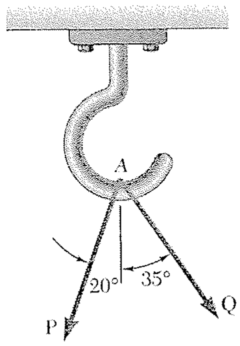
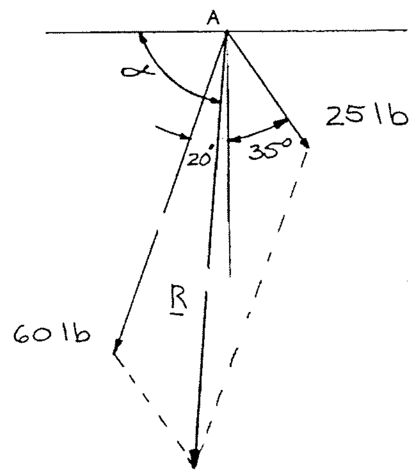


PROBLEM 2.2

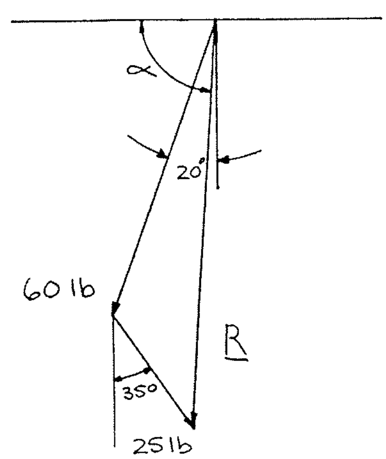
Two forces **P** and **Q** are applied as shown at Point A of a hook support. Knowing that $P = 60$ lb and $Q = 25$ lb, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

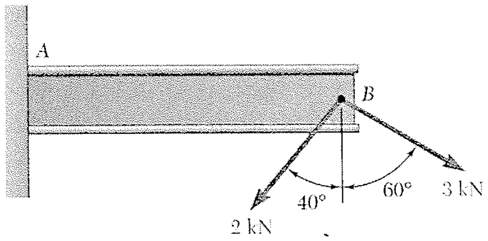
(*a*) Parallelogram law:



(*b*) Triangle rule:



We measure: $R = 77.1$ lb, $\alpha = 85.4°$ **R** $= 77.1$ lb $\measuredangle$ 85.4° ◀

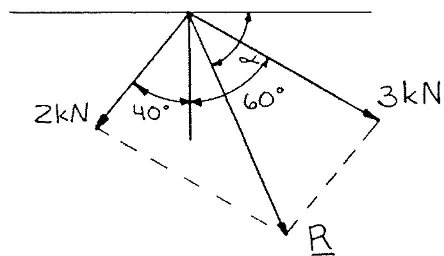


PROBLEM 2.4

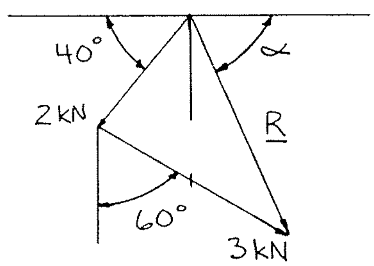
Two forces are applied at Point B of beam AB. Determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

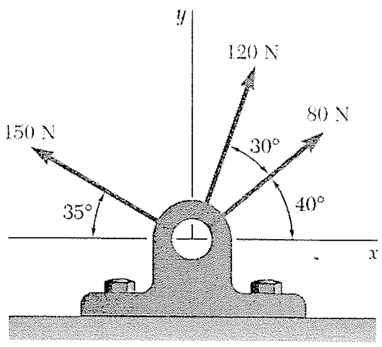
(a) Parallelogram law:



(b) Triangle rule:



We measure: $R = 3.30$ kN, $\alpha = 66.6°$ **R** $= 3.30$ kN ⦫ $66.6°$ ◀



PROBLEM 2.24

Determine the x and y components of each of the forces shown.

SOLUTION

80-N Force: $F_x = +(80\text{ N})\cos 40°$ $\quad F_x = 61.3\text{ N}$ ◀

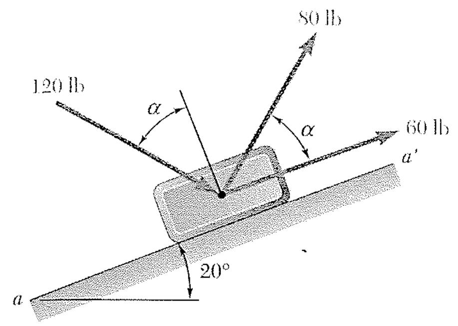
$F_y = +(80\text{ N})\sin 40°$ $\quad F_y = 51.4\text{ N}$ ◀

120-N Force: $F_x = +(120\text{ N})\cos 70°$ $\quad F_x = 41.0\text{ N}$ ◀

$F_y = +(120\text{ N})\sin 70°$ $\quad F_y = 112.8\text{ N}$ ◀

150-N Force: $F_x = -(150\text{ N})\cos 35°$ $\quad F_x = -122.9\text{ N}$ ◀

$F_y = +(150\text{ N})\sin 35°$ $\quad F_y = 86.0\text{ N}$ ◀



PROBLEM 2.37

Knowing that $\alpha = 40°$, determine the resultant of the three forces shown.

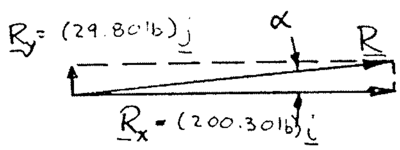
SOLUTION

60-lb Force:

$$F_x = (60\text{ lb})\cos 20° = 56.38\text{ lb}$$
$$F_y = (60\text{ lb})\sin 20° = 20.52\text{ lb}$$

80-lb Force:

$$F_x = (80\text{ lb})\cos 60° = 40.00\text{ lb}$$
$$F_y = (80\text{ lb})\sin 60° = 69.28\text{ lb}$$



120-lb Force:

$$F_x = (120\text{ lb})\cos 30° = 103.92\text{ lb}$$
$$F_y = -(120\text{ lb})\sin 30° = -60.00\text{ lb}$$

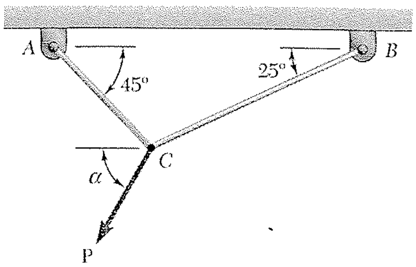
and

$$R_x = \Sigma F_x = 200.30\text{ lb}$$
$$R_y = \Sigma F_y = 29.80\text{ lb}$$
$$R = \sqrt{(200.30\text{ lb})^2 + (29.80\text{ lb})^2}$$
$$= 202.50\text{ lb}$$

Further:

$$\tan\alpha = \frac{29.80}{200.30}$$
$$\alpha = \tan^{-1}\frac{29.80}{200.30}$$
$$= 8.46°$$

R = 203 lb ∡ 8.46° ◀

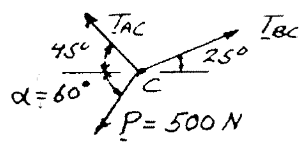


PROBLEM 2.45

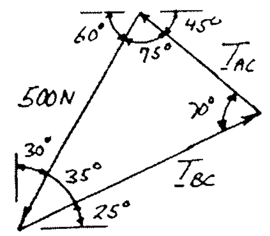
Two cables are tied together at C and are loaded as shown. Knowing that $\mathbf{P} = 500$ N and $\alpha = 60°$, determine the tension in (*a*) in cable AC, (*b*) in cable BC.

SOLUTION

Free-Body Diagram



Force Triangle



Law of sines:

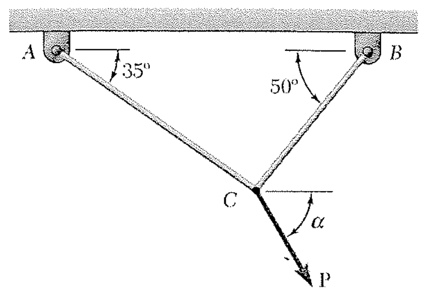
$$\frac{T_{AC}}{\sin 35°} = \frac{T_{BC}}{\sin 75°} = \frac{500 \text{ N}}{\sin 70°}$$

(*a*)

$$T_{AC} = \frac{500 \text{ N}}{\sin 70°} \sin 35° \qquad T_{AC} = 305 \text{ N} ◀$$

(*b*)

$$T_{BC} = \frac{500 \text{ N}}{\sin 70°} \sin 75° \qquad T_{BC} = 514 \text{ N} ◀$$

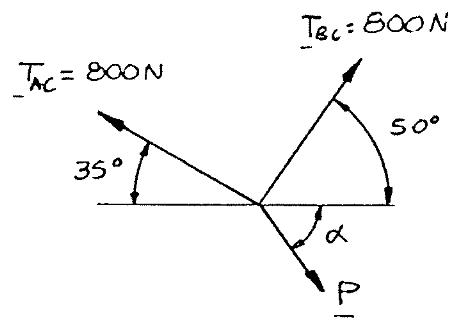


PROBLEM 2.61

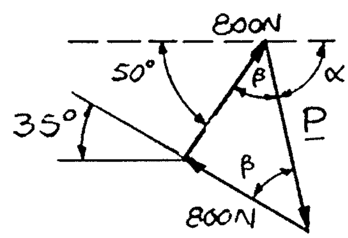
Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N, determine (a) the magnitude of the largest force **P** that can be applied at C, (b) the corresponding value of α.

SOLUTION

Free-Body Diagram: C



Force Triangle



Force triangle is isosceles with

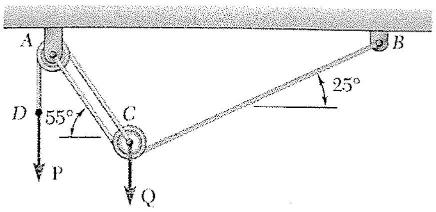
$$2\beta = 180° - 85°$$

$$\beta = 47.5°$$

(a) $P = 2(800\text{ N})\cos 47.5° = 1081\text{ N}$

Since $P > 0$, the solution is correct. $P = 1081\text{ N}$ ◀

(b) $\alpha = 180° - 50° - 47.5° = 82.5°$ $\alpha = 82.5°$ ◀

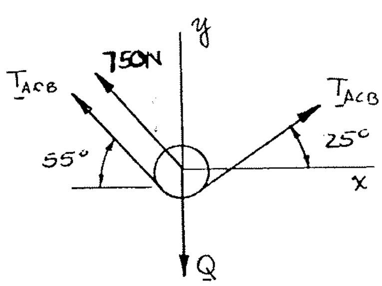


PROBLEM 2.69

A load **Q** is applied to the pulley C, which can roll on the cable ACB. The pulley is held in the position shown by a second cable CAD, which passes over the pulley A and supports a load **P**. Knowing that $P = 750$ N, determine (*a*) the tension in cable ACB, (*b*) the magnitude of load **Q**.

SOLUTION

Free-Body Diagram: Pulley C



(*a*) $\xrightarrow{+} \Sigma F_x = 0$: $T_{ACB}(\cos 25° - \cos 55°) - (750\text{ N})\cos 55° = 0$

Hence: $T_{ACB} = 1292.88$ N

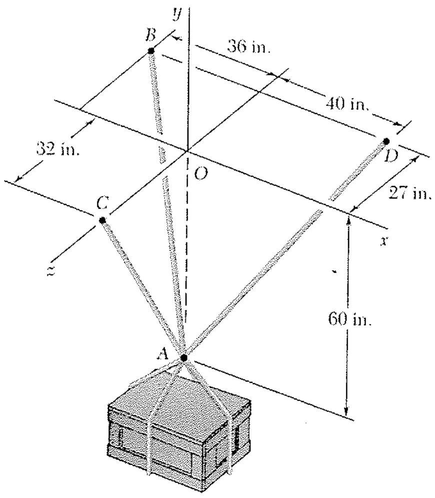
$T_{ACB} = 1293$ N ◀

(*b*) $+\uparrow \Sigma F_y = 0$: $T_{ACB}(\sin 25° + \sin 55°) + (750\text{ N})\sin 55° - Q = 0$

$$(1292.88\text{ N})(\sin 25° + \sin 55°) + (750\text{ N})\sin 55° - Q = 0$$

or $Q = 2219.8$ N $\qquad Q = 2220$ N ◀

PROBLEM 2.106



A 1600-lb crate is supported by three cables as shown. Determine the tension in each cable.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

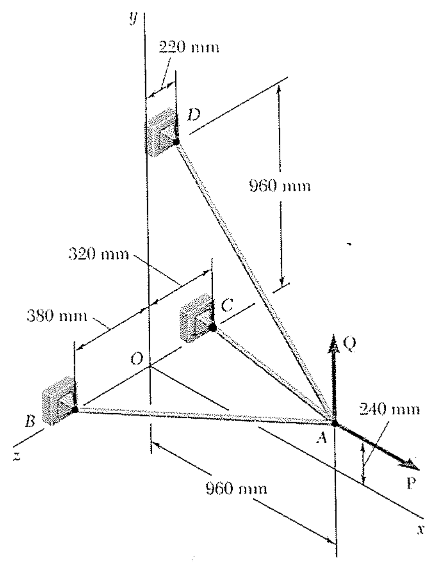
$$-0.48T_{AB} + 0.51948T_{AD} = 0$$
$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

Substituting $W = 1600$ lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$T_{AB} = 571$ lb ◀

$T_{AC} = 830$ lb ◀

$T_{AD} = 528$ lb ◀



PROBLEM 2.108

Three cables are connected at A, where the forces $\mathbf{P}$ and $\mathbf{Q}$ are applied as shown. Knowing that $P = 1200$ N, determine the values of Q for which cable AD is taut.

