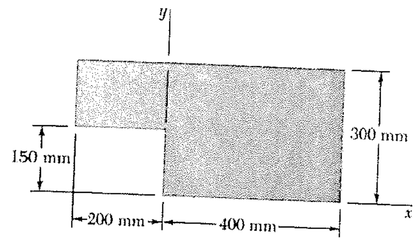
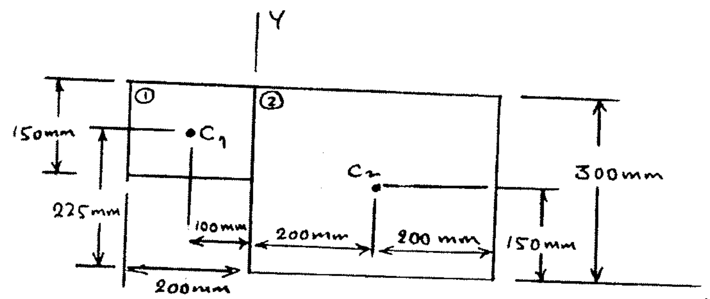
PROBLEM 5.1



Locate the centroid of the plane area shown.

SOLUTION



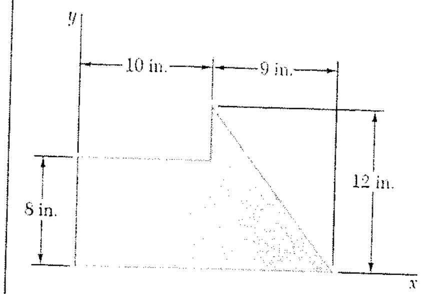
	A, mm^2	$\bar{x}, \text{mm}$	$\bar{y}, \text{mm}$	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$200 \times 150 = 30000$	−100	250	−30000000	6750000
2	$400 \times 300 = 120000$	200	150	24000000	18000000
Σ	150000			21000000	24750000

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{21000000}{150000} \text{mm}$

or $\bar{X} = 140.0 \text{ mm}$ ◀

$\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{24750000}{150000} \text{mm}$

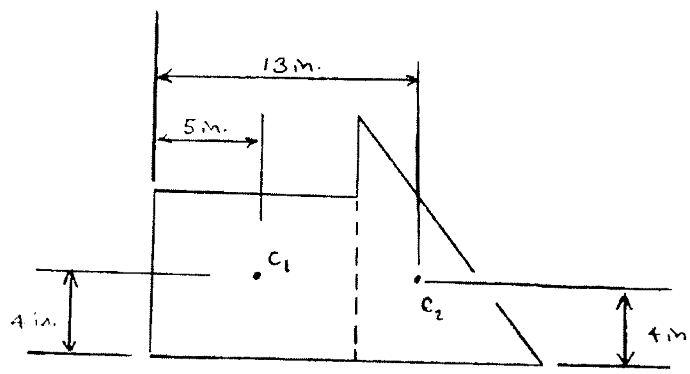
or $\bar{Y} = 165.0 \text{ mm}$ ◀



PROBLEM 5.2

Locate the centroid of the plane area shown.

SOLUTION



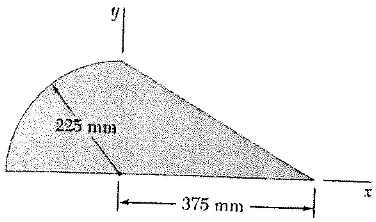
	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$10 \times 8 = 80$	5	4	400	320
2	$\frac{1}{2} \times 9 \times 12 = 54$	13	4	702	216
Σ	134			1102	536

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{1102}{134}$ or $\bar{X} = 8.22 \text{ in.}$ ◀

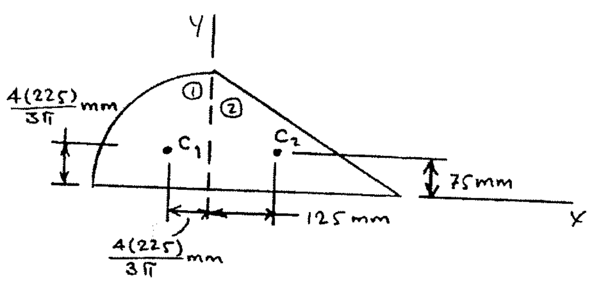
and $\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{1102}{134}$ or $\bar{Y} = 4.00 \text{ in.}$ ◀

PROBLEM 5.5

Locate the centroid of the plane area shown.



