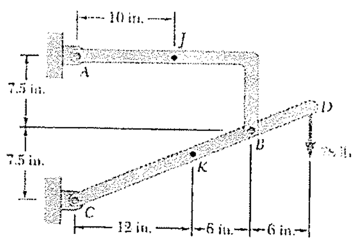
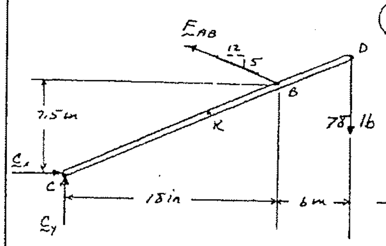
PROBLEM 7.6



Determine the internal forces at point K of the structure shown.

SOLUTION

FBD CD:



Note: AB is a two-force member

$$\circlearrowleft \Sigma M_C = 0: \quad (18 \text{ in.})\left(\frac{5}{13}F_{AB}\right) + (7.5 \text{ in.})\left(\frac{12}{13}F_{AB}\right) - (24 \text{ in.})(78 \text{ lb}) = 0$$

$$F_{AB} = 135.2 \text{ lb}$$

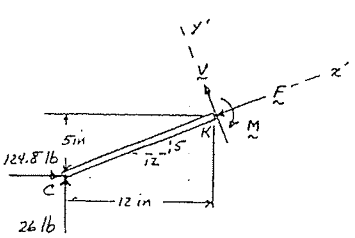
$$\rightarrow \Sigma F_x = 0: \quad C_x - \frac{12}{13}(135.2 \text{ lb}) = 0$$

$$\mathbf{C}_x = 124.8 \text{ lb} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: \quad C_y + \frac{5}{13}(135.2 \text{ lb}) - (78 \text{ lb}) = 0$$

$$\mathbf{C}_y = 26 \text{ lb} \uparrow$$

FBD CK:



$$\nearrow \Sigma F_{x'} = 0: \quad -F + \frac{12}{13}(124.8 \text{ lb}) + \frac{5}{13}(26 \text{ lb}) = 0$$

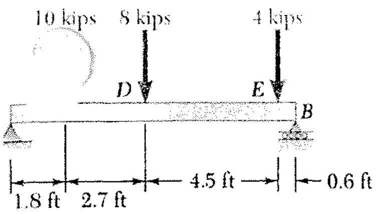
$$\mathbf{F} = 125.2 \text{ lb} \nearrow 22.6° \blacktriangleleft$$

$$\nwarrow \Sigma F_{y'} = 0: \quad V - \frac{5}{13}(124.8 \text{ lb}) + \frac{12}{13}(26 \text{ lb}) = 0$$

$$\mathbf{V} = 24.0 \text{ lb} \searrow 67.4° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: \quad (5 \text{ in.})(124.8 \text{ lb}) - (12 \text{ in.})(26 \text{ lb}) - M = 0$$

$$\mathbf{M} = 312 \text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

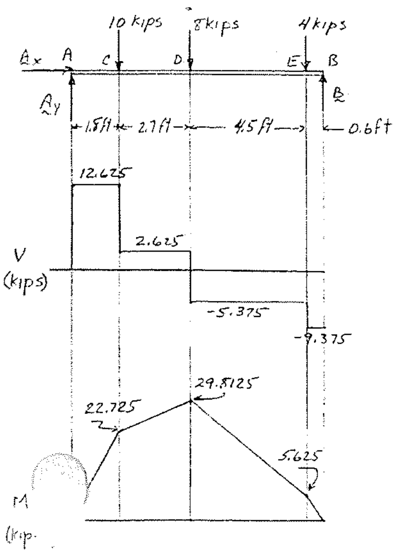


PROBLEM 7.34

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*)



FBD Beam:

$\circlearrowleft \Sigma M_B = 0$:

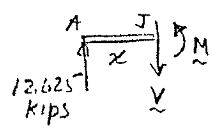
$$(.6\text{ ft})(4\text{ kips}) + (5.1\text{ ft})(8\text{ kips}) + (7.8\text{ ft})(10\text{ kips}) - (9.6\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 12.625\text{ kips}\uparrow$$

$\uparrow \Sigma F_y = 0$: $\quad 12.625\text{ kips} - 10\text{ kips} - 8\text{ kips} - 4\text{ kips} + B = 0$

$$\mathbf{B} = 9.375\text{ kips}\uparrow$$

Along AC:



$\uparrow \Sigma F_y = 0$: $\quad 12.625\text{ kips} - V = 0$

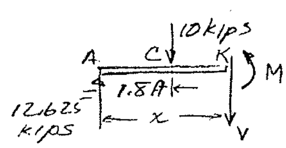
$$V = 12.625\text{ kips}$$

$\circlearrowleft \Sigma M_J = 0$: $\quad M - x(12.625\text{ kips}) = 0$

$$M = (12.625\text{ kips})x$$

$$M = 22.725\text{ kip}\cdot\text{ft at } C$$

Along CD:



$\uparrow \Sigma F_y = 0$: $\quad 12.625\text{ kips} - 10\text{ kips} - V = 0$

$$V = 2.625\text{ kips}$$

$\circlearrowleft \Sigma M_K = 0$: $\quad M + (x - 1.8\text{ ft})(10\text{ kips}) - x(12.625\text{ kips}) = 0$

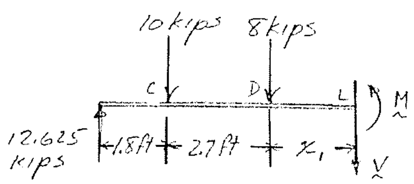
$$M = 18\text{ kip}\cdot\text{ft} + (2.625\text{ kips})\,x$$

$$M = 29.8125\text{ kip}\cdot\text{ft at } D\ (x = 4.5\text{ ft})$$

continued

PROBLEM 7.34 CONTINUED

Along DE:



$$\uparrow \Sigma F_y = 0: \quad (12.625 - 10 - 8)\text{ kips} - V = 0 \qquad V = -5.375\text{ kips}$$

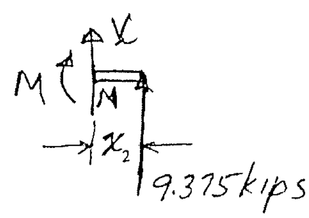
$$\circlearrowleft \Sigma M_L = 0: \; M + x_1(8\text{ kips}) + (2.7\text{ ft} + x_1)(10\text{ kips})$$

$$- (4.5\text{ ft} + x_1)(12.625\text{ kips}) = 0$$

$$M = 29.8125\text{ kip}\cdot\text{ft} - (5.375\text{ kips})\, x_1$$

$$M = 5.625\text{ kip}\cdot\text{ft at } E \; (x_1 = 4.5\text{ ft})$$

Along EB:



$$\uparrow \Sigma F_y = 0: \quad V + 9.375\text{ kips} = 0 \qquad V = 9.375\text{ kips}$$

$$\circlearrowleft \Sigma M_N = 0: \quad x_2(9.375\text{ kip}) - M = 0$$

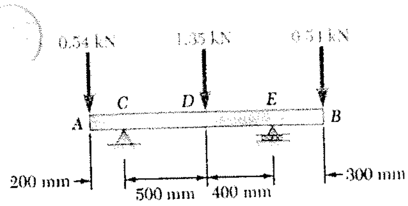
$$M = (9.375\text{ kips})\, x_2$$

$$M = 5.625\text{ kip}\cdot\text{ft at } E$$

(*b*) From diagrams: $|V|_{max} = 12.63\text{ kips on } AC$ ◀

$|M|_{max} = 29.8\text{ kip}\cdot\text{ft at } D$ ◀

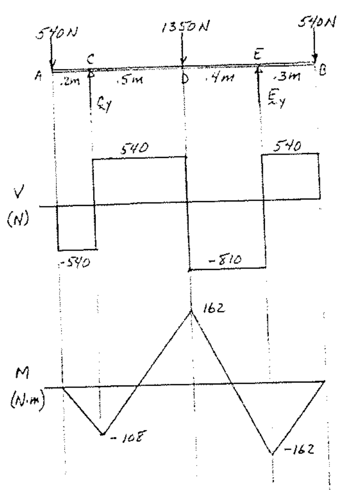
PROBLEM 7.35



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*)



FBD Beam: $\circlearrowleft \Sigma M_E = 0$:

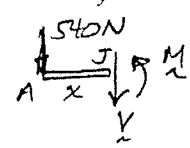
$$(1.1\text{ m})(540\text{ N}) - (0.9\text{ m})C_y + (0.4\text{ m})(1350\text{ N}) - (0.3\text{ m})(540\text{ N}) = 0$$

$$\mathbf{C}_y = 1080\text{ N}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad -540\text{ N} + 1080\text{ N} - 1350\text{ N}$$

$$-540\text{ N} + E_y = 0 \qquad \mathbf{E}_y = 1350\text{ N}\uparrow$$

Along AC:

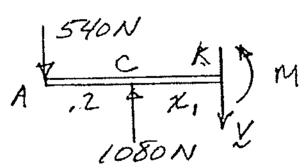


$$\uparrow \Sigma F_y = 0: \quad -540\text{ N} - V = 0$$

$$V = -540\text{ N}$$

$$\circlearrowleft \Sigma M_J = 0: \quad x(540\text{ N}) + M = 0 \qquad M = -(540\text{ N})x$$

Along CD:



$$\uparrow \Sigma F_y = 0: \quad -540\text{ N} + 1080\text{ N} - V = 0 \qquad V = 540\text{ N}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + (0.2\text{ m} + x_1)(540\text{ N}) - x_1(1080\text{ N}) = 0$$

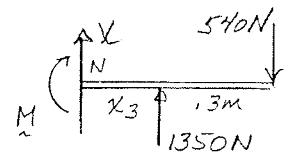
$$M = -108\text{ N}\cdot\text{m} + (540\text{ N})x_1$$

$$M = 162\text{ N}\cdot\text{m at } D\ (x_1 = 0.5\text{ m})$$

continued

PROBLEM 7.35 CONTINUED

Along DE:



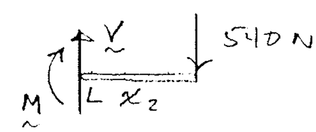
$\uparrow \Sigma F_y = 0$: $V + 1350\text{ N} - 540\text{ N} = 0$ $\quad V = -810\text{ N}$

$\circlearrowleft \Sigma M_N = 0$: $M + (x_3 + 0.3\text{ m})(540\text{ N}) - x_3(1350\text{ N}) = 0$

$$M = -162\text{ N}\cdot\text{m} + (810\text{ N})x_3$$

$$M = 162\text{ N}\cdot\text{m at } D\ (x_3 = 0.4)$$

Along EB:



$\uparrow \Sigma F_y = 0$: $V - 540\text{ N} = 0$ $\quad V = 540\text{ N}$

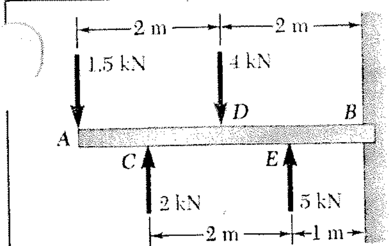
$\circlearrowleft \Sigma M_L = 0$: $M + x_2(540\text{ N}) = 0$ $\quad M = -540\text{ N}\,x_2$

$$M = -162\text{ N}\cdot\text{m at } E \quad (x_2 = 0.3\text{ m})$$

(*b*)

From diagrams $\quad |V|_{max} = 810\text{ N on } DE$ ◀

$|M|_{max} = 162.0\text{ N}\cdot\text{m at } D \text{ and } E$ ◀

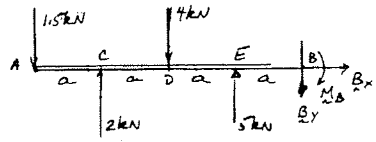


PROBLEM 7.36

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) **FBD Beam:**

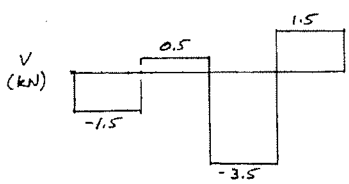


$a = 1\text{ m}$ $\quad \xrightarrow{} \Sigma F_x = 0: \quad B_x = 0$

$\uparrow \Sigma F_y = 0: \quad -1.5\text{ kN} + 2\text{ kN} - 4\text{ kN} + 5\text{ kN} - B_y = 0$

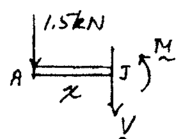
$$B_y = 1.5\text{ kN}\downarrow$$

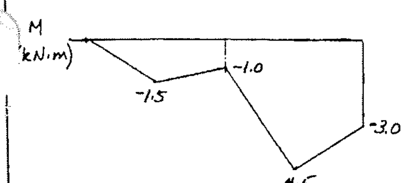
$\circlearrowleft \Sigma M_B = 0: \quad a\left[4(1.5\text{ kN}) - 3(2\text{ kN}) + 2(4\text{ kN}) - 1(5\text{ kN})\right] - M_B = 0$



Along AC:

$$M_B = (3\text{ kN})a = 3\text{ kN}\cdot\text{m}$$



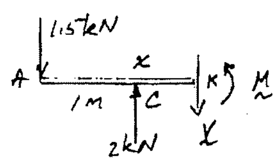


$\uparrow \Sigma F_y = 0: \quad -1.5\text{ kN} - V = 0 \quad V = -1.5\text{ kN}$

$\circlearrowleft \Sigma M_J = 0: \quad M - x(1.5\text{ kN}) = 0 \quad M = -(1.5\text{ kN})x$

Along CD:

$$M(1\text{ m}) = -1.5\text{ kN}\cdot\text{m}$$



$\uparrow \Sigma F_y = 0: \quad -1.5\text{ kN} + 2\text{ kN} - V = 0 \qquad V = 0.5\text{ kN}$

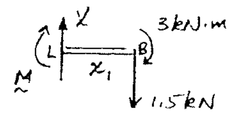
$\circlearrowleft \Sigma M_K = 0: \quad M + x(1.5\text{ kN}) - (x - 1\text{ m})(2\text{ kN}) = 0$

$$M = -2\text{ kN}\cdot\text{m} + (0.5\text{ kN})x \quad M(2\text{ m}) = -1\text{ kN}\cdot\text{m}$$

continued

PROBLEM 7.36 CONTINUED

Along EB:

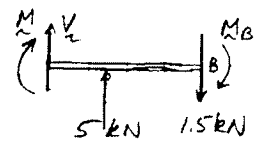


$+\uparrow \Sigma F_y = 0$: $V - 1.5 \text{ kN} = 0 \quad V = 1.5 \text{ kN}$

$+\circlearrowleft \Sigma M_L = 0$: $-M - x_1(1.5 \text{ kN}) - 3 \text{ kN}\cdot\text{m} = 0$

$$M = -3 \text{ kN}\cdot\text{m} - (1.5 \text{ kN})x_1, \quad M(1 \text{ m}) = -4.5 \text{ kN}\cdot\text{m}$$