SOLUTION

We assume that $T_{AD} = 0$ and write $\quad \Sigma \mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + Q\mathbf{j} + (1200\text{ N})\mathbf{i} = 0$

$$\overrightarrow{AB} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} + (380\text{ mm})\mathbf{k} \quad AB = 1060\text{ mm}$$

$$\overrightarrow{AC} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \quad AC = 1040\text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)T_{AC}$$

Substituting into $\Sigma \mathbf{F}_A = 0$, factoring $\mathbf{i}, \mathbf{j}, \mathbf{k}$, and setting each coefficient equal to ϕ gives:

$$\mathbf{i}: \quad -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} + 1200\text{ N} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + Q = 0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 0 \qquad (3)$$

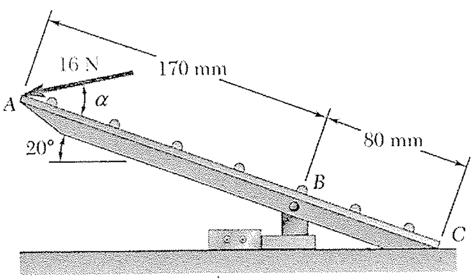
Solving the resulting system of linear equations using conventional algorithms gives:

$$T_{AB} = 605.71\text{ N}$$

$$T_{AC} = 705.71\text{ N}$$

$$Q = 300.00\text{ N} \qquad 0 \le Q < 300\text{ N} \blacktriangleleft$$

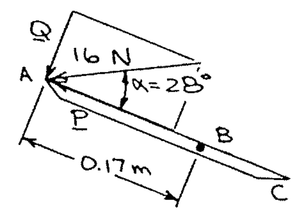
Note: This solution assumes that Q is directed upward as shown ($Q \ge 0$), if negative values of Q are considered, cable AD remains taut, but AC becomes slack for $Q = -460$ N.



PROBLEM 3.2

A foot valve for a pneumatic system is hinged at B. Knowing that $\alpha = 28°$, determine the moment of the 16-N force about Point B by resolving the force into components along ABC and in a direction perpendicular to ABC.

SOLUTION



First resolve the 4-lb force into components **P** and **Q**, where

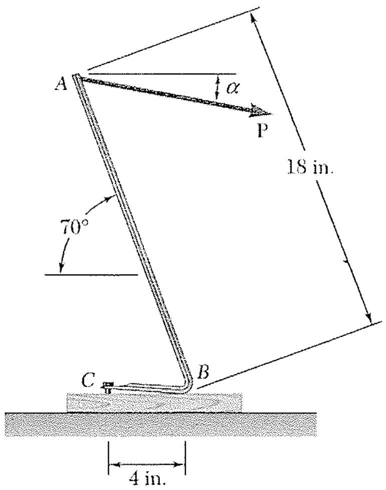
$$Q = (16\text{ N})\sin 28°$$
$$= 7.5115\text{ N}$$

Then

$$M_B = r_{A/B}Q$$
$$= (0.17\text{ m})(7.5115\text{ N})$$
$$= 1.277\text{ N}\cdot\text{m}$$

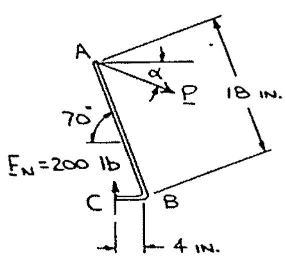
or $\mathbf{M}_B = 1.277\text{ N}\cdot\text{m}$ ↻ ◀

PROBLEM 3.8



It is known that a vertical force of 200 lb is required to remove the nail at C from the board. As the nail first starts moving, determine (a) the moment about B of the force exerted on the nail, (b) the magnitude of the force **P** that creates the same moment about B if $\alpha = 10°$, (c) the smallest force **P** that creates the same moment about B.

SOLUTION



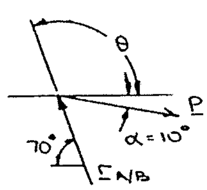
(a) We have $M_B = r_{C/B} F_N$

$$= (4 \text{ in.})(200 \text{ lb})$$
$$= 800 \text{ lb} \cdot \text{in.}$$

or $M_B = 800 \text{ lb} \cdot \text{in.}$ ↻ ◀

(b) By definition $M_B = r_{A/B} P \sin\theta$

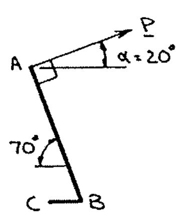
$$\theta = 10° + (180° - 70°)$$
$$= 120°$$



Then $800 \text{ lb} \cdot \text{in.} = (18 \text{ in.}) \times P \sin 120°$

or $P = 51.3$ lb ◀

(c) For **P** to be minimum, it must be perpendicular to the line joining Points A and B. Thus, **P** must be directed as shown.



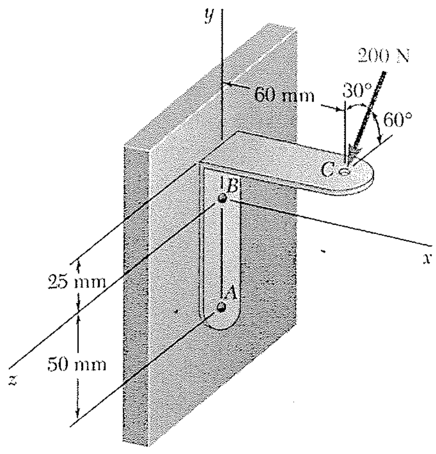
Thus $M_B = dP_{\min}$

$$d = r_{A/B}$$

or $800 \text{ lb} \cdot \text{in.} = (18 \text{ in.}) P_{\min}$

or $P_{\min} = 44.4$ lb $\mathbf{P}_{\min} = 44.4 \text{ lb} \measuredangle 20°$ ◀

PROBLEM 3.21



A 200-N force is applied as shown to the bracket ABC. Determine the moment of the force about A.

SOLUTION

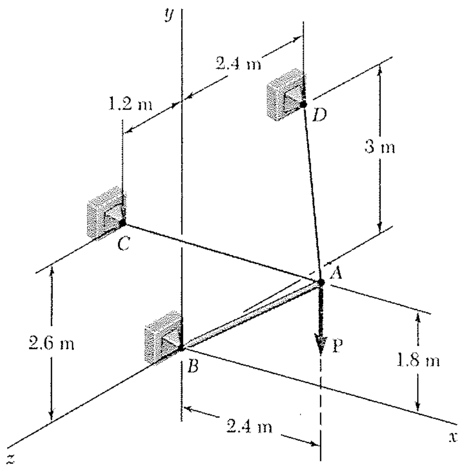
We have $\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}_C$

where $\mathbf{r}_{C/A} = (0.06\text{ m})\mathbf{i} + (0.075\text{ m})\mathbf{j}$

$\mathbf{F}_C = -(200\text{ N})\cos 30°\mathbf{j} + (200\text{ N})\sin 30°\mathbf{k}$

Then
$$\mathbf{M}_A = 200\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.06 & 0.075 & 0 \\ 0 & -\cos 30° & \sin 30° \end{vmatrix}$$
$$= 200[(0.075\sin 30°)\mathbf{i} - (0.06\sin 30°)\mathbf{j} - (0.06\cos 30°)\mathbf{k}]$$

or $\mathbf{M}_A = (7.50\text{ N}\cdot\text{m})\mathbf{i} - (6.00\text{ N}\cdot\text{m})\mathbf{j} - (10.39\text{ N}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.42

Knowing that the tension in cable *AD* is 405 N, determine (*a*) the angle between cable *AD* and the boom *AB*, (*b*) the projection on *AB* of the force exerted by cable *AD* at Point *A*.

SOLUTION

(*a*) First note

$$AD = \sqrt{(-2.4)^2 + (1.2)^2 + (-2.4)^2} = 3.6 \text{ m}$$

$$AB = \sqrt{(-2.4)^2 + (-1.8)^2 + (0)^2} = 3.0 \text{ m}$$

and

$$\mathbf{AD} = -(2.4 \text{ m})\mathbf{i} + (1.2 \text{ m})\mathbf{j} - (2.4 \text{ m})\mathbf{k}$$

$$\mathbf{AB} = -(2.4 \text{ m})\mathbf{i} - (1.8 \text{ m})\mathbf{j}$$

By definition,

$$\mathbf{AD} \cdot \mathbf{AB} = (AD)(AB)\cos\theta$$

$$(-2.4\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k}) \cdot (-2.4\mathbf{i} - 1.8\mathbf{j}) = (3.6)(3.0)\cos\theta$$

$$(-2.4)(-2.4) + (1.2)(-1.8) + (-2.4)(0) = 10.8\cos\theta$$

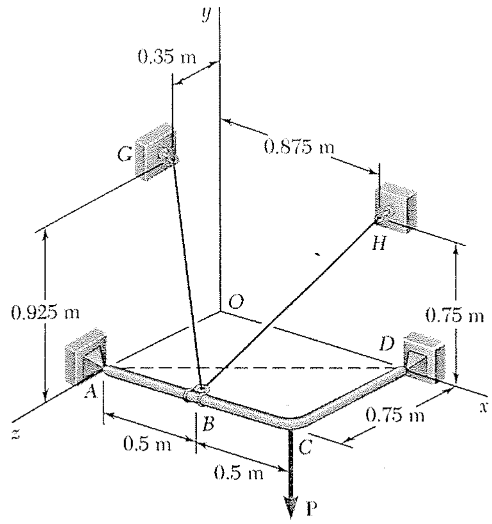
$$\cos\theta = \frac{1}{3}$$

$\theta = 70.5°$ ◀

(*b*)

$$(T_{AD})_{AB} = \mathbf{T}_{AD} \cdot \boldsymbol{\lambda}_{AB} = T_{AD}\cos\theta = (405 \text{ N})\left(\frac{1}{3}\right)$$

$(T_{AD})_{AB} = 135.0 \text{ N}$ ◀



PROBLEM 3.56

In Problem 3.55, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BG* of the cable.

SOLUTION

$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BG})$$

Where

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$\mathbf{r}_{B/A} = (0.5 \text{ m})\mathbf{j}$$

and

$$BG = \sqrt{(-0.5)^2 + (0.925)^2 + (-0.4)^2}$$

$$= 1.125 \text{ m}$$

Then

$$\bar{\mathbf{T}}_{BG} = \frac{450 \text{ N}}{1.125}(-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k})$$

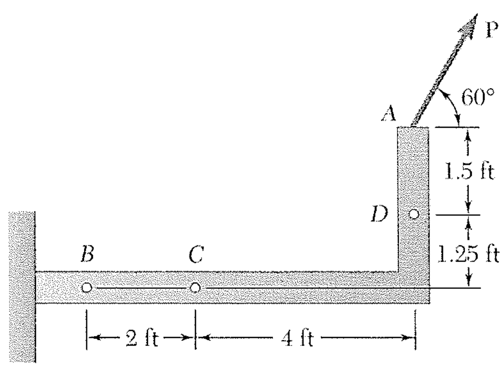
$$= -(200 \text{ N})\mathbf{i} + (370 \text{ N})\mathbf{j} - (160 \text{ N})\mathbf{k}$$

Finally

$$M_{AD} = \frac{1}{5}\begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ -200 & 370 & -160 \end{vmatrix}$$

$$= \frac{1}{5}[(-3)(0.5)(370)]$$

$$M_{AD} = -111.0 \text{ N} \cdot \text{m} \blacktriangleleft$$



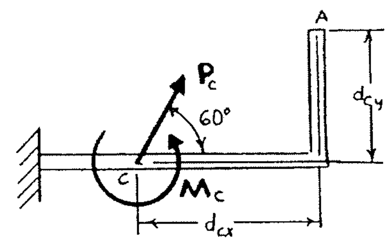
PROBLEM 3.82

A 160-lb force **P** is applied at Point A of a structural member. Replace **P** with (a) an equivalent force-couple system at C, (b) an equivalent system consisting of a vertical force at B and a second force at D.