SOLUTION

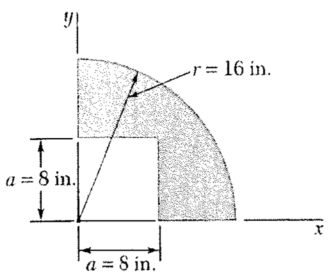


	A, mm^2	$\bar{x}, \text{mm}$	$\bar{y}, \text{mm}$	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$\dfrac{\pi(225)^2}{4} = 39\ 761$	$-\dfrac{4(225)}{3\pi} = -95.493$	95.493	−3 796 900	3 796 900
2	$\dfrac{1}{2}(375)(225) = 42\ 188$	125	75	5 273 500	3 164 100
Σ	81 949			1 476 600	6 961 000

Then $\bar{X} = \dfrac{\Sigma\bar{x}A}{\Sigma A} = \dfrac{1476600}{81949}\text{ mm}$ or $\bar{X} = 18.02\text{ mm}$ ◀

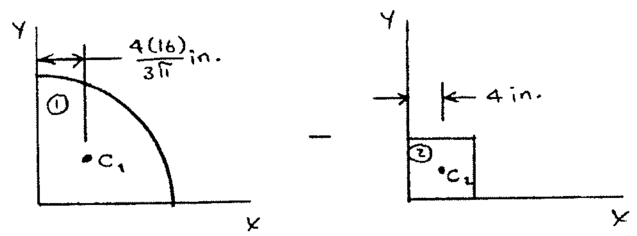
$\bar{Y} = \dfrac{\Sigma\bar{y}A}{\Sigma A} = \dfrac{6961000}{81949}\text{ mm}$ or $\bar{Y} = 84.9\text{ mm}$ ◀

PROBLEM 5.7



Locate the centroid of the plane area shown.

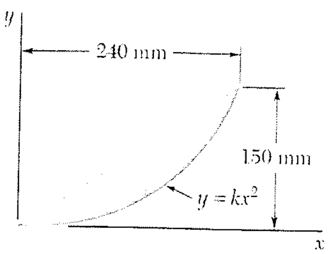
SOLUTION



	A, in^2	$\bar{x}, \text{in.}$	$\bar{x}A, \text{in}^3$
1	$\dfrac{\pi(16)^2}{4} = 201.06$	$\dfrac{4(16)}{3\pi} = 6.7906$	1365.32
2	$-(8)(8) = -64$	4	-256
Σ	137.06		1109.32

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{1109.32}{137.06}\text{ in.}$ or $\bar{X} = 8.09\text{ in.}$ ◀

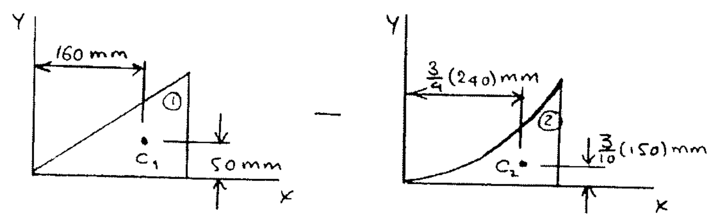
and $\bar{Y} = \bar{X}$ by symmetry or $\bar{Y} = 8.09\text{ in.}$ ◀



PROBLEM 5.12

Locate the centroid of the plane area shown.