Along DE:



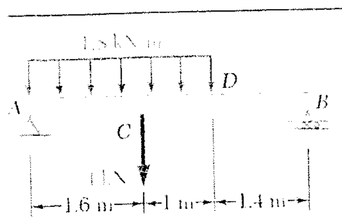
$+\uparrow \Sigma F_y = 0$: $V + 5 \text{ kN} - 1.5 \text{ kN} = 0 \qquad V = -3.5 \text{ kN}$

Also M is linear here

(*b*)

$|V|_{max} = 3.50 \text{ kN}$ along DE ◀

$|M|_{max} = 4.50 \text{ kN}\cdot\text{m}$ at E ◀

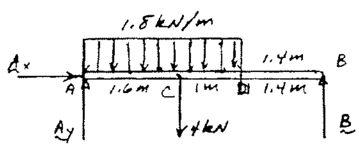


PROBLEM 7.37

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) **FBD Beam:**

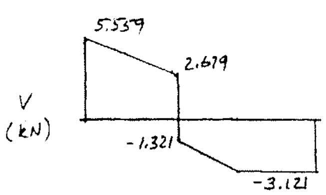


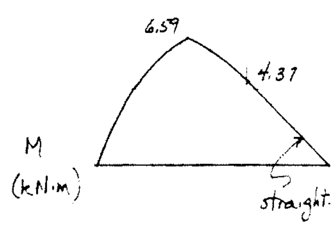
$$\circlearrowleft \Sigma M_A = 0: \quad -(1.3\text{ m})\big[(1.8\text{ kN/m})(2.6\text{ m})\big] - (1.6\text{ m})(4\text{ kN}) + (4\text{ m})B = 0$$

$$\mathbf{B} = 3.121\text{ kN} \uparrow$$

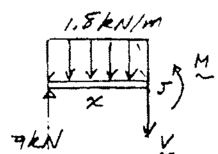
$$\uparrow \Sigma F_y = 0: \quad A_y - (1.8\text{ kN/m})(2.6\text{ m}) - 4\text{ kN} + 3.121\text{ kN} = 0$$

$$\mathbf{A}_y = 5.559\text{ kN} \uparrow$$





Along AC:



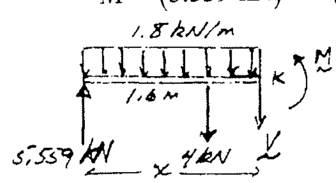
$$\uparrow \Sigma F_y = 0: \quad 5.559\text{ kN} - (1.8\text{ kN/m})x - V = 0$$

$$V = 5.559\text{ kN} - (1.8\text{ kN/m})x$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{x}{2}\big[(1.8\text{ km})x\big] - x(5.559\text{ kN}) = 0$$

$$M = (5.559\text{ kN})x - (0.9\text{ kN/m})x^2$$

Along CD:



$$\uparrow \Sigma F_y = 0: \quad 5.559\text{kN} - x(1.8\text{ kN/m}) - 4\text{ kN} - V = 0$$

$$V = (1.559\text{ kN}) - (1.8\text{ kN/m})x$$

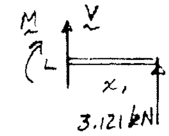
$$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 1.6\text{ m})(4\text{ kN}) + \frac{x}{2}\big[(1.8\text{ kN/m})x\big] - x(5.559\text{ kN}) = 0$$

$$M = 6.4\text{ kN}\cdot\text{m} + (1.559\text{ kN})x - (0.9\text{ kN/m})x^2$$

continued

PROBLEM 7.37 CONTINUED

Along DB:



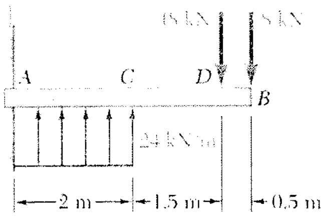
$\uparrow \Sigma F_y = 0$: $V + 3.121 \text{ kN} = 0$

$V = -3.121 \text{ kN}$

$\circlearrowleft \Sigma M_L = 0$: $-M + x_1(3.121 \text{ kN}) = 0$ $M = (3.121 \text{ kN})x_1$

(*b*) $|V|_{max} = 5.56 \text{ kN}$ at A ◀

$|M|_{max} = 6.59 \text{ kN·m}$ at C ◀

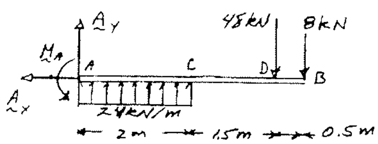


PROBLEM 7.38

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) **FBD Beam:**



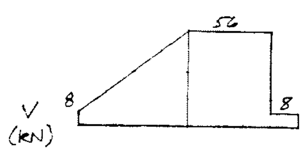
$\longleftarrow \Sigma F_x = 0: \quad \mathbf{A}_x = 0$

$\uparrow \Sigma F_y = 0: \quad A_y + (2\text{ m})(24\text{ kN/m}) - 48\text{ kN} - 8\text{ kN} = 0$