SOLUTION

(a) Based on ΣF: $P_C = P = 160 \text{ lb}$ or $\mathbf{P}_C = 160 \text{ lb} \measuredangle 60°$ ◀

$$\Sigma M_C: \quad M_C = -P_x d_{cy} + P_y d_{Cx}$$



where

$$P_x = (160 \text{ lb})\cos 60° = 80 \text{ lb}$$

$$P_y = (160 \text{ lb})\sin 60° = 138.564 \text{ lb}$$

$$d_{Cx} = 4 \text{ ft}$$

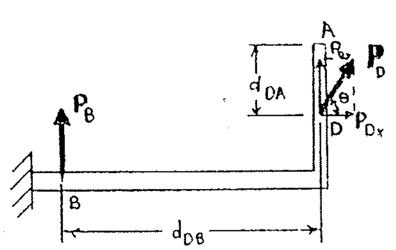
$$d_{Cy} = 2.75 \text{ ft}$$

$$M_C = (80 \text{ lb})(2.75 \text{ ft}) + (138.564 \text{ lb})(4 \text{ ft})$$
$$= 220 \text{ lb}\cdot\text{ft} + 554.26 \text{ lb}\cdot\text{ft}$$
$$= 334.26 \text{ lb}\cdot\text{ft}$$

or $\mathbf{M}_C = 334 \text{ lb}\cdot\text{ft}$ ↻ ◀

(b) Based on ΣF_x: $P_{Dx} = P\cos 60°$

$$= (160 \text{ lb})\cos 60°$$
$$= 80 \text{ lb}$$



$$\Sigma M_D: \quad (P\cos 60°)(d_{DA}) = P_B(d_{DB})$$

$$[(160 \text{ lb})\cos 60°](1.5 \text{ ft}) = P_B(6 \text{ ft})$$

$$P_B = 20.0 \text{ lb}$$

or $\mathbf{P}_B = 20.0 \text{ lb}\uparrow$ ◀

PROBLEM 3.82 (Continued)

$$\Sigma F_y: \quad P\sin 60° = P_B + P_{Dy}$$

$$(160 \text{ lb})\sin 60° = 20.0 \text{ lb} + P_{Dy}$$

$$P_{Dy} = 118.564 \text{ lb}$$

$$P_D = \sqrt{(P_{Dx})^2 + (P_{Dy})^2}$$

$$= \sqrt{(80)^2 + (118.564)^2}$$

$$= 143.029 \text{ lb}$$

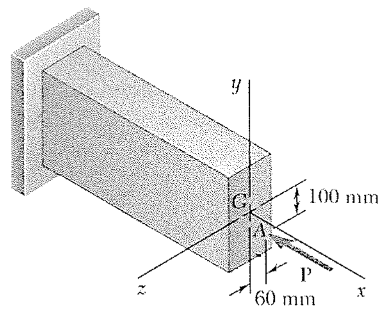
$$\theta = \tan^{-1}\left(\frac{P_{Dy}}{P_{Dx}}\right)$$

$$= \tan^{-1}\left(\frac{118.564}{80}\right)$$

$$= 55.991°$$

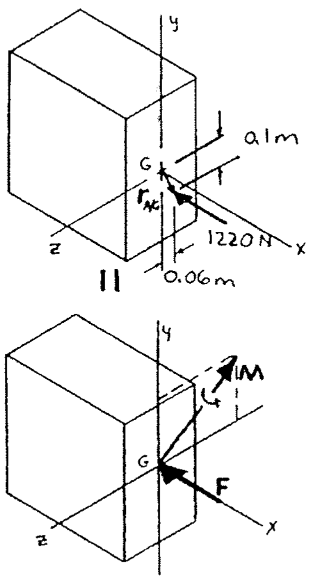
or $P_D = 143.0 \text{ lb} \measuredangle 56.0°$ ◀

PROBLEM 3.93



An eccentric, compressive 1220-N force **P** is applied to the end of a cantilever beam. Replace **P** with an equivalent force-couple system at G.

SOLUTION



We have

$$\Sigma \mathbf{F}: \quad -(1220\ \text{N})\mathbf{i} = \mathbf{F}$$

$$\mathbf{F} = -(1220\ \text{N})\mathbf{i} \blacktriangleleft$$

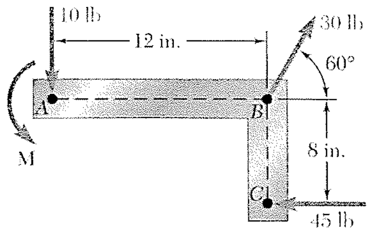
Also, we have

$$\Sigma \mathbf{M}_G: \quad \mathbf{r}_{A/G} \times \mathbf{P} = \mathbf{M}$$

$$1220\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -.1 & -.06 \\ -1 & 0 & 0 \end{vmatrix} \text{N} \cdot \text{m} = \mathbf{M}$$

$$\mathbf{M} = (1220\ \text{N} \cdot \text{m})[(-0.06)(-1)\mathbf{j} - (-0.1)(-1)\mathbf{k}]$$

$$\text{or} \quad \mathbf{M} = (73.2\ \text{N} \cdot \text{m})\mathbf{j} - (122\ \text{N} \cdot \text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.110

A couple **M** and the three forces shown are applied to an angle bracket. Find the moment of the couple if the line of action of the resultant of the force system is to pass through (*a*) Point *A*, (*b*) Point *B*, (*c*) Point *C*.

SOLUTION

In each case, must have $\mathbf{M}_1^R = 0$

(*a*) $+\circlearrowleft M_A^B = \Sigma M_A = M + (12 \text{ in.})[(30 \text{ lb})\sin 60°] - (8 \text{ in.})(45 \text{ lb}) = 0$

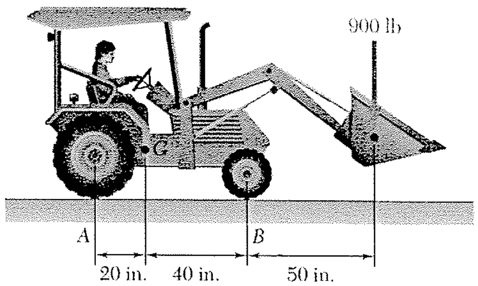
$M = +48.231 \text{ lb} \cdot \text{in.}$ $\quad\quad \mathbf{M} = 48.2 \text{ lb} \cdot \text{in.} \circlearrowleft$ ◀

(*b*) $+\circlearrowleft M_B^R = \Sigma M_B = M + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})(45 \text{ lb}) = 0$

$M = +240 \text{ lb} \cdot \text{in.}$ $\quad\quad \mathbf{M} = 240 \text{ lb} \cdot \text{in.} \circlearrowleft$ ◀

(*c*) $+\circlearrowleft M_C^R = \Sigma M_C = M + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})[(30 \text{ lb})\cos 60°] = 0$

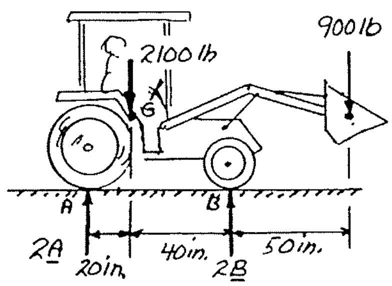
$M = 0$ $\quad\quad \mathbf{M} = 0$ ◀



PROBLEM 4.1

A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION



(*a*) Rear wheels

$$+\circlearrowleft \Sigma M_B = 0: \quad +(2100 \text{ lb})(40 \text{ in.}) - (900 \text{ lb})(50 \text{ in.}) + 2A(60 \text{ in.}) = 0$$

$$A = +325 \text{ lb} \qquad \mathbf{A} = 325 \text{ lb} \uparrow \blacktriangleleft$$

(*b*) Front wheels

$$+\circlearrowleft \Sigma M_A: \quad -(2100 \text{ lb})(20 \text{ in.}) - (900 \text{ lb})(110 \text{ in.}) - 2B(60 \text{ in.}) = 0$$

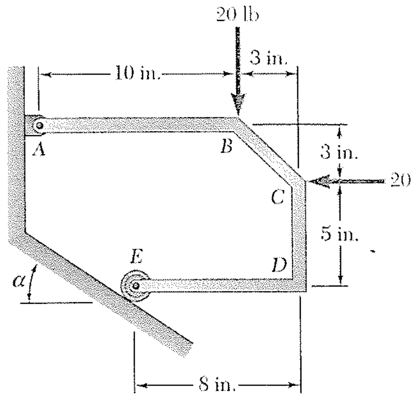
$$B = +1175 \text{ lb} \qquad \mathbf{B} = 1175 \text{ lb} \uparrow \blacktriangleleft$$

Check:

$$+\Sigma F_y = 0: \quad 2A + 2B - 2100 \text{ lb} - 900 \text{ lb} = 0$$

$$2(325 \text{ lb}) + 2(1175 \text{ lb}) - 2100 \text{ lb} - 900 = 0$$

$$0 = 0 \quad \text{(Checks)}$$

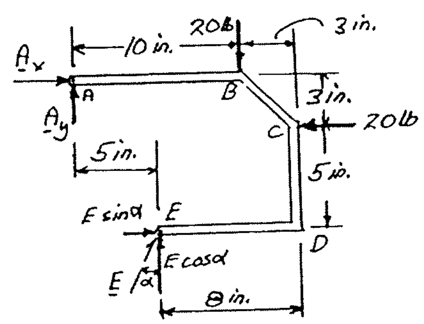


PROBLEM 4.22

For the frame and loading shown, determine the reactions at A and E when (*a*) $\alpha = 30°$, (*b*) $\alpha = 45°$.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (E\sin\alpha)(8\text{ in.}) + (E\cos\alpha)(5\text{ in.}) - (20\text{ lb})(10\text{ in.}) - (20\text{ lb})(3\text{ in.}) = 0$$

$$E = \frac{260}{8\sin\alpha + 5\cos\alpha}$$

(*a*) When $\alpha = 30°$:

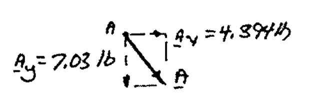
$$E = \frac{260}{8\sin 30° + 5\cos 30°} = 31.212\text{ lb}$$

$\mathbf{E} = 31.2\text{ lb} \measuredangle 60.0°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 20\text{ lb} + (31.212\text{ lb})\sin 30° = 0$$

$$A_x = +4.394\text{ lb}$$

$\mathbf{A}_x = 4.394\text{ lb} \longrightarrow$



$$+\uparrow \Sigma F_y = 0: \quad A_y - 20° + (31.212\text{ lb})\cos 30° = 0$$

$$A_y = -7.03\text{ lb}$$

$\mathbf{A}_y = 7.03\text{ lb} \downarrow$

$\mathbf{A} = 8.29\text{ lb} \;\nwarrow\; 58.0°$ ◀

PROBLEM 4.22 (Continued)

(*b*) When $\alpha = 45°$:

$$E = \frac{260}{8\sin 45° + 5\cos\alpha} = 28.28 \text{ lb}$$

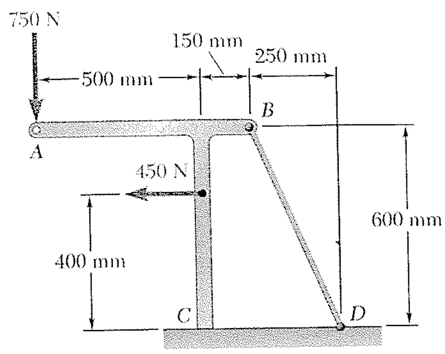
$\mathbf{E} = 28.3$ lb ⦣ 45.0° ◀

$\xrightarrow{+} \Sigma F_x = 0$: $A_x - 20 \text{ lb} + (28.28 \text{ lb})\sin 45° = 0$

$A_x = 0$ $\qquad$ $\mathbf{A}_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y - 20 \text{ lb} + (28.28 \text{ lb})\cos 45° = 0$

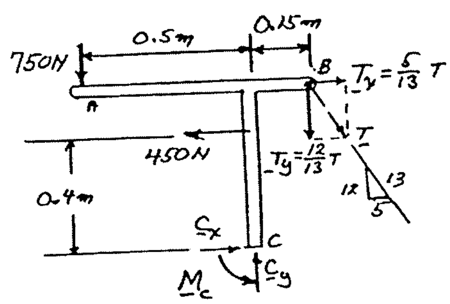
$A_y = 0$ $\qquad$ $A_y = 0$ $\qquad$ $\mathbf{A} = 0$ ◀



PROBLEM 4.50

Determine the range of allowable values of the tension in wire *BD* if the magnitude of the couple at the fixed support *C* is not to exceed 100 N · m.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad (750\text{ N})(0.5\text{ m}) + (450\text{ N})(0.4\text{ m}) - \left(\frac{5}{13}T\right)(0.6\text{ m})$$

$$-\left(\frac{12}{13}T\right)(0.15\text{ m}) + M_C = 0$$

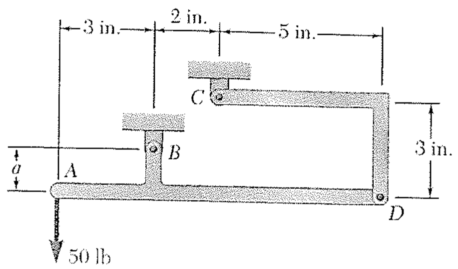
$$375\text{ N}\cdot\text{m} + 180\text{ N}\cdot\text{m} - \left(\frac{4.8}{13}\text{m}\right)T + M_C = 0$$

$$T = \frac{13}{4.8}(555 + M_C)$$

For $M_C = -100\text{ N}\cdot\text{m}: \quad T = \frac{13}{4.8}(555 - 100) = 1232\text{ N}$

For $M_C = +100\text{ N}\cdot\text{m}: \quad T = \frac{13}{4.8}(555 + 100) = 1774\text{ N}$

For $|M_C| \le 100\text{ N}\cdot\text{m}:$ $\quad 1.232\text{ kN} \le T \le 1.774\text{ kN}$ ◀



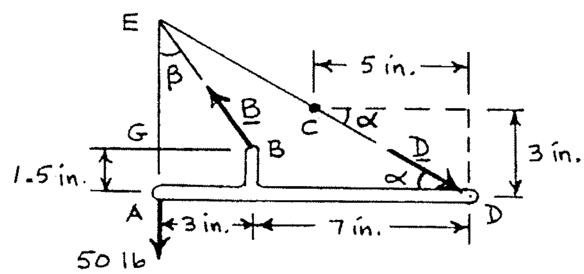
PROBLEM 4.68

Determine the reactions at B and C when $a = 1.5$ in.

SOLUTION

Since CD is a two-force member, the force it exerts on member ABD is directed along DC.

Free-Body Diagram of ABD: (Three-Force member)



The reaction at B must pass through E, where **D** and the 50-lb load intersect.