SOLUTION



	A, mm^2	$\bar{x}, \text{mm}$	$\bar{y}, \text{mm}$	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$\frac{1}{2}(240)(150) = 18\,000$	160	50	2 880 000	900 000
2	$-\frac{1}{3}(240)(150) = 12\,000$	$\frac{3}{4}(240) = 180$	$\frac{3}{10}(150) = 45$	−2 160 000	−540 000
Σ	6000			720 000	360 000

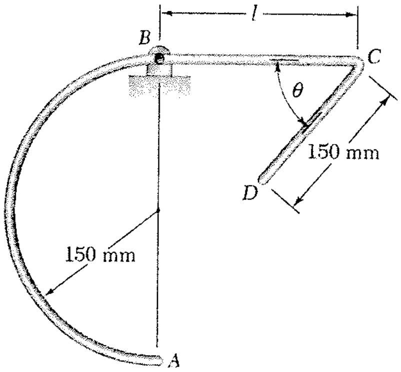
Then

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{720\,000}{6000} \text{ mm}$$

or $\bar{X} = 120.0$ mm ◀

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{360\,000}{6000} \text{ mm}$$

or $\bar{Y} = 60.0$ mm ◀

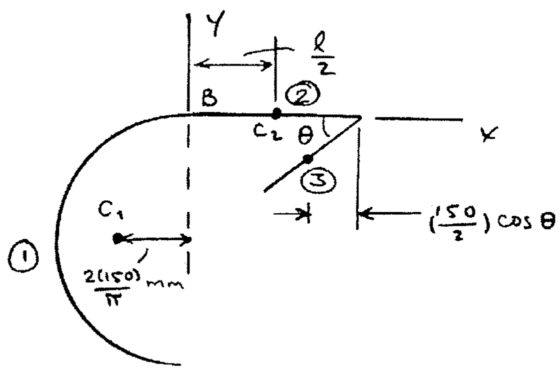


PROBLEM 5.26

The homogeneous wire *ABCD* is bent as shown and is supported by a pin at *B*. Knowing that $l = 200$ mm, determine the angle θ for which portion *BC* of the wire is horizontal.

SOLUTION

The wire supported only by the pin at *B* is a two-force body. For equilibrium the center of gravity of the wire must lie directly under *B*. Also, because the wire is homogeneous the center of gravity will coincide with the centroid. In other words, $\bar{x} = 0$, or $\Sigma \bar{x}L = 0$.



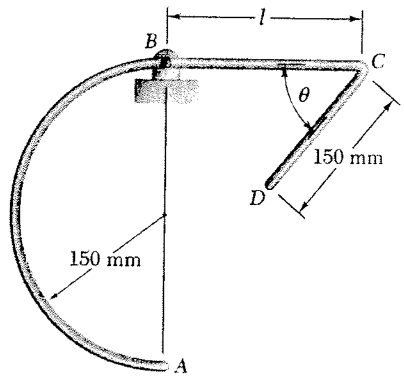
$$\Sigma \bar{x}L = -\frac{2(150\text{ mm})}{\pi}\left[\pi(150\text{ mm})\right] + \left(\frac{200\text{ mm}}{2}\right)(200\text{ mm}) + \left(200\text{ mm} - \frac{150\text{ mm}}{2}\cos\theta\right)(150\text{ mm}) = 0$$

or

$$\cos\theta = \frac{5000}{11250}$$

or $\theta = 63.6°$ ◀

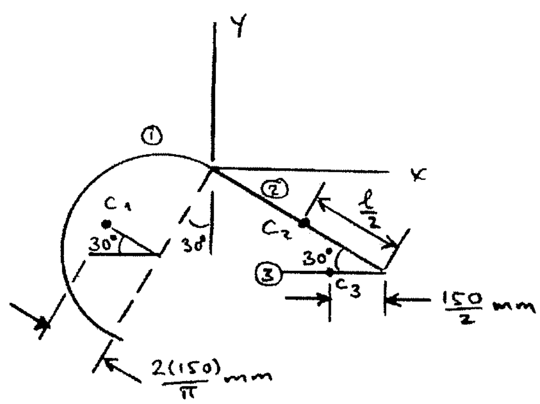
PROBLEM 5.27



The homogeneous wire $ABCD$ is bent as shown and is supported by a pin at B. Knowing that $\theta = 30°$, determine the length l for which portion CD of the wire is horizontal.