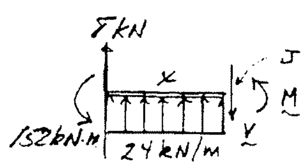
$$\mathbf{A}_y = 8\text{ kN}\uparrow$$

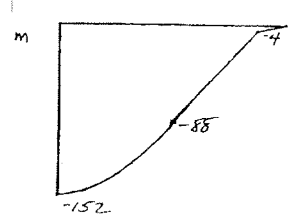
$\circlearrowleft \Sigma M_A = 0: \quad M_A + (1\text{ m})(2\text{ m})(24\text{ kN/m}) - (3.5\text{ m})(48\text{ kN})$

$$-(2\text{ m})(8\text{ kN}) = 0, \qquad \mathbf{M}_A = 152\text{ kN}\cdot\text{m}\circlearrowleft$$



Along AC:





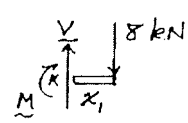
$\uparrow \Sigma F_y = 0: \quad 8\text{ kN} + x(24\text{ kN}\cdot\text{m}) - V = 0$

$$V = 8\text{ kN} + (24\text{ kN/m})x$$

$\circlearrowleft \Sigma M_J = 0: \quad M + 152\text{ kN}\cdot\text{m} - x(8\text{ kN}) - \frac{x}{2}(24\text{ kN/m})x = 0$

$$M = (12\text{ kN/m})x^2 + (8\text{ kN})x - 152\text{ kN}\cdot\text{m}$$

Along DB:



$\uparrow \Sigma F_y = 0: \quad V - 8\text{ kN} = 0$

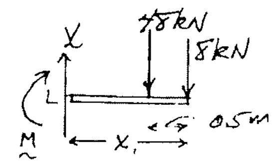
$$V = 8\text{ kN}$$

$\circlearrowleft \Sigma M_K = 0: \quad M + x_1(8\text{ kN}) = 0, \qquad M = -(8\text{ kN})x_1$

continued

PROBLEM 7.38 CONTINUED

Along CD:



$\uparrow \Sigma F_y = 0$: $V - 48\text{ kN} - 8\text{ kN} = 0$, $V = 56\text{ kN}$

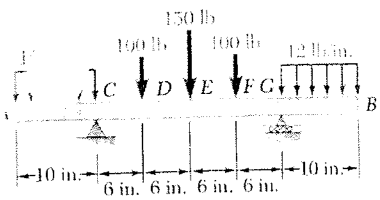
$\circlearrowleft \Sigma M_L = 0$: $M + (x_1 - 0.5\text{ m})(48\text{ kN}) + x_1(8\text{ kN}) = 0$

$$M = 24\text{ kN}\cdot\text{m} - (56\text{ kN})x_1$$

(*b*)

$|V|_{max} = 56.0\text{ kN}$ along *CD* ◀

$|M|_{max} = 152.0\text{ kN}\cdot\text{m}$ at *A* ◀

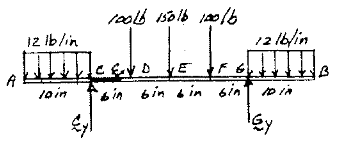


PROBLEM 7.39

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) **FBD Beam:**

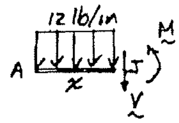


by symmetry, $\mathbf{C}_x = 0$, and

$$C_y = G_y = \frac{1}{2}\left[2(12\text{ lb/in})(10\text{ in}) + 2(100\text{ lb}) + (150\text{ lb})\right]$$

$$\mathbf{C}_y = \mathbf{G}_y = 295\text{ lb}\uparrow$$

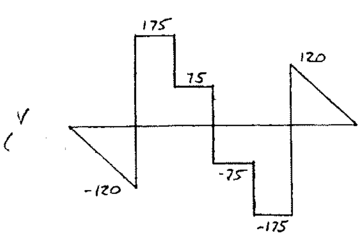
Along AC:

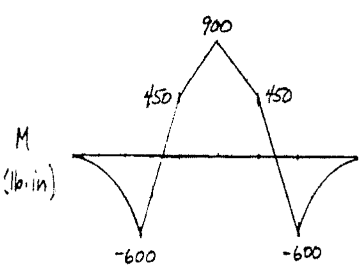


$$\uparrow \Sigma F_y = 0: \quad -(12\text{ lb/in.})x - V = 0$$

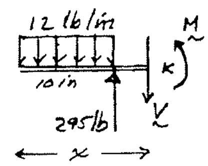
$$V = -\left(12\frac{\text{lb}}{\text{in.}}\right)x$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{x}{2}(12\text{ lb/in.})x = 0, \qquad M = -\left(6\frac{\text{lb}}{\text{in.}}\right)x^2$$





Along CD:



$$\uparrow \Sigma F_y = 0: \quad -(12\text{ lb/in.})(10\text{ in.}) + 295\text{ lb} - V = 0$$

$$V = 175\text{ lb}$$

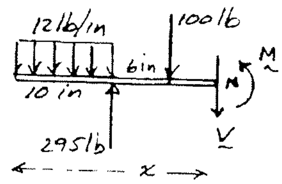
$$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 5\text{ in.})(12\text{ lb/in.})(10\text{ in.}) - (x - 10\text{ in.})(295\text{ lb}) = 0$$

$$M = -2350\text{ lb}\cdot\text{in.} + (175\text{ lb})x$$

continued

PROBLEM 7.39 CONTINUED

Along DE:



$+\uparrow \Sigma F_y = 0$: $\quad -(12\text{ lb/in.})(10\text{ in.}) + 295\text{lb} - 100\text{ lb} - V = 0, \qquad V = 75\text{ lb}$

$$\circlearrowleft \Sigma M_N = 0: \quad M + (x - 16\text{ in.})(100\text{ lb}) - (x - 10\text{ in.})(295\text{ lb})$$

$$+ (x - 5\text{ in.})(12\text{ lb/in.})(10\text{ in.}) = 0$$

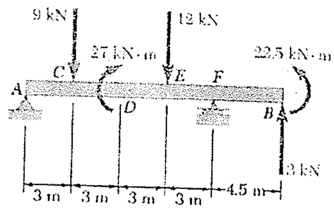
$$M = -750\text{ lb}\cdot\text{in} + (75\text{ lb})x$$

Complete diagrams using symmetry.

(*b*)

$|V|_{max} = 175.0\text{ lb}$ along CD and FG ◀

$|M|_{max} = 900\text{ lb}\cdot\text{in.}$ at E ◀

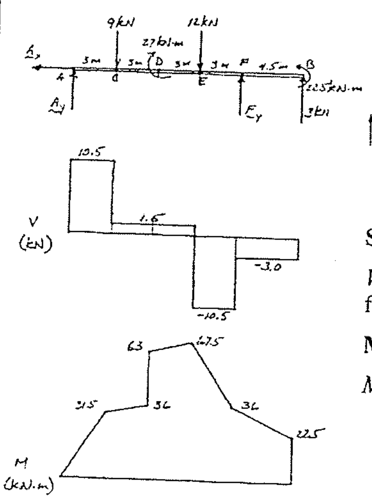


PROBLEM 7.71

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*)



$\circlearrowleft \Sigma M_A = 0$: $\quad -(3\text{ m})(9\text{ kN}) - 27\text{ kN}\cdot\text{m} - (9\text{ m})(12\text{ kN}) + (12\text{ m})F$

$$+(16.5\text{ m})(3\text{ kN}) + 22.5\text{ kN}\cdot\text{m} = 0$$

$$\mathbf{F} = 7.5\text{ kN} \uparrow$$

$\uparrow \Sigma F_y = 0$: $\quad A_y - 9\text{ kN} - 12\text{ kN} + 7.5\text{ kN} + 3\text{ kN} = 0$

$$\mathbf{A}_y = 10.5\text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant, with jumps at A, C, E, F, and B, equal to the forces there.

Moment Diag:

M is piecewise linear with jumps at D and B equal to the couples there.

$$M_C = (10.5\text{ kN})(3\text{ m}) = 31.5\text{ kN}\cdot\text{m}$$

$$M_{D^-} = 31.5\text{ kN}\cdot\text{m} + (1.5\text{ kN})(3\text{ m}) = 36.0\text{ kN}\cdot\text{m}$$

$$M_{D^+} = 36\text{ kN}\cdot\text{m} + 27\text{ kN}\cdot\text{m} = 63\text{ kN}\cdot\text{m}$$

$$M_E = 63\text{ kN}\cdot\text{m} + (1.5\text{ kN})(3\text{ m}) = 67.5\text{ kN}\cdot\text{m}$$

$$M_F = 67.5\text{ kN}\cdot\text{m} - (10.5\text{ kN})(3\text{ m}) = 36\text{ kN}\cdot\text{m}$$

$$M_{B^-} = 36\text{ kN}\cdot\text{m} - (3\text{ kN})(4.5\text{ m}) = 22.5\text{ kN}\cdot\text{m}$$

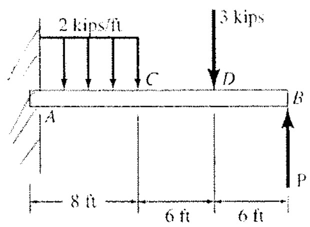
Finally M drops $22.5\text{ kN}\cdot\text{m}$ to zero at B

(*b*)

From the diagrams, $\quad |V|_{max} = 10.50\text{ kN}$ along AC and EF ◀

$|M|_{max} = 67.5\text{ kN}\cdot\text{m}$ at E ◀

Final Exam Review

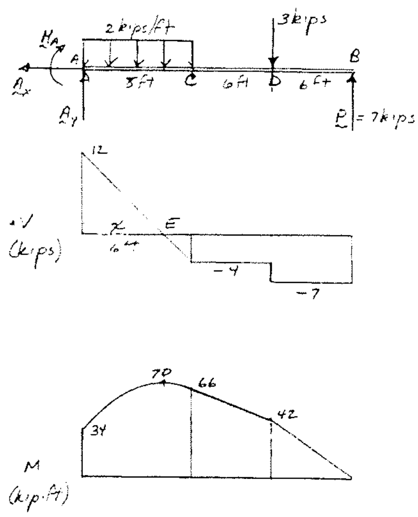


PROBLEM 7.74

For the beam shown, draw the shear and bending-moment diagrams, and determine the maximum absolute value of the bending moment knowing that (a) $P = 7$ kips, (b) $P = 10$ kips.