Triangle CFD:

$$\tan\alpha = \frac{3}{5} = 0.6$$

$$\alpha = 30.964°$$

Triangle EAD:

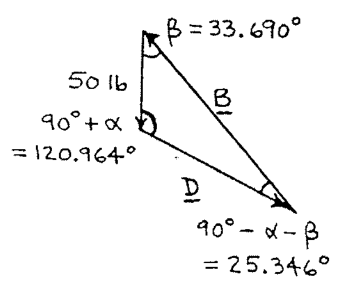
$$AE = 10\tan\alpha = 6 \text{ in.}$$

$$GE = AE - AG = 6 - 1.5 = 4.5 \text{ in.}$$

Triangle EGB:

$$\tan\beta = \frac{GB}{GE} = \frac{3}{4.5}$$

$$\beta = 33.690°$$



Force triangle

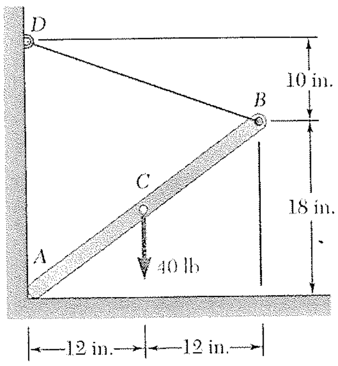
$$\frac{B}{\sin 120.964°} = \frac{D}{\sin 33.690°} = \frac{50 \text{ lb}}{\sin 25.346°}$$

$$B = 100.155 \text{ lb}$$

$$D = 64.789 \text{ lb}$$

$\mathbf{B} = 100.2$ lb ⦪ 56.3° ◀

$\mathbf{C} = \mathbf{D} = 64.8$ lb ⦩ 31.0° ◀

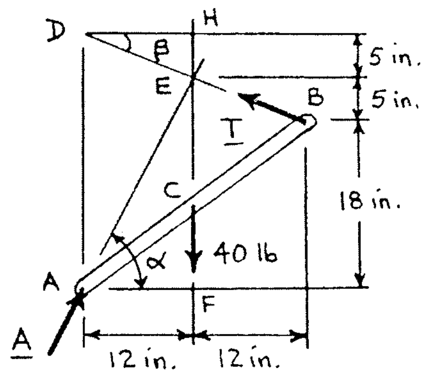


PROBLEM 4.71

One end of rod *AB* rests in the corner *A* and the other end is attached to cord *BD*. If the rod supports a 40-lb load at its midpoint *C*, find the reaction at *A* and the tension in the cord.

SOLUTION

Free-Body Diagram: (Three-Force body)



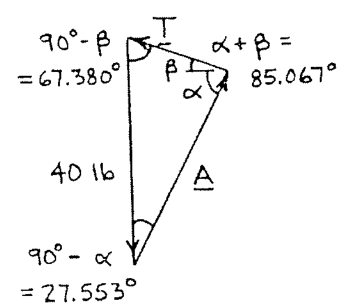
The line of action of reaction at *A* must pass through *E*, where **T** and the 40-lb load intersect.

$$\tan\alpha = \frac{EF}{AF} = \frac{23}{12}$$

$$\alpha = 62.447°$$

$$\tan\beta = \frac{EH}{DH} = \frac{5}{12}$$

$$\beta = 22.620°$$

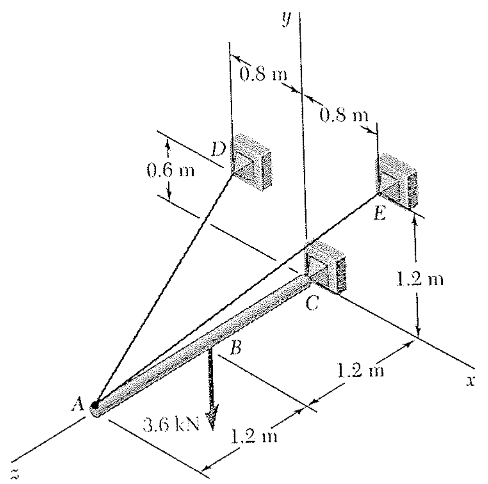


Force triangle

$$\frac{A}{\sin 67.380°} = \frac{T}{\sin 27.553°} = \frac{40 \text{ lb}}{\sin 85.067°}$$

A $= 37.1$ lb ∠ 62.4° ◀

$T = 18.57$ lb ◀

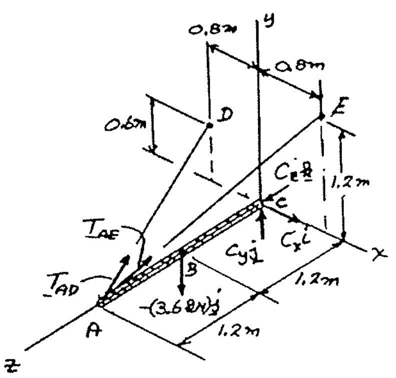


PROBLEM 4.106

A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram: Five Unknowns and six Eqs. of equilibrium. Equilibrium is maintained $(\Sigma M_{AC} = 0)$.



$$\mathbf{r}_B = 1.2\mathbf{k}$$

$$\mathbf{r}_A = 2.4\mathbf{k}$$

$$\overline{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6 \text{ m}$$

$$\overline{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overline{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$

$$T_{AE} = \frac{\overline{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times (-3 \text{ kN})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ -0.8 & 0.6 & -2.4 \end{vmatrix} \frac{T_{AD}}{2.6} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix} \frac{T_{AE}}{2.8} + 1.2\mathbf{k} \times (-3.6 \text{ kN})\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero.

$$\mathbf{i}: \quad -0.55385\,T_{AD} - 1.02857\,T_{AE} + 4.32 = 0 \qquad (1)$$

$$\mathbf{j}: \quad -0.73846\,T_{AD} + 0.68671\,T_{AE} = 0$$

$$T_{AD} = 0.92857\,T_{AE} \qquad (2)$$

Eq. (1):

$$-0.55385(0.92857)\,T_{AE} - 1.02857\,T_{AE} + 4.32 = 0$$

$$1.54286\,T_{AE} = 4.32$$

$$T_{AE} = 2.800 \text{ kN}$$

$T_{AE} = 2.80$ kN ◀

PROBLEM 4.106 (Continued)

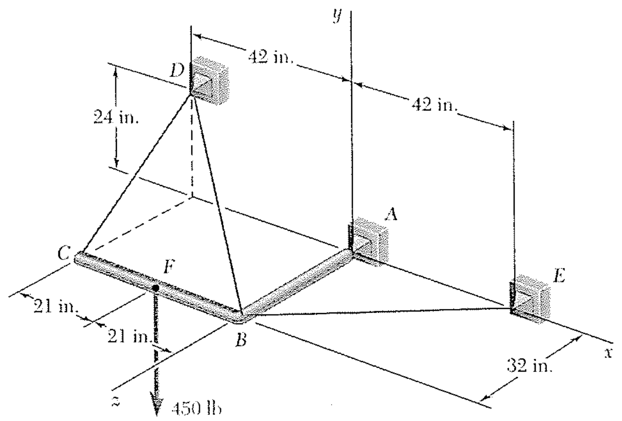
Eq. (2): $T_{AD} = 0.92857(2.80) = 2.600 \text{ kN}$ $\qquad T_{AD} = 2.60 \text{ kN}$ ◀

$\Sigma F_x = 0: \quad C_x - \frac{0.8}{2.6}(2.6 \text{ kN}) + \frac{0.8}{2.8}(2.8 \text{ kN}) = 0 \qquad C_x = 0$

$\Sigma F_y = 0: \quad C_y + \frac{0.6}{2.6}(2.6 \text{ kN}) + \frac{1.2}{2.8}(2.8 \text{ kN}) - (3.6 \text{ kN}) = 0 \qquad C_y = 1.800 \text{ kN}$

$\Sigma F_z = 0: \quad C_z - \frac{2.4}{2.6}(2.6 \text{ kN}) - \frac{2.4}{2.8}(2.8 \text{ kN}) = 0 \qquad C_z = 4.80 \text{ kN}$

$\mathbf{C} = (1.800 \text{ kN})\mathbf{j} + (4.80 \text{ kN})\mathbf{k}$ ◀

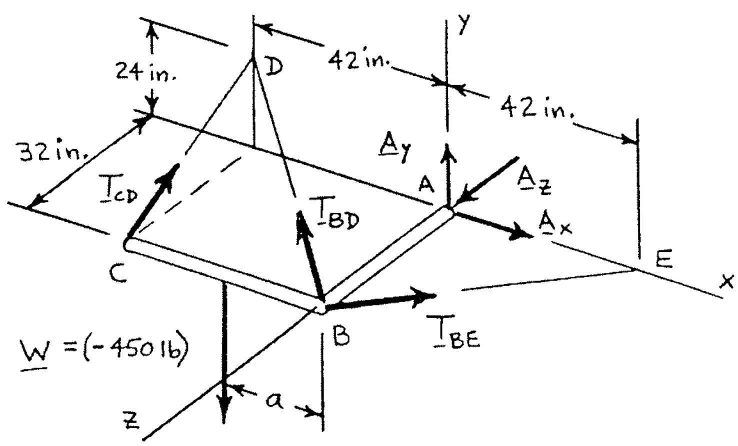


PROBLEM 4.123

The rigid L-shaped member ABC is supported by a ball-and-socket joint at A and by three cables. If a 450-lb load is applied at F, determine the tension in each cable.

SOLUTION

Free-Body Diagram:



In this problem: $a = 21$ in.

We have

$$\overrightarrow{CD} = (24 \text{ in.})\mathbf{j} - (32 \text{ in.})\mathbf{k} \quad CD = 40 \text{ in.}$$

$$\overrightarrow{BD} = -(42 \text{ in.})\mathbf{i} + (24 \text{ in.})\mathbf{j} - (32 \text{ in.})\mathbf{k} \quad BD = 58 \text{ in.}$$

$$\overrightarrow{BE} = (42 \text{ in.})\mathbf{i} - (32 \text{ in.})\mathbf{k} \quad BE = 52.802 \text{ in.}$$

Thus

$$\mathbf{T}_{CD} = T_{CD}\frac{\overrightarrow{CD}}{CD} = T_{CD}(0.6\mathbf{j} - 0.8\mathbf{k})$$

$$\mathbf{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = T_{BD}(-0.72414\mathbf{i} + 0.41379\mathbf{j} - 0.55172\mathbf{k})$$

$$\mathbf{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = T_{BE}(0.79542\mathbf{i} - 0.60604\mathbf{k})$$

$$\Sigma \mathbf{M}_A = 0: \quad (\mathbf{r}_C \times \mathbf{T}_{CD}) + (\mathbf{r}_B \times \mathbf{T}_{BD}) + (\mathbf{r}_B \times \mathbf{T}_{BE}) + (\mathbf{r}_W \times \mathbf{W}) = 0$$

PROBLEM 4.123 (Continued)

Noting that

$$\mathbf{r}_C = -(42 \text{ in.})\mathbf{i} + (32 \text{ in.})\mathbf{k}$$
$$\mathbf{r}_B = (32 \text{ in.})\mathbf{k}$$
$$\mathbf{r}_W = -a\mathbf{i} + (32 \text{ in.})\mathbf{k}$$

and using determinants, we write

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -42 & 0 & 32 \\ 0 & 0.6 & -0.8 \end{vmatrix} T_{CD} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 32 \\ -0.72414 & 0.41379 & -0.55172 \end{vmatrix} T_{BD}$$
$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 32 \\ 0.79542 & 0 & -0.60604 \end{vmatrix} T_{BE} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -a & 0 & 32 \\ 0 & -450 & 0 \end{vmatrix} = 0$$

Equating to zero the coefficients of the unit vectors:

$$\mathbf{i}: \quad -19.2T_{CD} - 13.241T_{BD} + 14400 = 0 \qquad (1)$$

$$\mathbf{j}: \quad -33.6T_{CD} - 23.172T_{BD} + 25.453T_{BE} = 0 \qquad (2)$$

$$\mathbf{k}: \quad -25.2T_{CD} + 450a = 0 \qquad (3)$$

Recalling that $a = 21$ in., Eq. (3) yields

$$T_{CD} = \frac{450(21)}{25.2} = 375 \text{ lb} \qquad T_{CD} = 375 \text{ lb} \blacktriangleleft$$

From (1):

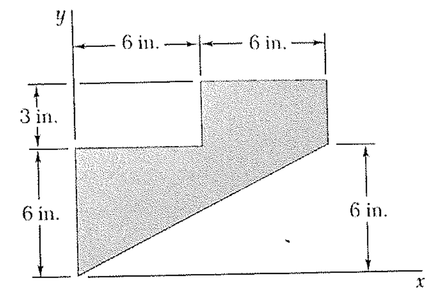
$$-19.2(375) - 13.241T_{BD} + 14400 = 0$$

$$T_{BD} = 543.77 \text{ lb} \qquad T_{BD} = 544 \text{ lb} \blacktriangleleft$$

From (2):

$$-33.6(375) - 23.172(543.77) + 25.453T_{BE} = 0$$

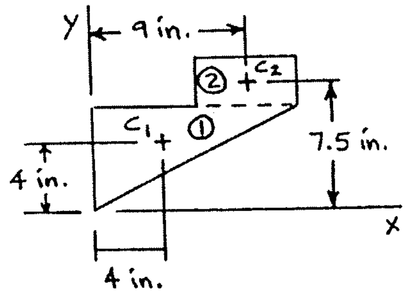
$$T_{BE} = 990.07 \text{ lb} \qquad T_{BE} = 990 \text{ lb} \blacktriangleleft$$



PROBLEM 5.4

Locate the centroid of the plane area shown.

