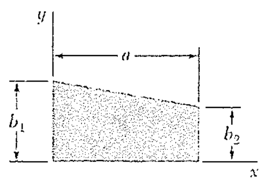


PROBLEM 9.1

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION

First note: $y = \dfrac{b_2 - b_1}{a}x + b_1$

Have $I_y = \int x^2 dA$

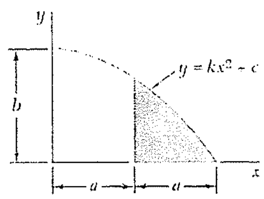
$$= \int_0^a \int_0^{\frac{b_2-b_1}{a}x+b_1} x^2 dy dx$$

$$= \int_0^a x^2\left(\frac{b_2 - b_1}{a}x + b_1\right)dx$$

$$= \left(\frac{1}{4}\frac{b_2 - b_1}{a}x^4 + \frac{1}{3}b_1x^3\right)\bigg|_0^a$$

$$= \frac{1}{12}a^3(b_1 + 3b_2)$$

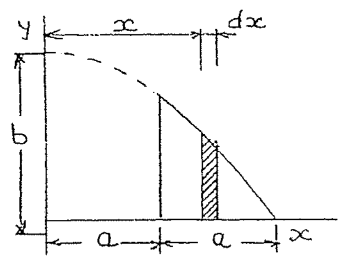
$$I_y = \frac{1}{12}a^3(b_1 + 3b_2) \blacktriangleleft$$



PROBLEM 9.8

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



Have $y = kx^2 + c$

At $x = 0,\ \ y = b{:}\ \ b = k(0) + c$

or $c = b$

At $x = 2a,\ \ y = 0{:}\ \ 0 = k(2a)^2 + b$

or $k = -\dfrac{b}{4a^2}$

Then $y = \dfrac{b}{4a^2}\left(4a^2 - x^2\right)$

Now $dI_x = \dfrac{1}{3}y^3dx$

$$= \frac{1}{3}\frac{b^3}{64a^6}\left(4a^2 - x^2\right)^3 dx$$

PROBLEM 9.8 CONTINUED

Then

$$I_x = \int dI_x$$

$$= \frac{1}{3}\frac{b^3}{64a^6}\int_a^{2a}\left(4a^2 - x^2\right)^3 dx$$

$$= \frac{b^3}{192a^6}\int_a^{2a}\left(64a^6 - 48a^4x^2 + 12a^2x^4 - x^6\right)dx$$

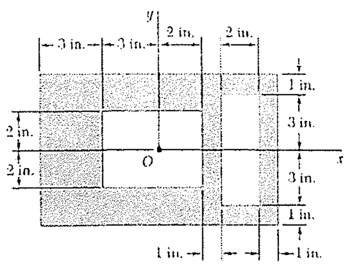
$$= \frac{b^3}{192a^6}\left[64a^6x - 16a^4x^3 + \frac{12}{5}a^2x^5 - \frac{x^7}{7}\right]_a^{2a}$$

$$= \frac{b^3}{192a^6}\left[64a^7(2-1) - 16a^7(8-1)\right.$$

$$\left. + \frac{12}{5}a^7(32-1) - \frac{1}{7}(128-1)\right]$$

$$= \frac{ab^3}{192}\left(64 - 112 + \frac{372}{5} - \frac{127}{7}\right) = 0.043006ab^3$$

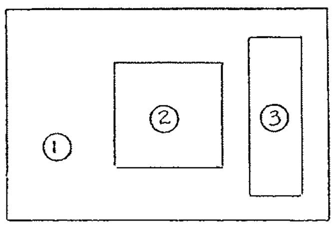
$I_x = 0.0430ab^3$ ◀



PROBLEM 9.34

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the y axis.