SOLUTION

The wire supported only by the pin at B is a two-force body. For equilibrium the center of gravity of the wire must lie directly under B. Also, because the wire is homogeneous the center of gravity will coincide with the centroid. In other words, $\bar{x} = 0$, or $\Sigma \bar{x}L = 0$.



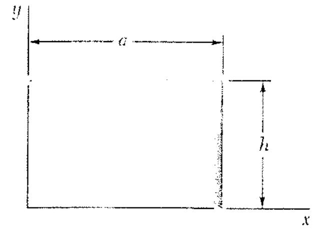
$$\Sigma \bar{x}L = -\frac{2(150\text{ mm})}{\pi}\left[\pi(150\text{ mm})\right] + \left(\frac{200\text{ mm}}{2}\right)(200\text{ mm}) + \left(200\text{ mm} - \frac{150\text{ mm}}{2}\cos\theta\right)(150\text{ mm}) = 0$$

or

$l^2 + 300l - 197602 = 0.$

Solving for l: $l = 319.15$, and $l = -619.15$, and discarding the negative root

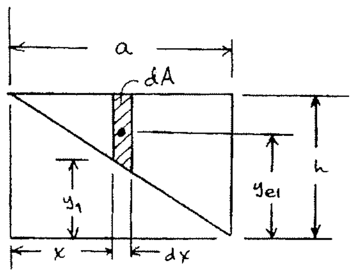
$l = 319$ mm ◀



PROBLEM 5.31

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.

SOLUTION



Note that $y_1 = -\dfrac{h}{a}x + h$

$= \dfrac{h}{a}(a - x)$

Choose the area element (EL) as

$$dA = (h - y_1)dx = \frac{h}{a}xdx$$

Then $$A = \frac{h}{a}\int_0^a xdx = \frac{h}{a}\left[\frac{1}{2}x^2\right]_0^a = \frac{1}{2}ah$$

Now, noting that $\overline{x}_{EL} = x$, and $\overline{y}_{EL} = \dfrac{1}{2}(h + y_1)$

$$\overline{x} = \frac{1}{A}\int xdA = \frac{2}{ah}\int_0^a x\left(\frac{h}{a}xdx\right) = \frac{2}{a^2}\left[\frac{1}{3}x^3\right]_0^a = \frac{2}{3}a$$

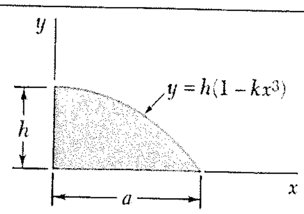
$$\overline{y} = \frac{1}{A}\int\frac{1}{2}(h + y_1)dA = \frac{2}{ah}\int_0^h\left[\frac{1}{2}(h + y_1)\right]\left[(h - y_1)dx\right] = \frac{2}{ah}\left(\frac{1}{2}\right)\int_0^a\left(h^2 - y_1^2\right)dx$$

$$= \frac{1}{ah}\int_0^a\left[h^2 - \frac{h^2}{a^2}(a - x)^2\right]dx = \frac{h}{a}\left[x + \left(\frac{1}{3}\right)\left(\frac{1}{a^2}\right)(a - x)^3\right]_0^A = \frac{h}{a}\left(a - \frac{1}{3}a\right) = \frac{2}{3}h$$

Therefore:

$\overline{x} = \dfrac{2}{3}a$ ◀

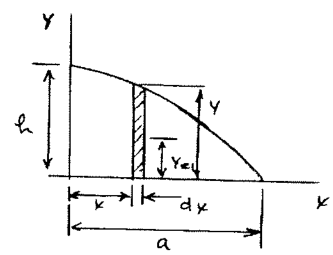
$\overline{y} = \dfrac{2}{3}h$ ◀



PROBLEM 5.32

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.