SOLUTION

(a)



$\uparrow \Sigma F_y = 0: \quad A_y - (2 \text{ kips/ft})(8 \text{ ft}) - 3 \text{ kips} + 7 \text{ kips} = 0$

$\mathbf{A}_y = 12 \text{ kips} \uparrow$

$\circlearrowleft \Sigma M_A = 0: \quad M_A + (4 \text{ ft})(2 \text{ kips/ft})(8 \text{ ft}) - (14 \text{ ft})(3 \text{ kips})$

$- (20 \text{ ft})(7 \text{ kips}) = 0 \qquad \mathbf{M}_A = 34 \text{ kip}\cdot\text{ft}$

Shear Diag:

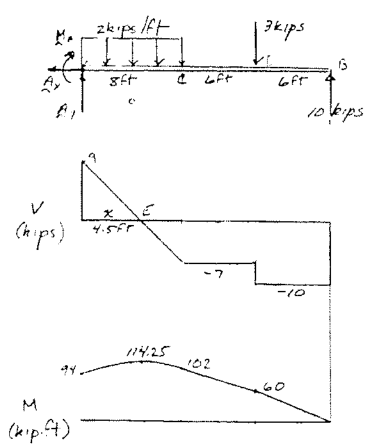
V jumps to 12 kips at A, then decreases at 2 kips/ft to -4 kips at C to D. V drops 3 kips to -7 kips from D to B and jumps 7 kips to zero. Note: $V = 0$ where $12 \text{ kips} - (2 \text{ kips/ft})x = 0, \ x = 6 \text{ ft}$.

Moment Diag:

M jumps to 34 kip·ft at A and then increases with decreasing slope to $34 \text{ kip}\cdot\text{ft} + \frac{1}{2}(12 \text{ kips})(6 \text{ ft}) = 70 \text{ kip}\cdot\text{ft}$ at E, and decreases by $\frac{1}{2}(4 \text{ kips})(2 \text{ ft}) = 4 \text{ kip}\cdot\text{ft}$, to 66 kip·ft at C. M then decreases by $(4 \text{ kips})(6 \text{ ft})$ to 42 kip·ft at D, and by $(7 \text{ kips})(6 \text{ ft})$ to zero at B.

$|M|_{max} = 70 \text{ kip}\cdot\text{ft}$ at E ◀

(b)



$\uparrow \Sigma F_y = 0: \quad A_y - (2 \text{ kips/ft})(8 \text{ ft}) - 3 \text{ kips} + 10 \text{ kips} = 0$

$\mathbf{A}_y = 9 \text{ kips} \uparrow$

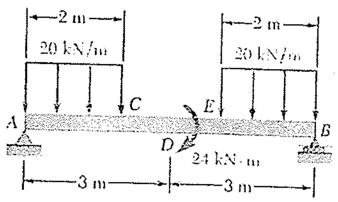
$\circlearrowleft \Sigma M_A = 0: \quad M_A + (4 \text{ ft})(2 \text{ kips/ft})(8 \text{ ft}) + (14 \text{ ft})(3 \text{ kips})$

$- (20 \text{ ft})(10 \text{ kips}) = 0 \qquad \mathbf{M}_A = 94 \text{ kip}\cdot\text{ft}$

Shear Diag:

V jumps to 9 kips at A, then decreases, at 2 kips/ft, to -7 kips at C to D, drops 3 kips to -10 kips from D to B and jumps 10 kips to 0.

Note: $V = 0$ where $9 \text{ kips} - (2 \text{ kips/ft})x = 0, \ x = 4.5 \text{ ft}$.

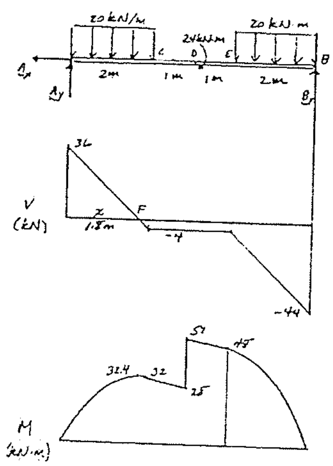


PROBLEM 7.76

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the location and magnitude of the maximum absolute value of the bending moment.

SOLUTION

(*a*)



$$\circlearrowleft \Sigma M_A = 0: \ (6 \text{ m})B_y - (1 \text{ m})(20 \text{ kN/m})(2 \text{ m}) - 24 \text{ kN}\cdot\text{m}$$

$$- (5 \text{ m})(20 \text{ kN/m})(2 \text{ m}) = 0 \qquad \mathbf{B}_y = 44 \text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: \ A_y - 2(20 \text{ kN/m})(2 \text{ m}) + 44 \text{ kN} = 0$$

$$\mathbf{A}_y = 36 \text{ kN}\uparrow$$

Shear Diag:

V jumps to 36 kN at A, then decreases with slope -20 kN/m to -4 kN at C, is constant to E, then decreases with slope -20 kN/m to -44 kN at B.

