Chapter 8 : Friction

Laws of Friction, Coefficient of Friction

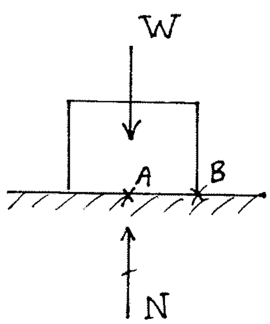
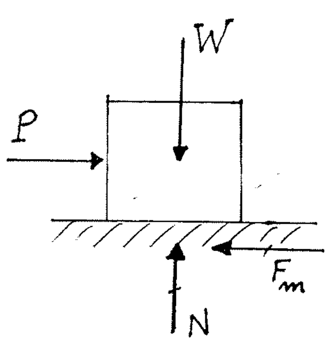
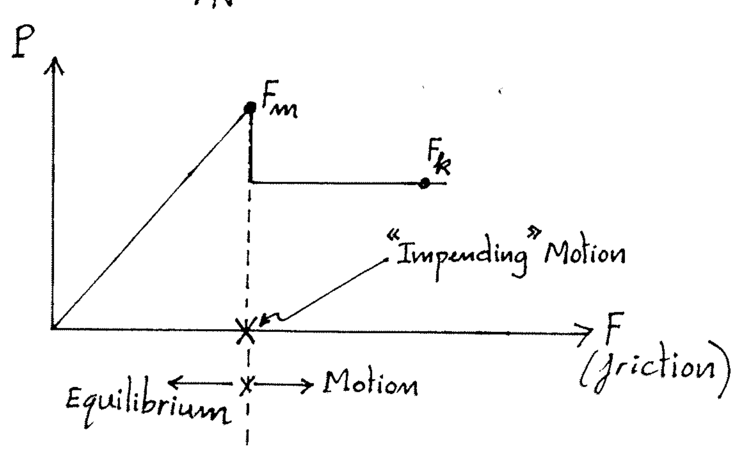


Table 8.1 Approx. Values of Coeff. of Static Friction (μ_s) for Dry Surface	
Metal on metal	0.15 – 0.60
Metal on wood	0.20 – 0.60
⋮	⋮
Wood on leather	0.25 – 0.50
⋮	⋮
Rubber on Concrete	0.60 – 0.90





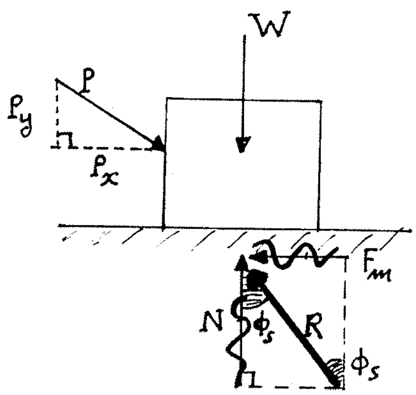
$$F_m = \mu_s N \quad \ldots\ (8.1)$$

$$F_k = \mu_k N \quad \ldots\ (8.2)$$

where : $\mu_k < \mu_s$

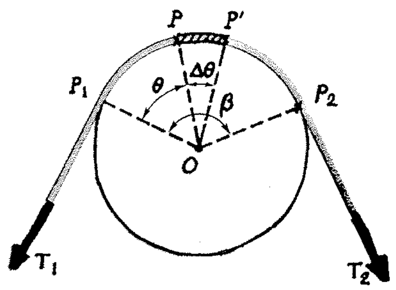
Angles of Friction

ϕ_s = Angle of friction (statics)
ϕ_k = Angle of friction (Kinematics)

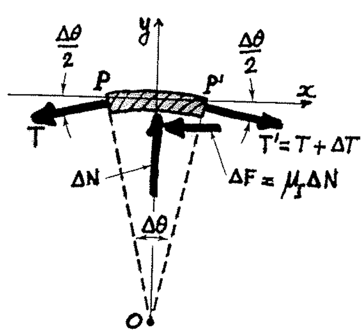


$$\boxed{\begin{aligned} \tan(\phi_s) &= \frac{F_m}{N} = \frac{\mu_s \cancel{N}}{\cancel{N}} = \mu_s \quad \cdots\cdots (8.3) \\ \tan(\phi_k) &= \frac{F_k}{N} = \frac{\mu_k \cancel{N}}{\cancel{N}} = \mu_k \quad \cdots\cdots (8.4) \end{aligned}}$$

Belt Friction



(a)



(b)

$$\overset{+}{\rightarrow}\ \Sigma F_x = 0 = (T+\Delta T)\cos\left(\frac{\Delta\theta}{2}\right) - T\cos\left(\frac{\Delta\theta}{2}\right) - \mu_s \Delta N$$

$$\uparrow + \ \Sigma F_y = 0 = \Delta N - (T+\Delta T)\sin\left(\frac{\Delta\theta}{2}\right) - T\sin\left(\frac{\Delta\theta}{2}\right)$$

First

solve for $\Delta N = (T+\Delta T)\sin\left(\frac{\Delta\theta}{2}\right) + T\sin\left(\frac{\Delta\theta}{2}\right)$

The

Finally:

$$\Delta T\cos\left(\frac{\Delta\theta}{2}\right) - \mu_s(2T+\Delta T)\sin\left(\frac{\Delta\theta}{2}\right) = 0$$

or $$\Delta T\cos\left(\frac{\Delta\theta}{2}\right) - \frac{\mu_s\left(T+\frac{\Delta T}{2}\right)\sin\left(\frac{\Delta\theta}{2}\right)}{\left(\frac{1}{2}\right)} = 0$$

Now, divide both sides of above equation by $\Delta\theta$, one gets:

$$\frac{\Delta T}{\Delta\theta} * \cos\left(\frac{\Delta\theta}{2}\right) - \frac{\mu_s\left(T+\frac{\Delta T}{2}\right) * \sin\left(\frac{\Delta\theta}{2}\right)}{\left(\frac{\Delta\theta}{2}\right)} = 0$$

Since $\Delta\theta$ = small angle $\Rightarrow$ we let $\Delta\theta \to 0^{rad}$, then above Eq. becomes:

$$\frac{dT}{d\theta} * \cos(0^{rad}) - \mu_s\left(T+\cancel{\frac{0}{2}}\right) * \frac{\left(\frac{\Delta\theta}{2}\right)}{\left(\frac{\Delta\theta}{2}\right)} = 0$$

<u>Note</u>: sin(small angle) ≈ tan(small angle) ≈ "small angle itself" (in radian)

$$\frac{dT}{d\theta} - \mu_s T = 0 \Longrightarrow \frac{dT}{T} = \mu_s\, d\theta \Longrightarrow \int_{T_1}^{T_2}\frac{dT}{T} = \int_0^{\beta}\mu_s\, d\theta \Longrightarrow \left[\ln(T)\right]_{T_1}^{T_2} = \mu_s\left[\theta\right]_0^{\beta}$$

or $$\ln(T_2) - \ln(T_1) = \mu_s\beta \Longrightarrow \ln\left(\frac{T_2}{T_1}\right) = \mu_s\beta \Longrightarrow \boxed{\frac{T_2}{T_1} = e^{\mu_s\beta}} \quad \ldots\ldots (8.14)$$