SOLUTION



First determine k:

For $x = a$, $y = 0$ and therefore

$$0 = h\left(1 - ka^3\right) \text{ or } k = a^{-3}, \text{ and therefore}$$

$$y = h\left(1 - \frac{x^3}{a^3}\right)$$

Choosing an area element as in the figure:

$$x_{EL} = x, \qquad y_{EL} = \frac{y}{2}, \qquad \text{and } dA = ydx$$

$$A = \int dA = \int_0^a ydx = \int_0^a h\left(1 - \frac{x^3}{a^3}\right)dx = h\left[x - \frac{x^4}{4a^3}\right]_0^a = \frac{3}{4}ah$$

$$\int x_{EL}dA = \int_0^a xydx = \int_0^a h\left(x - \frac{x^4}{a^3}\right)dx = b\left[\frac{x^2}{2} - \frac{x^5}{5a^3}\right]_0^a = \frac{3}{10}a^2b$$

$$\int y_{EL}dA = \int_0^a\left(\frac{y}{2}\right)ydx = \frac{1}{2}\int_0^a h^2\left(x - \frac{x^3}{a^3}\right)^2 dx = \frac{b^2}{2}\int_0^a\left(1 - \frac{2x^3}{a^3} + \frac{x^6}{a^6}\right)dx = \frac{b^2}{2}\left[x - \frac{x^4}{2a^3} + \frac{x^7}{7a^6}\right]_0^a = \frac{9}{28}ab^2$$

Now

$$\bar{x} = \frac{1}{A}\int x_{EL}dA = \frac{4}{3ab}\frac{3a^2b}{10} = \frac{2}{5}a$$

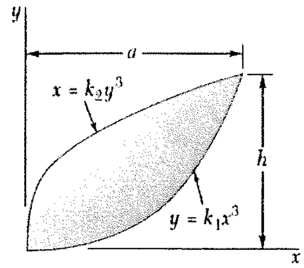
and

$$\bar{y} = \frac{1}{A}\int y_{EL}dA = \frac{4}{3ab}\frac{9ab^2}{28} = \frac{3}{7}b$$

Therefore:

$$\bar{x} = \frac{2}{5}a \blacktriangleleft$$

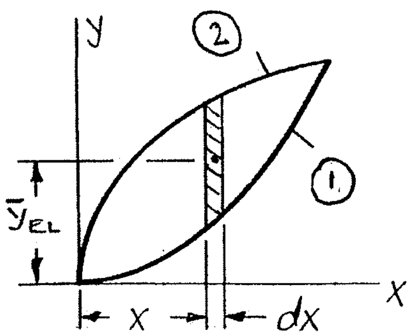
$$\bar{y} = \frac{3}{7}b \blacktriangleleft$$



PROBLEM 5.33

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.

SOLUTION



For the element (EL) shown

At $\quad x = a, y = h$: $\quad h = k_1 a^3 \quad$ or $\quad k_1 = \dfrac{h}{a^3}$

$a = k_2 h^3 \quad$ or $\quad k_2 = \dfrac{a}{h^3}$

Hence, on line 1

$$y = \frac{h}{a^3}x^3$$

and on line 2

$$y = \frac{h}{a^{1/3}}x^{1/3}$$

Then

$$dA = \left(\frac{h}{a^{1/3}}x^{1/3} - \frac{h}{a^3}x^3\right)dx \quad \text{and} \quad \bar{y}_{EL} = \frac{1}{2}\left(\frac{h}{a^{1/3}}x^{1/3} + \frac{h}{a^3}x^3\right)$$

$$\therefore\ A = \int dA = \int_0^a\left(\frac{h}{a^{1/3}}x^{1/3} - \frac{h}{a^3}x^3\right)dx = h\left(\frac{3}{4a^{1/3}}x^{4/3} - \frac{1}{4a^3}x^4\right)\Bigg|_0^a = \frac{1}{2}ah$$