SOLUTION



Have $A = A_1 - A_2 - A_3$

$$= \left[(12)(8) - (5)(4) - (2)(6)\right] \text{in}^2$$

$$= 64 \text{ in}^2$$

Have $I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$

$$= \left\{\left[\frac{1}{12}(8)(12)^3\right] - \left[\frac{1}{12}(4)(5)^3 + 20\left(\frac{1}{2}\right)^2\right] - \left[\frac{1}{12}(6)(2)^3 + 12(4)^2\right]\right\} \text{in}^4$$

$$= \left[(1152) - (41.667 + 5) - (4 + 192)\right] \text{in}^4$$

$$= 909.33 \text{ in}^4$$

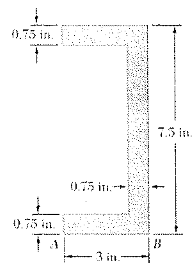
$I_y = 909 \text{ in}^4$ ◀

And $k_y = \sqrt{\dfrac{I_y}{A}}$

$$= \sqrt{\frac{909.33 \text{ in}^4}{64 \text{ in}^2}}$$

$$= 3.77 \text{ in.}$$

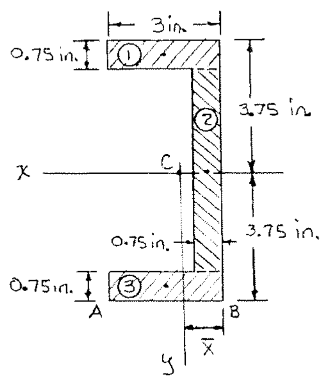
$k_y = 3.77$ in. ◀



PROBLEM 9.41

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION



Determination of centroid:

$\bar{y} = 0$ by symmetry.

Part	Area (in^2)	$\bar{x}$ (in.)	$\bar{x}A(\text{in}^3)$
1	$3(0.75) = 2.25$	1.5	3.375
2	$6(0.75) = 4.50$	0.375	1.6875
3	$3(0.75) = 2.25$	1.5	3.375
Σ	9.00		8.4375

$$\bar{x} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{8.4375 \text{ in}^3}{9.00 \text{ in}^2} = 0.9375 \text{ in.}$$

Determination of $\bar{I}_x$:

Part ①: $\bar{I}_x = \frac{1}{12}(3 \text{ in.})(0.75 \text{ in.})^3 + (3 \text{ in.})(0.75 \text{ in.})(3.375 \text{ in.})^2 = 25.734 \text{ in}^4$

Part ②: $\bar{I}_x = \frac{1}{12}(0.75 \text{ in.})(6 \text{ in.})^3 = 13.50 \text{ in}^4$

Part ③: (Same as Part ①) $\bar{I}_x = 25.734 \text{ in}^4$

Entire Section: $\bar{I}_x = (25.734 + 13.50 + 25.734) \text{ in}^4$

$= 64.97 \text{ in}^4$ $\qquad \bar{I}_x = 65.0 \text{ in}^4$ ◀

Determination of $\bar{I}_y$:

Part ①: $\bar{I}_y = \frac{1}{12}(0.75 \text{ in.})(3 \text{ in.})^3 + (0.75 \text{ in.})(3 \text{ in.})[(1.5 - 0.9375) \text{ in.}]^2$

$= 2.3994 \text{ in}^4$

Part ②: $\bar{I}_y = \frac{1}{12}(6 \text{ in.})(0.75 \text{ in.})^3 + (6 \text{ in.})(0.75 \text{ in.})[(0.9375 - 0.375) \text{ in.}]^2$

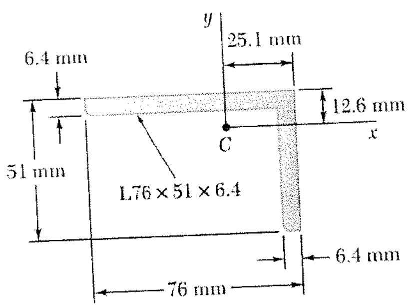
$= 1.6348 \text{ in}^4$

Part ③: (Same as Part ①) $\bar{I}_y = 2.3994 \text{ in}^4$

Entire Section: $\bar{I}_y = (2.3994 + 1.6348 + 2.3994) \text{ in}^4$

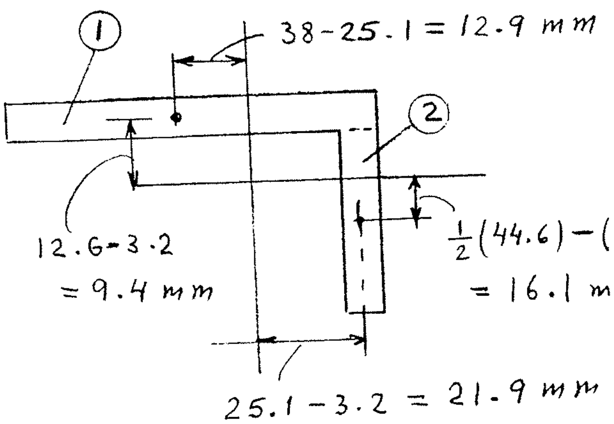
$= 6.434 \text{ in}^4$ $\qquad \bar{I}_y = 6.43 \text{ in}^4$ ◀

