
(B) Relations Between (Distributed) Loads "w", Internal Shear Force "V" and Bending Moment "M"



Consider the FBD of beam segment between cut-sections (2)-(2) and (3)-(3). For equilibrium, one has:

$$* \quad \sum F_y \overset{\downarrow +}{=} 0 = \cancel{-V} + (w * \Delta x) + (\cancel{V} + \Delta V)$$

$$0 = \frac{w \cancel{\Delta x}}{\cancel{\Delta x}} + \frac{\Delta V}{\Delta x} \Rightarrow \frac{\Delta V}{\Delta x} = -w$$

Let $\Delta x \to 0$; hence: $\boxed{\frac{dV}{dx} = -w}$

$$* \quad \sum M_{(2)} \overset{\curvearrowleft +}{=} 0 = \cancel{-M} - (w\Delta x)\left(\frac{\Delta x}{2}\right) + (\cancel{M} + \Delta M) - (V + \Delta V)\Delta x$$

$$0 = -w\left(\frac{\Delta x}{2}\right) + \frac{\Delta M}{\Delta x} - (V + \Delta V)$$

Let $\Delta x \to 0$; hence $\Delta V \to 0$; and above eq. becomes:

$$0 = +\frac{dM}{dx} - V \quad ; \text{ or } \quad \boxed{\frac{dM}{dx} = V}$$

or $$\int_{(2)}^{(3)} dM = \int_{(2)}^{(3)} V\,dx$$

$M_{(3)} - M_{(2)}$ = Area under shear curve, between cut-sections (2) & (3)