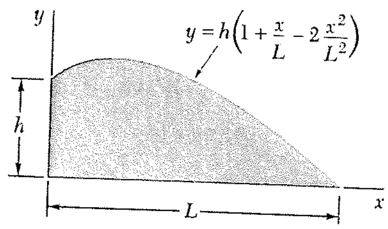
$$\int \bar{x}_{EL}dA = \int_0^a x\left(\frac{h}{a^{1/3}}x^{1/3} - \frac{h}{a^3}x^3\right)dx = h\left(\frac{3}{7a^{1/3}}x^{7/3} - \frac{1}{5a^3}x^5\right)\Bigg|_0^a = \frac{8}{35}a^2h$$

$$\int \bar{y}_{EL}dA = \int_0^a \frac{1}{2}\left(\frac{h}{a^{1/3}}x^{1/3} + \frac{h}{a^3}x^3\right)\left(\frac{h}{a^{1/3}}x^{1/3} - \frac{h}{a^3}x^3\right)dx$$

$$= \frac{h^2}{2}\int_0^a\left(\frac{x^{2/3}}{a^{2/3}} - \frac{x^6}{a^6}\right)dx = \frac{h^2}{2}\left(\frac{3}{5}\frac{x^{5/3}}{a^{5/3}} - \frac{1}{7}\frac{x^6}{a^6}\right)\Bigg|_0^a = \frac{8}{35}ah^2$$

From $\bar{x}A = \int \bar{x}_{EL}dA$: $\quad \bar{x}\left(\dfrac{ah}{2}\right) = \dfrac{8}{35}a^2h \quad$ or $\bar{x} = \dfrac{16}{35}a$ ◀

and $\quad \bar{y}A = \int \bar{y}_{EL}dA$: $\quad \bar{y}\left(\dfrac{ah}{2}\right) = \dfrac{8}{35}ah^2 \quad$ or $\bar{y} = \dfrac{16}{35}h$ ◀

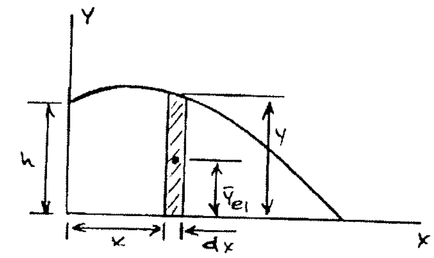


PROBLEM 5.39

Determine by direct integration the centroid of the area shown.

SOLUTION

Using the area element shown:



$\bar{x}_{EL} = x, \qquad \bar{y}_{EL} = \frac{y}{2}, \qquad$ and $dA = ydx$

$$A = \int dA = \int_0^L h\left(1 + \frac{x}{L} - 2\frac{x^2}{L^2}\right)dx = h\left[x + \frac{x^2}{2L} - \frac{2}{3}\frac{x^3}{L^2}\right]_0^L = \frac{5}{6}hL$$

$$\int \bar{x}_{EL}dA = \int_0^L xh\left(1 + \frac{x}{L} - 2\frac{x^2}{L^2}\right)dx = h\int_0^L\left(x + \frac{x^2}{L} - 2\frac{x^3}{L^2}\right)dx = h\left[\frac{x^2}{2} + \frac{1}{3}\frac{x^3}{L} - \frac{2}{4}\frac{x^4}{L^2}\right]_0^L = \frac{1}{3}hL^2$$

$$\int \bar{y}_{EL}dA = \frac{1}{2}\int y^2dx = \frac{h^2}{2}\int_0^L\left(1 + \frac{x}{L} - 2\frac{x^2}{L^2}\right)^2 dx = \frac{h^2}{2}\int_0^L\left(1 + \frac{x^2}{L^2} + 4\frac{x^4}{L^4} + 2\frac{x}{L} - 4\frac{x^2}{L^2} - 4\frac{x^3}{L^3}\right)dx$$

$$= \frac{h^2}{2}\left[x + \frac{x^3}{3L^2} + \frac{4x^5}{5L^4} + \frac{x^2}{L} - \frac{4x^3}{3L^2} - \frac{x^4}{L^3}\right]_0^L = \frac{4}{10}h^2L$$