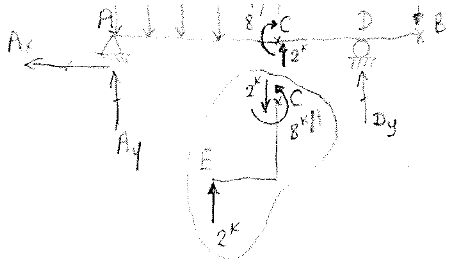
Note: $V = 0$ at F where $36 \text{ kN} - (20 \text{ kN/m})x = 0, \ x = 1.8 \text{ m}$.

Moment Diag:

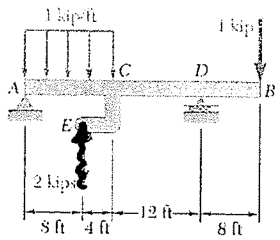
Starting at zero M increases with decreasing slope to $\frac{1}{2}(36 \text{ kN})(1.8 \text{ m})$ $= 32.4 \text{ kN}\cdot\text{m}$ at F, decreases by $\frac{1}{2}(4 \text{ kN})(0.2 \text{ m})$ to $32 \text{ kN}\cdot\text{m}$ at C, then with slope -4 kN to $28 \text{ kN}\cdot\text{m}$ at D, where it jumps to $52 \text{ kN}\cdot\text{m}$, M decreases with slope -4 kN to $48 \text{ kN}\cdot\text{m}$ at E, then with increasingly negative slope by $\left(\frac{4 + 44}{2} \text{ kN}\right)(2 \text{ m})$ to zero at B.

(*b*)

$$|M|_{max} = 52 \text{ kN}\cdot\text{m (at } D) \blacktriangleleft$$



Good Example ☺



PROBLEM 7.79

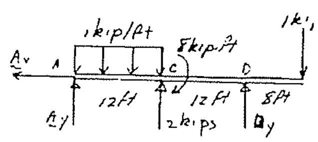
Solve Prob. 7.78 assuming that the 2-kip force applied at E is directed upward.

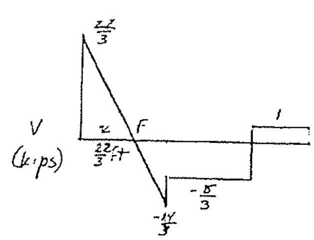
Problem 7.78: For beam AB, (a) draw the shear and bending-moment diagrams, (b) determine the location and magnitude of the maximum absolute value of the bending moment.

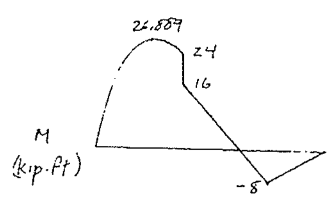
Also, find $M(x) = ??$ between A & C

SOLUTION

(a)







Note: The 2 kip force at E has been replaced by the equivalent force and couple at C.

$$+\circlearrowleft \Sigma M_A = 0: \quad -(6\text{ ft})(1\text{ kip/ft})(12\text{ ft}) + (12\text{ ft})(2\text{ kips}) - 8\text{ kip}\cdot\text{ft}$$

$$+ (24\text{ ft})D_y - (32\text{ ft})(1\text{ kip}) = 0 \qquad \mathbf{D}_y = \frac{11}{3}\text{ kips}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - (1\text{ kip/ft})(12\text{ ft}) + 2\text{ kips} - \frac{11}{3}\text{ kips} - 1\text{ kip} = 0$$

$$\mathbf{A}_y = \frac{22}{3}\text{ kips}\uparrow$$

Shear Diag:

Starting at $\frac{22}{3}$ kips at A, V decreases with slope -1 kip/ft to $-\frac{14}{3}$ kips at C, jumps 2 kips and remains constant at $-\frac{8}{3}$ kips to D, jumps $\frac{11}{3}$ kips and remains constant at 1 kip to B, drops to zero.

$V = 0$ at F, where $\frac{22}{3}\text{kip} - (1\text{ kip/ft})x = 0, \quad x = \frac{22}{3}\text{ft}.$

Moment Diag:

Starting from zero, M increases with decreasing slope to $\frac{1}{2}\left(\frac{22}{3}\text{ kips}\right)\left(\frac{22}{3}\text{ ft}\right) = 26.889\text{ kip}\cdot\text{ft}$ at F. M then decreases by $\frac{1}{2}\left(\frac{14}{3}\text{ kips}\right)\left(\frac{14}{3}\text{ ft}\right)$ to 16 kip·ft at C, jumps to 24 kip·ft, decreases with slope $-\frac{8}{3}$ kips to -8 kip·ft at D, and finally increases with slope 1 kip to zero at B.

(b)

$$|M|_{max} = 26.9\text{ kip}\cdot\text{ft at } F\ (7.33\text{ ft from } A) \blacktriangleleft$$