SOLUTION



	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}
1	$\frac{1}{2}(12)(6) = 36$	4	4	144	144
2	$(6)(3) = 18$	9	7.5	162	135
Σ	54			306	279

Then

$$\bar{X}A = \Sigma\bar{x}A$$

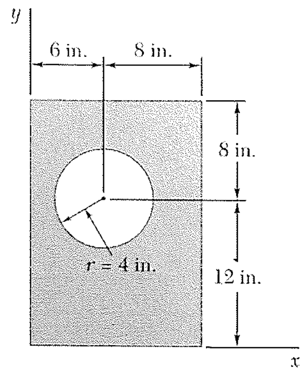
$$\bar{X}(54) = 306$$

$\bar{X} = 5.67$ in. ◀

$$\bar{Y}A = \Sigma\bar{y}A$$

$$\bar{Y}(54) = 279$$

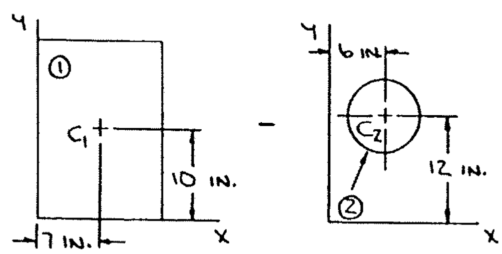
$\bar{Y} = 5.17$ in. ◀



PROBLEM 5.5

Locate the centroid of the plane area shown.

SOLUTION



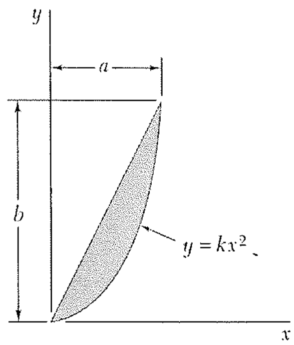
	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}
1	$14 \times 20 = 280$	7	10	1960	2800
2	$-\pi(4)^2 = -16\pi$	6	12	–301.59	–603.19
Σ	229.73			1658.41	2196.8

Then

$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{1658.41}{229.73} \qquad \bar{X} = 7.22 \text{ in.} \blacktriangleleft$$

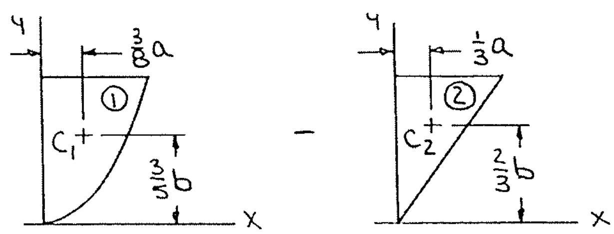
$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{2196.8}{229.73} \qquad \bar{Y} = 9.56 \text{ in.} \blacktriangleleft$$

PROBLEM 5.18



For the area shown, determine the ratio a/b for which $\bar{x} = \bar{y}$.

SOLUTION



	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	$\frac{2}{3}ab$	$\frac{3}{8}a$	$\frac{3}{5}b$	$\frac{a^2b}{4}$	$\frac{2ab^2}{5}$
2	$-\frac{1}{2}ab$	$\frac{1}{3}a$	$\frac{2}{3}b$	$-\frac{a^2b}{6}$	$-\frac{ab^2}{3}$
Σ	$\frac{1}{6}ab$			$\frac{a^2b}{12}$	$\frac{ab^2}{15}$

Then
$$\bar{X}\Sigma A = \Sigma \bar{x} A$$
$$\bar{X}\left(\frac{1}{6}ab\right) = \frac{a^2b}{12}$$

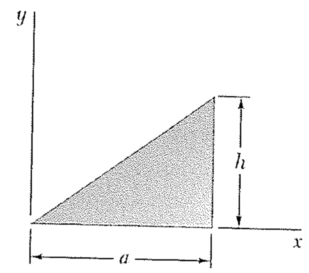
or
$$\bar{X} = \frac{1}{2}a$$
$$\bar{Y}\Sigma A = \Sigma \bar{y} A$$
$$\bar{Y}\left(\frac{1}{6}ab\right) = \frac{ab^2}{15}$$

or
$$\bar{Y} = \frac{2}{5}b$$

Now
$$\bar{X} = \bar{Y} \Rightarrow \quad \frac{1}{2}a = \frac{2}{5}b$$

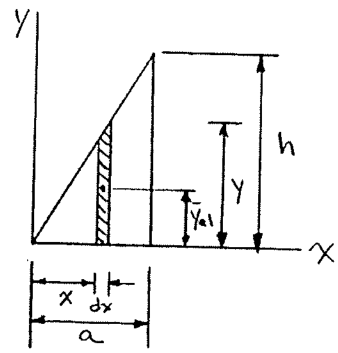
or $\frac{a}{b} = \frac{4}{5}$ ◀

PROBLEM 5.34



Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.

SOLUTION



$$\frac{y}{x} = \frac{h}{a}$$

$$y = \frac{h}{a}x$$

$$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}y$$

$$dA = ydx$$

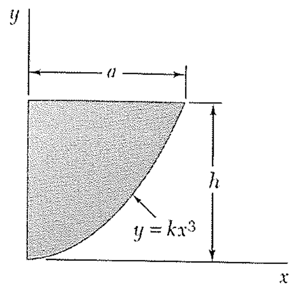
$$A = \int_0^a ydx = \int_0^a \left(\frac{h}{a}x\right)dx = \frac{1}{2}ah$$

$$\int \bar{x}_{EL}dA = \int xydx = \int_0^a x\left(\frac{h}{a}x\right)dx = \frac{h}{a}\left[\frac{x^3}{3}\right]_0^a = \frac{1}{3}ha^2$$

$$\int \bar{y}_{EL}dA = \int_0^a \left(\frac{1}{2}y\right)ydx = \frac{1}{2}\int_0^a \left(\frac{h}{a}x\right)^2 dx = \frac{1}{2}\frac{h^2}{b^2}\left[\frac{x^3}{3}\right]_0^a = \frac{1}{6}h^2a$$

$$\bar{x}A = \int \bar{x}_{EL}dA: \quad \bar{x}\left(\frac{1}{2}ah\right) = \frac{1}{3}ha^2 \qquad \bar{x} = \frac{2}{3}a \blacktriangleleft$$

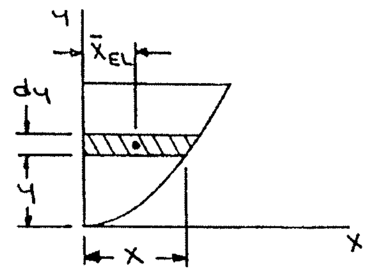
$$\bar{y}A = \int \bar{y}_{EL}dA: \quad \bar{y}\left(\frac{1}{2}ah\right) = \frac{1}{6}h^2a \qquad \bar{y} = \frac{1}{3}h \blacktriangleleft$$



PROBLEM 5.36

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.

SOLUTION



For the element (EL) shown

At $\quad x = a, \quad y = h: \quad h = ka^3 \quad \text{or} \quad k = \dfrac{h}{a^3}$

Then $\quad x = \dfrac{a}{h^{1/3}} y^{1/3}$

Now

$$dA = x\,dy = \frac{a}{h^{1/3}} y^{1/3}\, dy$$

$$\bar{x}_{EL} = \frac{1}{2}x = \frac{1}{2}\frac{a}{h^{1/3}} y^{1/3}$$

$$\bar{y}_{EL} = y$$

Then

$$A = \int dA = \int_0^h \frac{a}{h^{1/3}} y^{1/3} dy = \frac{3}{4}\frac{a}{h^{1/3}}\left(y^{4/3}\right)\Big|_0^h = \frac{3}{4}ah$$

and

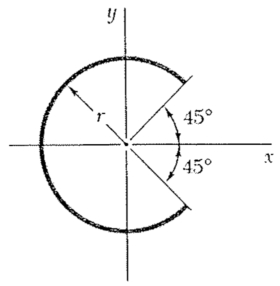
$$\int \bar{x}_{EL}\, dA = \int_0^h \frac{1}{2}\frac{a}{h^{1/3}} y^{1/3}\left(\frac{a}{h^{1/3}} y^{1/3} dy\right) = \frac{1}{2}\frac{a}{h^{2/3}}\left(\frac{3}{5}y^{5/3}\right)\Big|_0^h = \frac{3}{10}a^2 h$$

$$\int \bar{y}_{EL}\, dA = \int_0^h y\left(\frac{a}{h^{1/3}} y^{1/3} dy\right) = \frac{a}{h^{1/3}}\left(\frac{3}{7}y^{7/3}\right)\Big|_0^h = \frac{3}{7}ah^2$$

Hence

$$\bar{x}A = \int \bar{x}_{EL}\, dA: \quad \bar{x}\left(\frac{3}{4}ah\right) = \frac{3}{10}a^2 h \qquad \bar{x} = \frac{2}{5}a \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL}\, dA: \quad \bar{y}\left(\frac{3}{4}ah\right) = \frac{3}{7}ah^2 \qquad \bar{y} = \frac{4}{7}h \blacktriangleleft$$

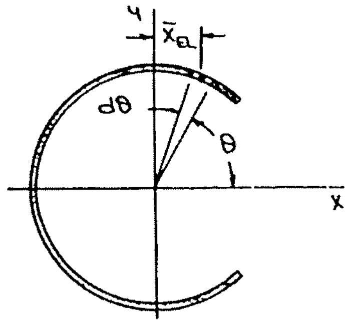


PROBLEM 5.46

A homogeneous wire is bent into the shape shown. Determine by direct integration the x coordinate of its centroid.

SOLUTION

First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line



Now $\qquad \bar{x}_{EL} = r\cos\theta \quad \text{and} \quad dL = r d\theta$

Then $\qquad L = \int dL = \int_{\pi/4}^{7\pi/4} r d\theta = r[\theta]_{\pi/4}^{7\pi/4} = \frac{3}{2}\pi r$

and

$$\int \bar{x}_{EL} dL = \int_{\pi/4}^{7\pi/4} r\cos\theta(r d\theta)$$

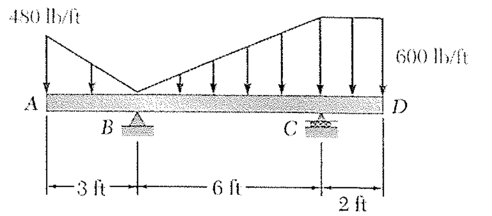
$$= r^2[\sin\theta]_{\pi/4}^{7\pi/4}$$

$$= r^2\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= -r^2\sqrt{2}$$

Thus $\qquad \bar{x}L = \int \bar{x} dL: \quad \bar{x}\left(\frac{3}{2}\pi r\right) = -r^2\sqrt{2}$

$$\bar{x} = -\frac{2\sqrt{2}}{3\pi} r \blacktriangleleft$$



PROBLEM 5.69

Determine the reactions at the beam supports for the given loading.

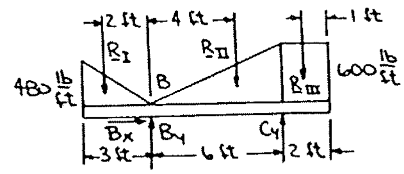
SOLUTION

We have

$$R_{\text{I}} = \frac{1}{2}(3\text{ ft})(480\text{ lb/ft}) = 720\text{ lb}$$

$$R_{\text{II}} = \frac{1}{2}(6\text{ ft})(600\text{ lb/ft}) = 1800\text{ lb}$$

$$R_{\text{III}} = (2\text{ ft})(600\text{ lb/ft}) = 1200\text{ lb}$$



Then

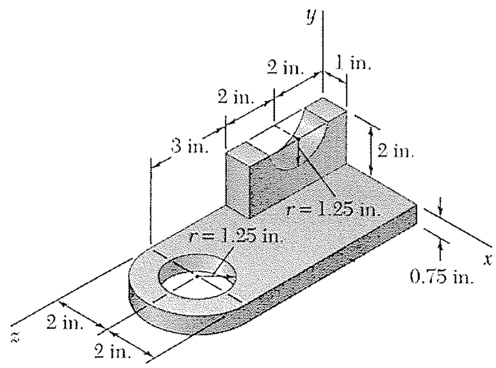
$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (2\text{ ft})(720\text{ lb}) - (4\text{ ft})(1800\text{ lb}) + (6\text{ ft})C_y - (7\text{ ft})(1200\text{ lb}) = 0$$

or $C_y = 2360\text{ lb}$ $\qquad$ $\mathbf{C} = 2360\text{ lb}\uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad -720\text{ lb} + B_y - 1800\text{ lb} + 2360\text{ lb} - 1200\text{ lb} = 0$$

or $B_y = 1360\text{ lb}$ $\qquad$ $\mathbf{B} = 1360\text{ lb}\uparrow$ ◀

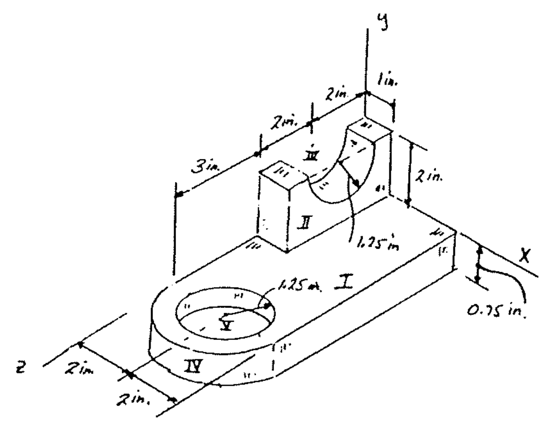


PROBLEM 5.103

For the machine element shown, locate the y coordinate of the center of gravity.

SOLUTION

For half cylindrical hole:



$$r = 1.25 \text{ in.}$$

$$\bar{y}_{\text{III}} = 2 - \frac{4(1.25)}{3\pi} = 1.470 \text{ in.}$$

For half cylindrical plate: $r = 2$ in.