PROBLEM 9.74



Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



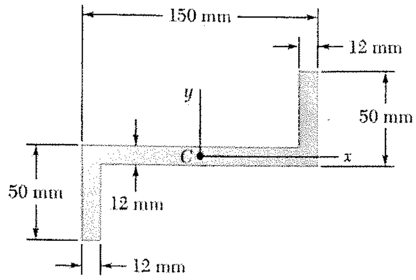
Have $\qquad I_{xy} = (I_{xy})_1 + (I_{xy})_2$

For each rectangle $\qquad I_{xy} = \bar{I}_{x'y'} + A\bar{x}\,\bar{y} \quad$ and $\quad \bar{I}_{x'y'} = 0$ (symmetry)

Thus $\qquad \bar{I}_{xy} = \Sigma \bar{x}\,\bar{y}\,A$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$A\bar{x}\,\bar{y}$, mm^4
1	$76(6.4) = 486.4$	-12.9	9.4	$-58\,980.86$
2	$44.6(6.4) = 285.44$	21.9	-16.1	$-100\,643.29$
Σ				$-159\,624.15$

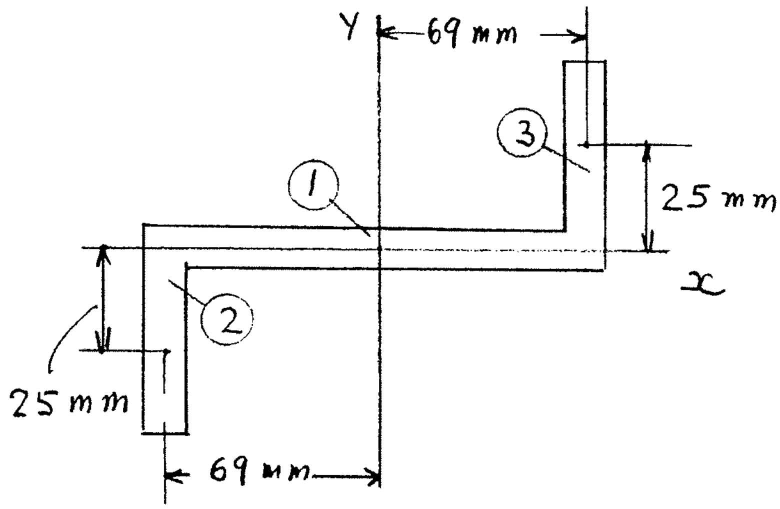
or $\bar{I}_{xy} = -0.1596 \times 10^6$ mm^4 ◀



PROBLEM 9.75

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



Have $\quad \bar{I}_{xy} = \left(\bar{I}_{xy}\right)_1 + \left(I_{xy}\right)_2 + \left(I_{xy}\right)_3$

Now symmetry implies $\quad \left(\bar{I}_{xy}\right)_1 = 0$

and for the other rectangles $\quad I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A \quad$ where $\quad \bar{I}_{x'y'} = 0$ (symmetry)

Thus

$$\bar{I}_{xy} = \left(\bar{x}\,\bar{y}A\right)_2 + \left(\bar{x}\,\bar{y}A\right)_3$$

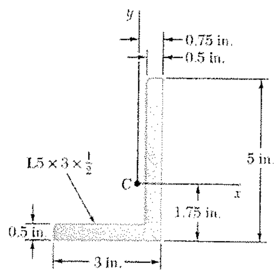
$$= (-69\text{ mm})(-25\text{ mm})\left[(12\text{ mm})(38\text{ mm})\right]$$

$$+ (69\text{ mm})(25\text{ mm})\left[(12\text{ mm})(38\text{ mm})\right]$$

$$= (786\ 600 + 786\ 600)\text{ mm}^4 = 1\ 573\ 200\text{ mm}^4$$

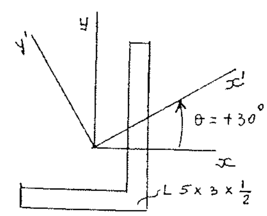
or $\bar{I}_{xy} = 1.573 \times 10^6\text{ mm}^4$ ◀

PROBLEM 9.84



Determine the moments of inertia and the product of inertia of the L5 × 3 × $\frac{1}{2}$ - in. angle cross section of Prob. 9.78 with respect to new centroidal axes obtained by rotating the x and y axes through 30° counterclockwise.