Now

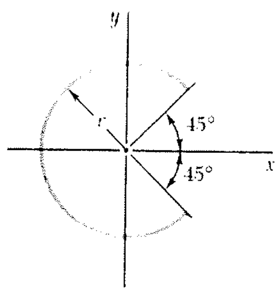
$$\bar{x} = \frac{1}{A}\int x_{EL}dA = \frac{6}{5hL}\left(\frac{1}{3}hL^2\right) = \frac{2}{5}L \text{ and}$$

$$\bar{y} = \frac{1}{A}\int y_{EL}dA = \frac{6}{5hL}\left(\frac{4}{10}h^2L\right) = \frac{12}{25}h$$

Therefore:

$\bar{x} = \frac{2}{5}L$ ◀

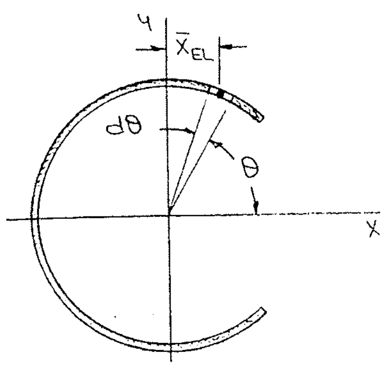
$\bar{y} = \frac{12}{25}h$ ◀



PROBLEM 5.42

A homogeneous wire is bent into the shape shown. Determine by direct integration the x coordinate of its centroid.

SOLUTION



First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line

Now $\quad \bar{x}_{EL} = r\cos\theta \quad$ and $\quad dL = rd\theta$

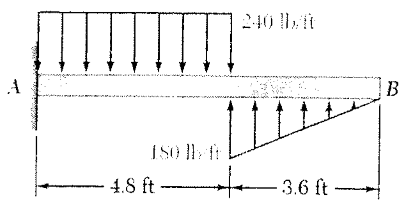
Then $$L = \int dL = \int_{\pi/4}^{7\pi/4} rd\theta = r[\theta]_{\pi/4}^{7\pi/4} = \frac{3}{2}\pi r$$

and $$\int \bar{x}_{EL} dL = \int_{\pi/4}^{7\pi/4} r\cos\theta(rd\theta)$$

$$= r^2[\sin\theta]_{\pi/4}^{7\pi/4} = r^2\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = -r^2\sqrt{2}$$

Thus $$\bar{x}L = \int \bar{x}dL: \quad \bar{x}\left(\frac{3}{2}\pi r\right) = -r^2\sqrt{2}$$

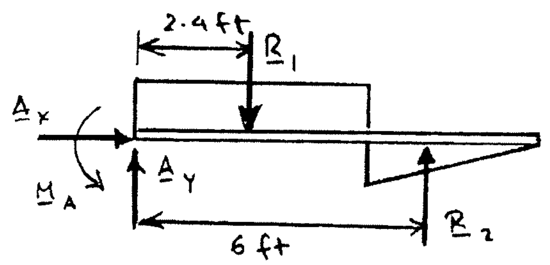
$$\bar{x} = -\frac{2\sqrt{2}}{3\pi}r \blacktriangleleft$$



PROBLEM 5.65

Determine the reactions at the beam supports for the given loading.

SOLUTION



$$R_1 = \left(240\frac{\text{lb}}{\text{ft}}\right)(4.8\text{ ft}) = 1152\text{ lb}$$

$$R_2 = \frac{1}{2}\left(180\frac{\text{lb}}{\text{ft}}\right)(3.6\text{ ft}) = 324\text{ lb}$$

Equilibrium:

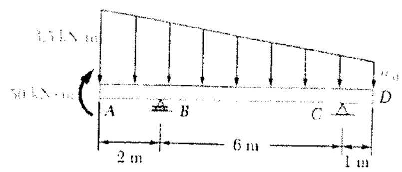
$+\Sigma F_x = 0$: $\quad A_x = 0$

$+\Sigma F_y = 0$: $\quad A_y - 1152\text{ lb} + 324\text{ lb} = 0$

$A_y = 828.00\text{ lb}$ $\qquad$ $\mathbf{A} = 828\text{ lb}\uparrow$ ◀

$+\Sigma M_A = 0$: $\quad M_A - (2.4\text{ ft})(1152\text{ lb}) + (6\text{ ft})(324\text{ lb}) = 0$