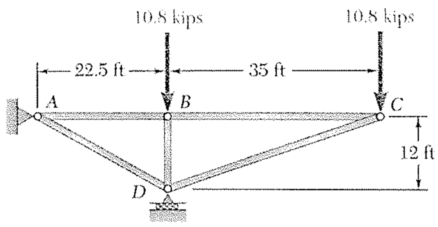
$$\bar{z}_{\text{IV}} = 7 + \frac{4(2)}{3\pi} = 7.85 \text{ in.}$$

		V, in.^{3}	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{y}V$, in.^{4}	$\bar{z}V$, in.^{4}
I	Rectangular plate	$(7)(4)(0.75) = 21.0$	–0.375	3.5	–7.875	73.50
II	Rectangular plate	$(4)(2)(1) = 8.0$	1.0	2	8.000	16.00
III	–(Half cylinder)	$-\frac{\pi}{2}(1.25)^2(1) = 2.454$	1.470	2	–3.607	–4.908
IV	Half cylinder	$\frac{\pi}{2}(2)^2(0.75) = 4.712$	–0.375	–7.85	–1.767	36.99
V	–(Cylinder)	$-\pi(1.25)^2(0.75) = -3.682$	–0.375	7	1.381	–25.77
	Σ	27.58			–3.868	95.81

$$\bar{Y}\Sigma V = \Sigma \bar{y} V$$

$$\bar{Y}(27.58 \text{ in.}^3) = -3.868 \text{ in.}^4$$

$\bar{Y} = -0.1403$ in. ◀

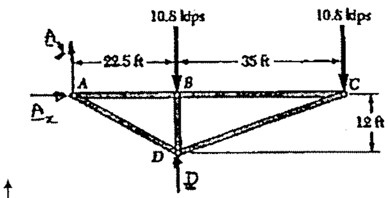


PROBLEM 6.5

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



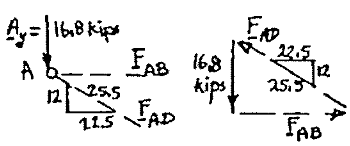
$\xrightarrow{+} \Sigma F_x = 0$: $\mathbf{A}_x = 0$

$+\circlearrowleft \Sigma M_A = 0$: $D(22.5) - (10.8 \text{ kips})(22.5) - (10.8 \text{ kips})(57.5) = 0$

$\mathbf{D} = 38.4 \text{ kips} \uparrow$

$\Sigma F_y = 0$: $\mathbf{A}_y = 16.8 \text{ kips} \downarrow$

Free body: Joint A:

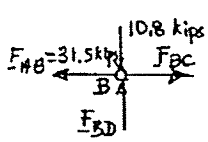


$$\frac{F_{AB}}{22.5} = \frac{F_{AD}}{25.5} = \frac{16.8 \text{ kips}}{12}$$

$F_{AB} = 31.5 \text{ kips} \quad T$ ◄

$F_{AD} = 35.7 \text{ kips} \quad C$ ◄

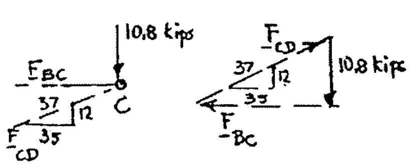
Free body: Joint B:



$\Sigma F_x = 0$: $F_{BC} = 31.5 \text{ kips} \quad T$ ◄

$\Sigma F_y = 0$: $F_{BD} = 10.80 \text{ kips} \quad C$ ◄

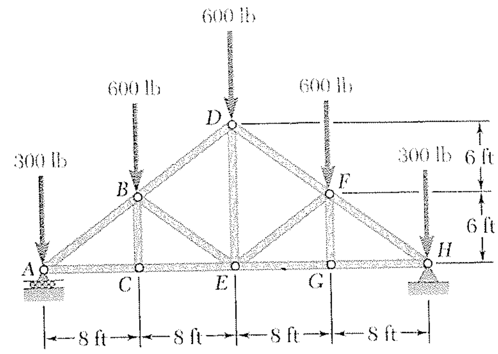
Free body: Joint C:



$$\frac{F_{CD}}{37} = \frac{F_{BC}}{35} = \frac{10.8 \text{ kips}}{12}$$

$F_{CD} = 33.3 \text{ kips} \quad C$ ◄

$F_{BC} = 31.5 \text{ kips} \quad T$ (Checks)

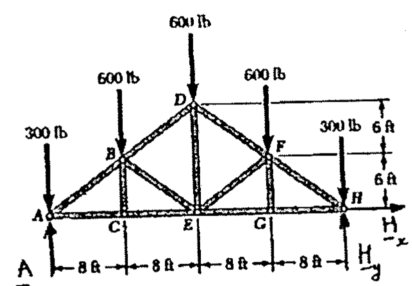


PROBLEM 6.11

Determine the force in each member of the Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



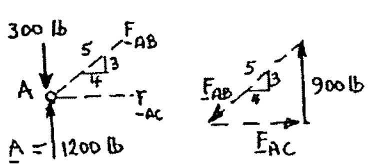
$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

Because of the symmetry of the truss and loading:

$$A = H_y = \frac{1}{2} \text{ Total load}$$

$$\mathbf{A} = \mathbf{H}_y = 1200 \text{ lb}\uparrow$$

Free body: Joint A:



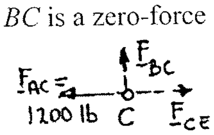
$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3}$$

$\mathbf{F}_{AB} = 1500 \text{ lb} \quad C$ ◀

$\mathbf{F}_{AC} = 1200 \text{ lb} \quad T$ ◀

Free body: Joint C:

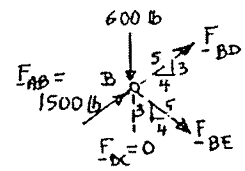
BC is a zero-force member



$\mathbf{F}_{BC} = 0$

$\mathbf{F}_{CE} = 1200 \text{ lb} \quad T$ ◀

Free body: Joint B:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{BD} + \frac{4}{5}F_{BC} + \frac{4}{5}(1500 \text{ lb}) = 0$$

or

$$F_{BD} + F_{BE} = -1500 \text{ lb} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - \frac{3}{5}F_{BE} + \frac{3}{5}(1500 \text{ lb}) - 600 \text{ lb} = 0$$

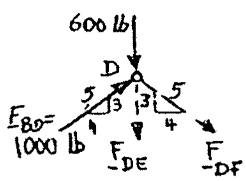
or

$$F_{BD} - F_{BE} = -500 \text{ lb} \qquad (2)$$

Add Eqs. (1) and (2): $2F_{BD} = -2000 \text{ lb}$ $\qquad F_{BD} = 1000 \text{ lb} \quad C$ ◀

Subtract (2) from (1): $2F_{BE} = -1000 \text{ lb}$ $\qquad F_{BE} = 500 \text{ lb} \quad C$ ◀

Free Body: Joint D:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}(1000 \text{ lb}) + \frac{4}{5}F_{DF} = 0$$

$F_{DF} = -1000 \text{ lb}$ $\qquad F_{DF} = 1000 \text{ lb} \quad C$ ◀

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}(1000 \text{ lb}) - \frac{3}{5}(-1000 \text{ lb}) - 600 \text{ lb} - F_{DE} = 0$$

$F_{DE} = +600 \text{ lb}$ $\qquad F_{DE} = 600 \text{ lb} \quad T$ ◀

Because of the symmetry of the truss and loading, we deduce that

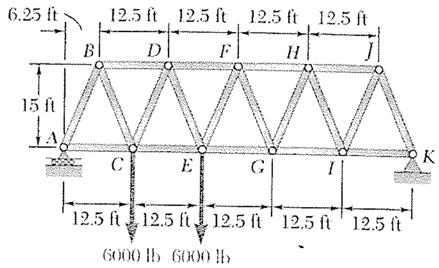
$F_{EF} = F_{BE}$ $\qquad F_{EF} = 500 \text{ lb} \quad C$ ◀

$F_{EG} = F_{CE}$ $\qquad F_{EG} = 1200 \text{ lb} \quad T$ ◀

$F_{FG} = F_{BC}$ $\qquad F_{FG} = 0$ ◀

$F_{FH} = F_{AB}$ $\qquad F_{FH} = 1500 \text{ lb} \quad C$ ◀

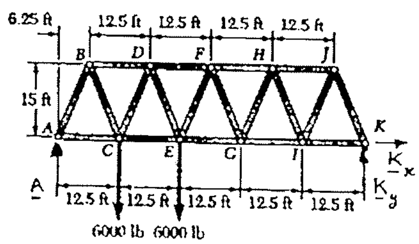
$F_{GH} = F_{AC}$ $\qquad F_{GH} = 1200 \text{ lb} \quad T$ ◀



PROBLEM 6.43

A Warren bridge truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

SOLUTION



Free body: Truss

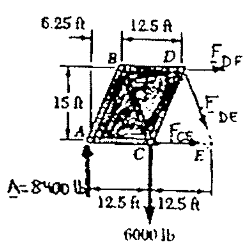
$\xrightarrow{+} \Sigma F_x = 0: \quad \mathbf{k}_x = 0$

$+\circlearrowleft \Sigma M_A = 0: \quad k_y(62.5 \text{ ft}) - (6000 \text{ lb})(12.5 \text{ ft}) - (6000 \text{ lb})(25 \text{ ft}) = 0$

$\mathbf{k} = \mathbf{k}_y = 3600 \text{ lb} \uparrow \triangleleft$

$+\uparrow \Sigma F_y = 0: \quad A + 3600 \text{ lb} - 6000 \text{ lb} - 6000 \text{ lb} = 0$

$\mathbf{A} = 8400 \text{ lb} \uparrow \triangleleft$



We pass a section through members *CE*, *DE*, and *DF* and use the free body shown.

$+\circlearrowleft \Sigma M_D = 0: \quad F_{CE}(15 \text{ ft}) - (8400 \text{ lb})(18.75 \text{ ft}) + (6000 \text{ lb})(6.25 \text{ ft}) = 0$

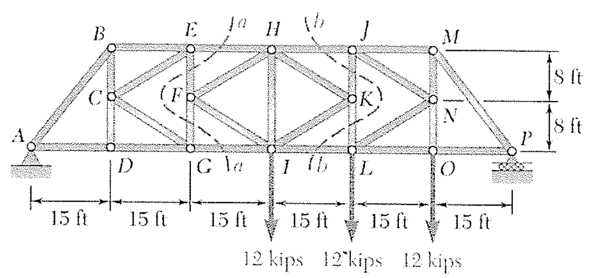
$F_{CE} = +8000 \text{ lb}$ $\qquad F_{CE} = 8000 \text{ lb} \quad T$ ◄

$+\uparrow \Sigma F_y = 0: \quad 8400 \text{ lb} - 6000 \text{ lb} - \dfrac{15}{16.25} F_{DE} = 0$

$F_{DE} = +2600 \text{ lb}$ $\qquad F_{DE} = 2600 \text{ lb} \quad T$ ◄

$+\circlearrowleft \Sigma M_E = 0: \quad 6000 \text{ lb}(12.5 \text{ ft}) - (8400 \text{ lb})(25 \text{ ft}) - F_{DF}(15 \text{ ft}) = 0$

$F_{DF} = -9000 \text{ lb}$ $\qquad F_{DF} = 9000 \text{ lb} \quad C$ ◄

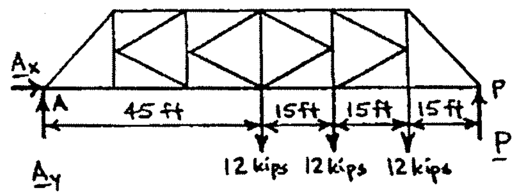


PROBLEM 6.63

Determine the force in members *EH* and *GI* of the truss shown. (*Hint:* Use section *aa*.)

SOLUTION

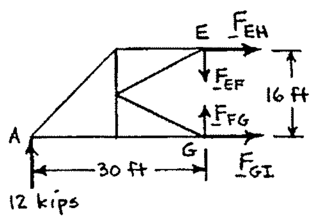
Reactions:



$\Sigma F_x = 0$: $A_x = 0$

$+\circlearrowleft \Sigma M_P = 0$: $12(45) + 12(30) + 12(15) - A_y(90) = 0$

$\mathbf{A}_y = 12 \text{ kips} \uparrow$

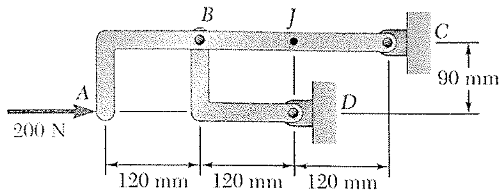


$+\uparrow \Sigma F_y = 0$: $12 - 12 - 12 - 12 + P = 0$ $\quad \mathbf{P} = 24 \text{ kips} \uparrow$

$+\circlearrowleft \Sigma M_G = 0$: $-(12 \text{ kips})(30 \text{ ft}) - F_{EH}(16 \text{ ft}) = 0$

$F_{EH} = -22.5 \text{ kips}$ $\quad F_{EH} = 22.5 \text{ kips} \quad C$ ◀

$\overset{+}{\rightarrow} \Sigma F_x = 0$: $F_{GI} - 22.5 \text{ kips} = 0$ $\quad F_{GI} = 22.5 \text{ kips} \quad T$ ◀

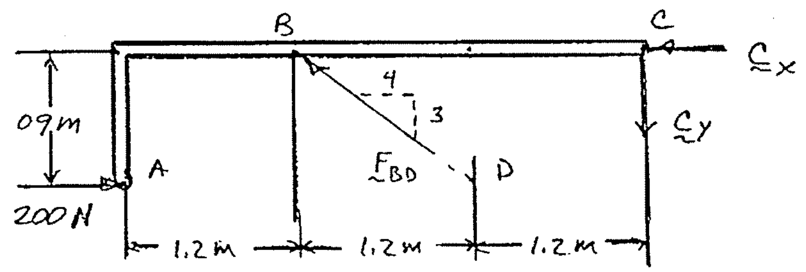


PROBLEM 6.75

For the frame and loading shown, determine the force acting on member *ABC* (*a*) at *B*, (*b*) at *C*.