SOLUTION



From Problem 9.78

$$\bar{I}_{xy} = 2.8125 \text{ in}^4$$

From Figure 9.13

$$\bar{I}_x = 9.45 \text{ in}^4, \qquad \bar{I}_y = 2.58 \text{ in}^4$$

Now

$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 6.015 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = 3.435 \text{ in}^4$$

Using Equations (9.18), (9.19), and (9.20)

Equation (9.18):

$$\bar{I}_{x'} = \frac{\bar{I}_x + \bar{I}_y}{2} + \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= \left[6.015 + 3.435\cos(60°) - 2.8125\sin(60°)\right]\text{in}^4 = 5.2968 \text{ in}^4$$

or $\bar{I}_{x'} = 5.30 \text{ in}^4$ ◀

Equation (9.19):

$$\bar{I}_{y'} = \frac{\bar{I}_x + \bar{I}_y}{2} - \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= \left[6.015 - 3.435\cos(60°) + 2.8125\sin(60°)\right]\text{in}^4 = 6.7332 \text{ in}^4$$

or $\bar{I}_{y'} = 6.73 \text{ in}^4$ ◀

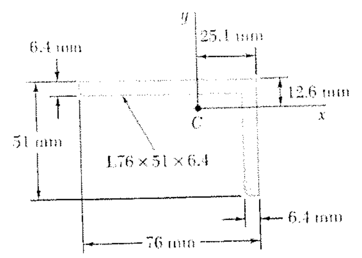
Equation (9.20):

$$\bar{I}_{x'y'} = \frac{\bar{I}_x - \bar{I}_y}{2}\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= \left[3.435\sin(60°) + 2.8125\cos(60°)\right]\text{in}^4 = 4.3810 \text{ in}^4$$

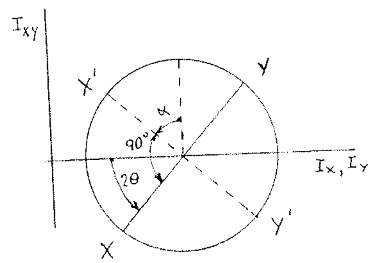
or $\bar{I}_{x'y'} = 4.38 \text{ in}^4$ ◀

PROBLEM 9.95



Using Mohr's circle, determine the moments of inertia and the product of inertia of the L76 × 51 × 6.4-mm angle cross section of Prob. 9.74 with respect to new centroidal axes obtained by rotating the x and y axes through 45° clockwise.

SOLUTION



From Problems 9.74 and 9.83

$$\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = -0.1596 \times 10^6 \text{ mm}^4$$

Now $$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 0.3095 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= 0.21463 \times 10^6 \text{ mm}^4$$

Then $$2\theta_m = \tan^{-1}\left[\frac{-2(-0.1596)}{0.166 - 0.453}\right] = -48.04°$$

and $$\alpha + 90° - 2\theta = 90°; \ \alpha = 2\theta_m$$

Then

$$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\sin\alpha = \left(0.3095 - 0.21463\sin 48.04°\right) \times 10^6 \text{ mm}^4$$

$$= 0.14989 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 0.1499 \times 10^6 \text{ mm}^4$ ◀

and

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\sin\alpha = \left(0.3095 + 0.21463\sin 48.04°\right) \times 10^6 \text{ mm}^4$$

$$= 0.46910 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{y'} = 0.4690 \times 10^6 \text{ mm}^4$ ◀

and

$$\bar{I}_{x'y'} = R\cos\alpha = 0.21463\cos 48.04° = 0.1435 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'y'} = 0.1435 \times 10^6 \text{ mm}^4$ ◀