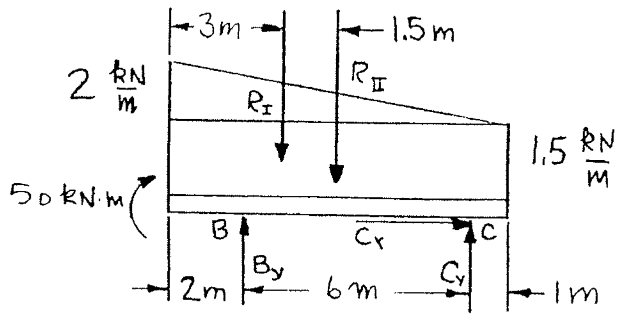
$M_A = 820.80\text{ lb}\cdot\text{ft}$ $\qquad$ $\mathbf{M}_A = 821\text{ lb}\cdot\text{ft}$ ↻ ◀



PROBLEM 5.69

Determine the reactions at the beam supports for the given loading when $w_0 = 1.5$ kN/m.

SOLUTION



Have

$$R_I = \frac{1}{2}(9\text{ m})(2\text{ kN/m}) = 9\text{ kN}$$

$$R_{II} = (9\text{ m})(1.5\text{ kN/m}) = 13.5\text{ kN}$$

Then

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -50\text{ kN}\cdot\text{m} - (1\text{ m})(9\text{ kN}) - (2.5\text{ m})(13.5\text{ kN}) + (6\text{ m})C_y = 0$$

or

$$C_y = 15.4583\text{ kN}$$

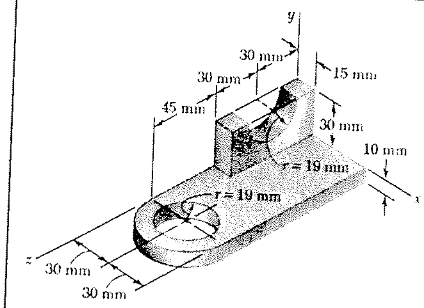
C = 15.46 kN ↑ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y - 9\text{ kN} - 13.5\text{ kN} + 15.4583 = 0$$

or

$$B_y = 7.0417\text{ kN}$$

B = 7.04 kN ↑ ◀

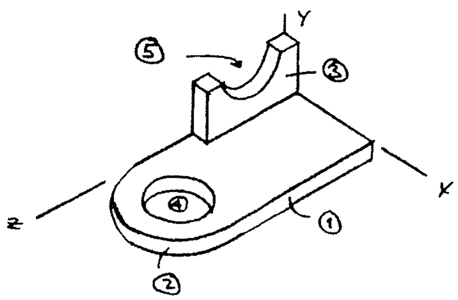


PROBLEM 5.95

For the machine element shown, locate the z coordinate of the center of gravity.

SOLUTION

Assume that the machine element is homogeneous so that its center of gravity coincides with the centroid of the volume.



	V, mm^3	$\bar{z}$, mm	$\bar{z}V$, mm^4
1	$(60)(105)(10) = 63000$	52.5	3 307 500
2	$\frac{1}{2}\pi(30)^2(10) = 14\,137.2$	$105 + \frac{4(30)}{3\pi} = 117.732$	1 664 400
3	$(15)(30)(60) = 27000$	30	810 000
4	$-\pi(19)^2(10) = -11\,341.1$	105	−1 190 820
5	$-\frac{1}{2}\pi(19)^2(15) = -8505.9$	30	−255 180
Σ	84 290		4 335 900

Then $\bar{Z} = \frac{\Sigma \bar{z}V}{\Sigma V} = \frac{4\,335\,900}{84\,290}$ mm

or $\bar{Z} = 51.4$ mm ◀