SOLUTION

FBD ABC:



Note: *BD* is two-force member

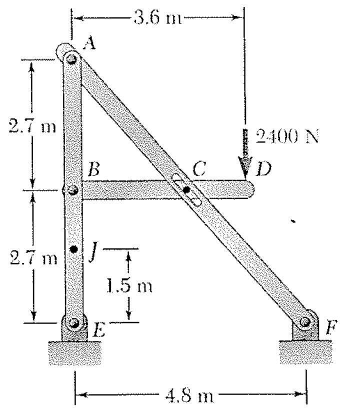
(*a*) $$\circlearrowleft \Sigma M_C = 0: \quad (0.09\text{ m})(200\text{ N}) - (2.4\text{ m})\left(\frac{3}{5}F_{BD}\right) = 0$$

$$\mathbf{F}_{BD} = 125.0\text{ N} \searrow 36.9° \blacktriangleleft$$

(*b*) $$\rightarrow \Sigma F_x = 0: \quad 200\text{ N} - \frac{4}{5}(125\text{ N}) - C_x = 0 \qquad \mathbf{C}_x = 100\text{ N} \leftarrow$$

$$\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - C_y = 0 \qquad \mathbf{C}_y = \frac{3}{5}(125\text{ N}) = 75\text{ N} \downarrow$$

$$\mathbf{C} = 125.0\text{ N} \nearrow 36.9° \blacktriangleleft$$



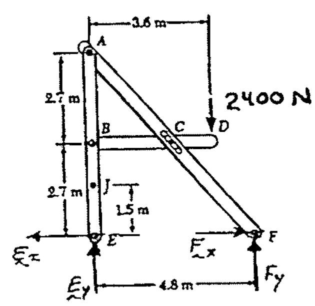
PROBLEM 6.102

For the frame and loading shown, determine the components of all forces acting on member *ABE*.

PROBLEM 6.101 For the frame and loading shown, determine the components of all forces acting on member *ABE*.

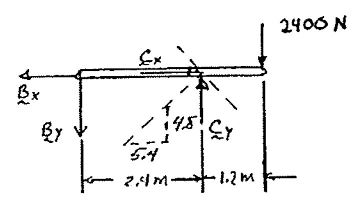
SOLUTION

FBD Frame:



$\circlearrowleft \Sigma M_F = 0: \quad (1.2\text{ m})(2400\text{ N}) - (4.8\text{ m})E_y = 0$ $\qquad \mathbf{E}_y = 600\text{ N}\uparrow$ ◀

FBD member *BC*:



$$C_y = \frac{4.8}{5.4}C_x = \frac{8}{9}C_x$$

$\circlearrowleft \Sigma M_C = 0: \quad (2.4\text{ m})B_y - (1.2\text{ m})(2400\text{ N}) = 0 \quad B_y = 1200\text{ N}\downarrow$

on *ABE*: $\qquad \mathbf{B}_y = 1200\text{ N}\uparrow$ ◀

$\uparrow \Sigma F_y = 0: \quad -1200\text{ N} + C_y - 2400\text{ N} = 0 \quad C_y = 3600\text{ N}\uparrow$

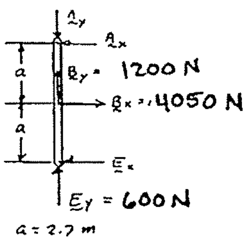
so $\qquad C_x = \frac{9}{8}C_y \quad C_x = 4050\text{ N}\longrightarrow$

$\longrightarrow \Sigma F_x = 0: \quad -B_x + C_x = 0 \quad B_x = 4050\text{ N}\longleftarrow$ on *BC*

on *ABE*: $\qquad \mathbf{B}_x = 4050\text{ N}\longrightarrow$ ◀

PROBLEM 6.102 (Continued)

FBD member *AB0E*:

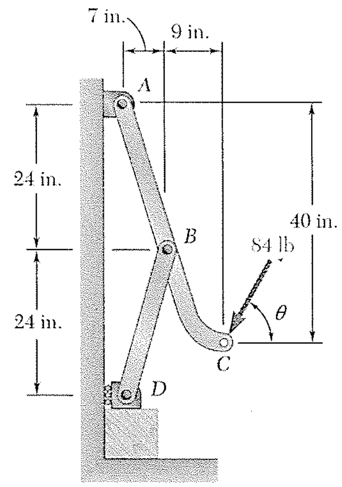


$\circlearrowleft \Sigma M_A = 0$: $a(4050\text{ N}) - 2aE_x = 0$

$E_x = 2025\text{ N}$ $\qquad \mathbf{E}_x = 2025\text{ N} \longleftarrow$ ◀

$\longrightarrow \Sigma F_x = 0$: $-A_x + (4050 - 2025)\text{ N} = 0$ $\qquad \mathbf{A}_x = 2025\text{ N} \longleftarrow$ ◀

$\uparrow \Sigma F_y = 0$: $600\text{ N} + 1200\text{ N} - A_y = 0$ $\qquad \mathbf{A}_y = 1800\text{ N} \downarrow$ ◀



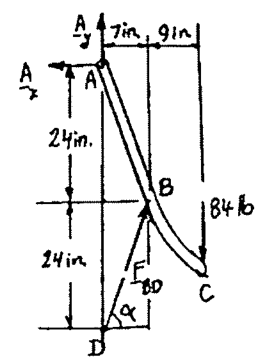
PROBLEM 6.122

An 84-lb force is applied to the toggle vise at C. Knowing that $\theta = 90°$, determine (a) the vertical force exerted on the block at D, (b) the force exerted on member ABC at B.

SOLUTION

We note that BD is a two-force member.

<u>Free body: Member ABC</u>



We have
$$BD = \sqrt{(7)^2 + (24)^2} = 25 \text{ in.}$$

$$(F_{BD})_x = \frac{7}{25}F_{BD}, \quad (F_{BD})_y = \frac{24}{25}F_{BD}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (F_{BD})_x(24) + (F_{BD})_y(7) - 84(16) = 0$$

$$\left(\frac{7}{25}F_{BD}\right)(24) + \left(\frac{24}{25}F_{BD}\right)(7) = 84(16)$$

$$\frac{336}{25}F_{BD} = 1344$$

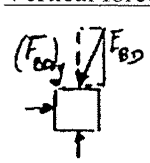
$$F_{BD} = 100 \text{ lb}$$

$$\tan\alpha = \frac{24}{7} \quad \alpha = 73.7°$$

(b) <u>Force exerted at B:</u>

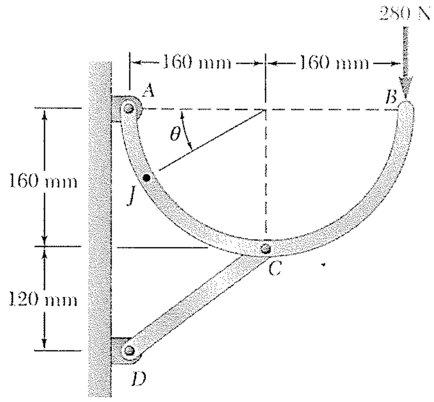
$\mathbf{F}_{BD} = 100.0 \text{ lb} \measuredangle 73.7°$ ◀

(a) <u>Vertical force exerted on block</u>



$$(F_{BD})_y = \frac{24}{25}F_{BD} = \frac{24}{25}(100 \text{ lb}) = 96 \text{ lb}$$

$(\mathbf{F}_{BD})_y = 96.0 \text{ lb} \downarrow$ ◀

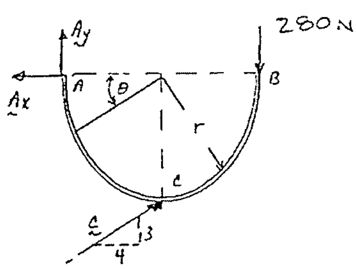


PROBLEM 7.13

A semicircular rod is loaded as shown. Determine the internal forces at Point J knowing that $\theta = 30°$.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(280\text{ N}) = 0$$

$$\mathbf{C} = 400\text{ N} \nearrow$$

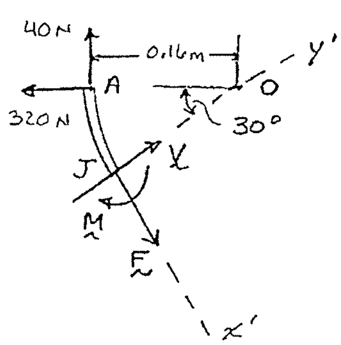
$$\rightarrow \Sigma F_x = 0: \quad -A_x + \frac{4}{5}(400\text{ N}) = 0$$

$$\mathbf{A}_x = 320\text{ N} \leftarrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y + \frac{3}{5}(400\text{ N}) - 280\text{ N} = 0$$

$$\mathbf{A}_y = 40.0\text{ N} \uparrow$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0: \quad F - (320\text{ N})\sin 30° - (40.0\text{ N})\cos 30° = 0$$

$$F = 194.641\text{ N}$$

$$\mathbf{F} = 194.6\text{ N} \nwarrow 60.0° \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: \quad V - (320\text{ N})\cos 30° + (40\text{ N})\sin 30° = 0$$

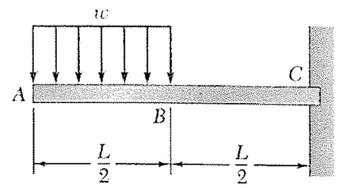
$$V = 257.13\text{ N}$$

$$\mathbf{V} = 257\text{ N} \measuredangle 30.0° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: \quad (0.160\text{ m})(194.641\text{ N}) - (0.160\text{ m})(40.0\text{ N}) - M = 0$$

$$M = 24.743$$

$$\mathbf{M} = 24.7\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

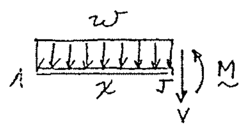


PROBLEM 7.32

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) **Along AB:**

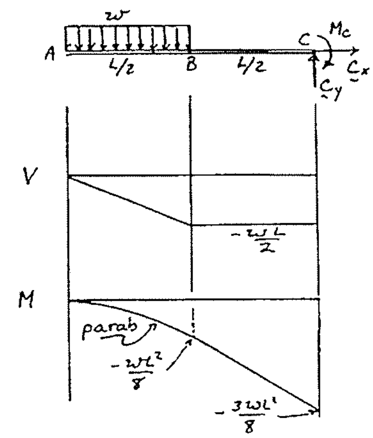


$$+\uparrow \Sigma F_y = 0: \quad -wx - V = 0 \qquad V = -wx$$

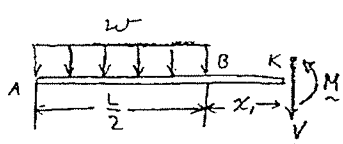
Straight with $V = -\dfrac{wL}{2}$ at $x = \dfrac{L}{2}$

$$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{x}{2}wx = 0 \quad M = -\frac{1}{2}wx^2$$

Parabola with $M = -\dfrac{wL^2}{8}$ at $x = \dfrac{L}{2}$



Along BC:



$$+\uparrow \Sigma F_y = 0: \quad -w\frac{L}{2} - V = 0 \qquad V = -\frac{1}{2}wL$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + \left(x_1 + \frac{L}{4}\right)w\frac{L}{2} = 0$$

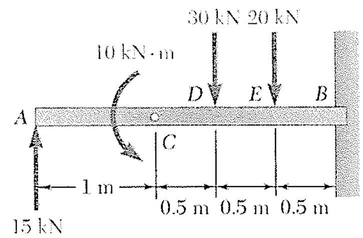
$$M = -\frac{wL}{2}\left(\frac{L}{4} + x_1\right)$$

Straight with $M = -\dfrac{3}{8}wL^2$ at $x_1 = \dfrac{L}{2}$

(*b*) From diagrams:

$$|V|_{\max} = \frac{wL}{2} \text{ on } BC \blacktriangleleft$$

$$|M|_{\max} = \frac{3wL^2}{8} \text{ at } C \blacktriangleleft$$



PROBLEM 7.35

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) Just to the right of *A*:

$$+\uparrow \Sigma F_y = 0 \quad V_1 = +15 \text{ kN} \quad M_1 = 0$$

Just to the left of *C*:

$$V_2 = +15 \text{ kN} \quad M_2 = +15 \text{ kN} \cdot \text{m}$$

Just to the right of *C*:

$$V_3 = +15 \text{ kN} \quad M_3 = +5 \text{ kN} \cdot \text{m}$$

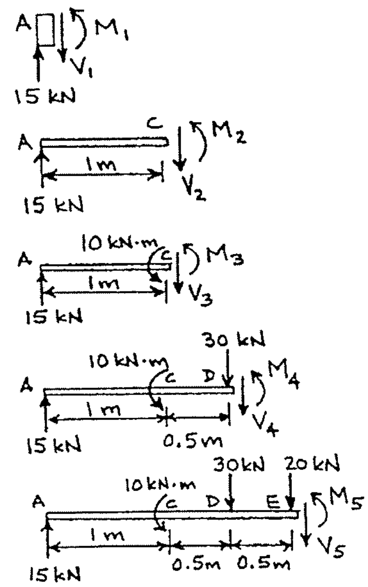
Just to the right of *D*:

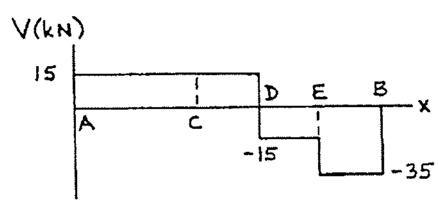
$$V_4 = -15 \text{ kN} \quad M_4 = +12.5 \text{ kN} \cdot \text{m}$$

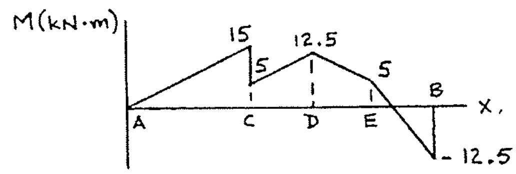
Just to the right of *E*:

$$V_5 = -35 \text{ kN} \quad M_5 = +5 \text{ kN} \cdot \text{m}$$

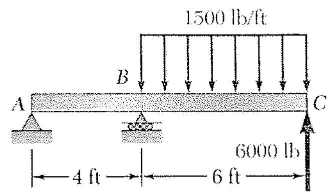
At *B*: $M_B = -12.5 \text{ kN} \cdot \text{m}$







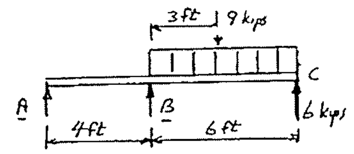
(*b*) $|V|_{max} = 35.0 \text{ kN}$ $|M|_{max} = 12.50 \text{ kN} \cdot \text{m}$ ◀



PROBLEM 7.80

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad (6 \text{ kips})(10 \text{ ft}) - (9 \text{ kips})(7 \text{ ft}) + 8(4 \text{ ft}) = 0$$

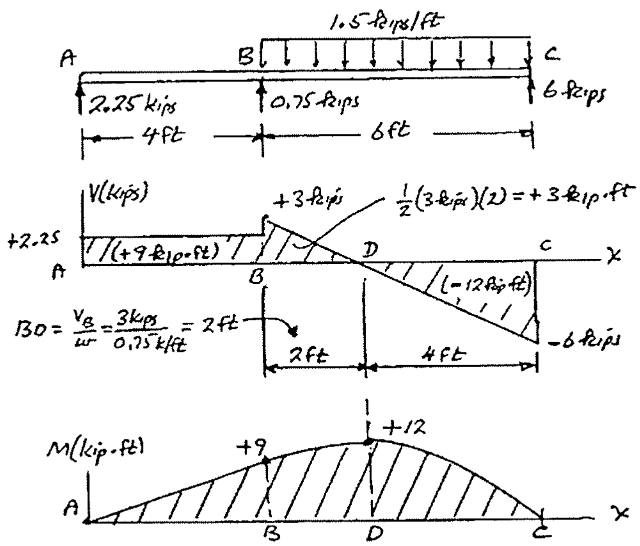
$$B = +0.75 \text{ kips}$$

$$\mathbf{B} = 0.75 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad A + 0.75 \text{ kips} - 9 \text{ kips} + 6 \text{ kips} = 0$$

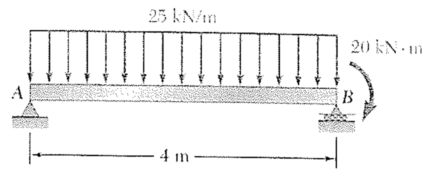
$$A = +2.25 \text{ kips}$$

$$\mathbf{A} = 2.25 \text{ kips} \uparrow$$



$M_{max} = 12.00 \text{ kip} \cdot \text{ft}$ ◀

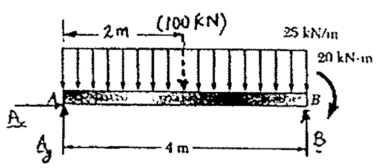
6.00 ft from A



PROBLEM 7.83

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Free body: Beam

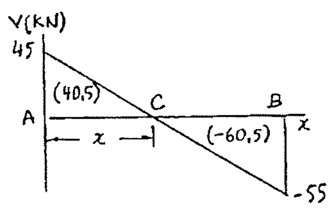
$$+\circlearrowleft \Sigma M_A = 0: \quad B(4\text{ m}) - (100\text{ kN})(2\text{ m}) - 20\text{ kN}\cdot\text{m} = 0$$

$$B = +55\text{ kN} \triangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 55 - 100 = 0$$

$$A_y = +45\text{ kN} \triangleleft$$



Shear diagram

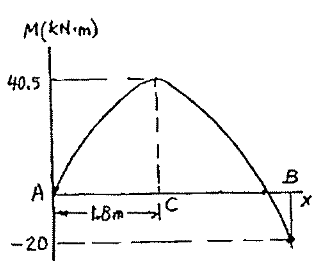
At A: $\quad V_A = A_y = +45\text{ kN}$

To determine Point C where $V = 0$:

$$V_C - V_A = -wx$$
$$0 - 45\text{ kN} = -(25\text{ kN}\cdot\text{m})x$$

$$x = 1.8\text{ m} \triangleleft$$

We compute all areas bending-moment



Bending-moment diagram

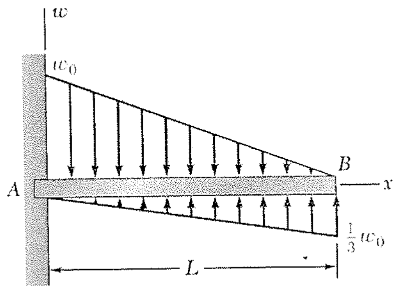
At A: $\quad M_A = 0$

At B: $\quad M_B = -20\text{ kN}\cdot\text{m}$

$$|M|_{max} = 40.5\text{ kN}\cdot\text{m} \blacktriangleleft$$

$$1.800\text{ m from } A \blacktriangleleft$$

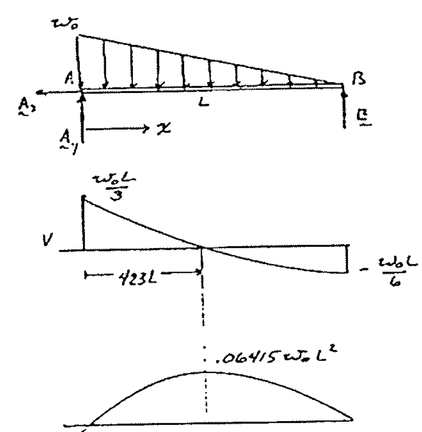
Single arc of parabola



PROBLEM 7.85

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION



Distributed load

$$w = w_0\left(1-\frac{x}{L}\right) \quad \left(\text{Total} = \frac{1}{2}w_0L\right)$$

$$\circlearrowleft \Sigma M_A = 0: \quad \frac{L}{3}\left(\frac{1}{2}w_0L\right) - LB = 0 \qquad \mathbf{B} = \frac{w_0L}{6}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - \frac{1}{2}w_0L + \frac{w_0L}{6} = 0 \quad \mathbf{A}_y = \frac{w_0L}{3}\uparrow$$

Shear: $\quad V_A = A_y = \dfrac{w_0L}{3}$

Then $\quad \dfrac{dV}{dx} = -w \rightarrow V$

$$= V_A - \int_0^x w_0\left(1-\frac{x}{L}\right)dx$$

$$V = \left(\frac{w_0L}{3}\right) - w_0x + \frac{1}{2}\frac{w_0}{L}x^2$$

$$= w_0L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]$$

Note: At $\quad x = L$

$$V = -\frac{w_0L}{6}$$

$$V = 0 \text{ at } \left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right) + \frac{2}{3}$$

$$= 0 \rightarrow \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}$$

Moment: $M_A = 0$

Then $$\left(\frac{dM}{dx}\right) = V \rightarrow M = \int_0^x V dx = L\int_0^{x/L} V\left(\frac{x}{L}\right) d\left(\frac{x}{L}\right)$$

$$M = w_0 L^2 \int_0^{x/L} \left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right] d\left(\frac{x}{L}\right)$$

$$M = w_0 L^2 \left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right]$$

$$M_{\max}\left(\text{at } \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}\right) = 0.06415 w_0 L^2$$

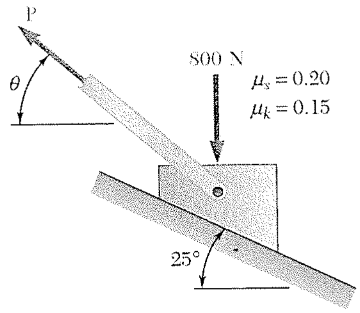
(*a*) $$V = w_0 L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right] ◀$$

$$M = w_0 L^2 \left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right] ◀$$

(*c*) $$M_{\max} = 0.0642\, w_0 L^2 ◀$$

at $x = 0.423L$ ◀

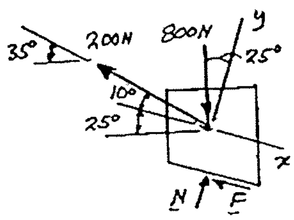
PROBLEM 8.4



Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 35°$ and $P = 200$ N.

SOLUTION

Assume equilibrium:



$+\nearrow \Sigma F_y = 0$: $N - (800 \text{ N})\cos 25° + (200 \text{ N})\sin 10° = 0$

$N = 690.3$ N $\qquad$ $\mathbf{N} = 690.3 \text{ N}$ ↑

$\searrow^{+} \Sigma F_x = 0$: $-F + (800 \text{ N})\sin 25° - (200 \text{ N})\cos 10° = 0$

$F = 141.13$ N $\qquad$ $\mathbf{F} = 141.13 \text{ N}$ ↖

Maximum friction force:

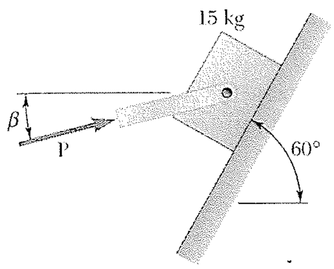
$$F_m = \mu_s N$$
$$= (0.20)(690.3 \text{ N})$$
$$= 138.06 \text{ N}$$

Since $F > F_m$, $\qquad$ Block moves down ↘ ◀

Friction force:

$$F = \mu_k N$$
$$= (0.15)(690.3 \text{ N})$$
$$= 103.547 \text{ N}$$

$\mathbf{F} = 103.5 \text{ N}$ ↖ ◀

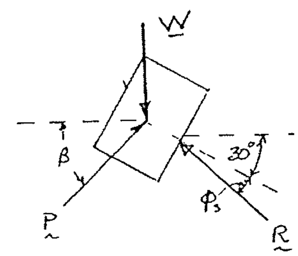


PROBLEM 8.7

Knowing that the coefficient of friction between the 15-kg block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of P required to maintain the block in equilibrium, (*b*) the corresponding value of β.

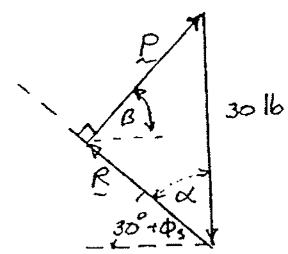
SOLUTION

FBD block (Impending motion downward):



$$W = (15\text{ kg})(9.81\text{ m/s}^2)$$
$$= 147.150\text{ N}$$
$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1}(0.25)$$
$$= 14.036°$$

(*a*) *Note:* For minimum P, $\quad \mathbf{P} \perp \mathbf{R}$



So $\quad \beta = \alpha$

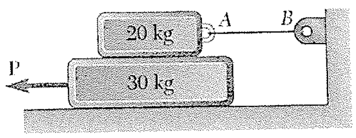
$$= 90° - (30° + 14.036°)$$
$$= 45.964°$$

and $\quad P = (147.150\text{ N})\sin\alpha$

$$= (147.150\text{ N})\sin(45.964°)$$

$P = 108.8\text{ N}$ ◀

(*b*) $\beta = 46.0°$ ◀

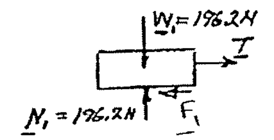


PROBLEM 8.12

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable AB (*a*) is attached as shown, (*b*) is removed.

SOLUTION

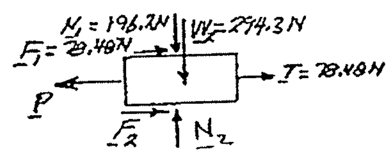
(*a*) Free body: 20-kg block



$$W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$
$$F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N}$$

$$\xrightarrow{+} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N}$$

Free body: 30-kg block



$$W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$
$$N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N}$$
$$F_2 = \mu_s N_2 = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

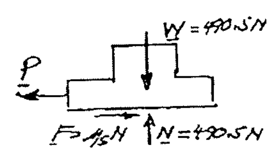
$$\xleftarrow{+} \Sigma F = 0: \quad P - F_1 - F_2 = 0$$

$$P = 78.48 \text{ N} + 196.2 \text{ N} = 274.7 \text{ N}$$

P $= 275$ N ⟵ ◀

(*b*) Free body: Both blocks

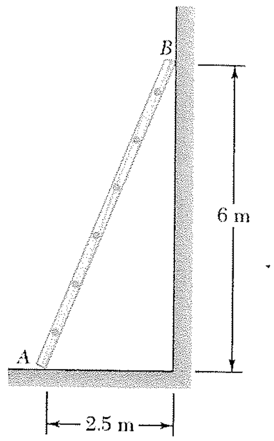
Blocks move together



$$W = (50 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 490.5 \text{ N}$$

$$\xleftarrow{+} \Sigma F = 0: \quad P - F = 0$$
$$P = \mu_s N = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

P $= 196.2$ N ⟵ ◀



PROBLEM 8.22

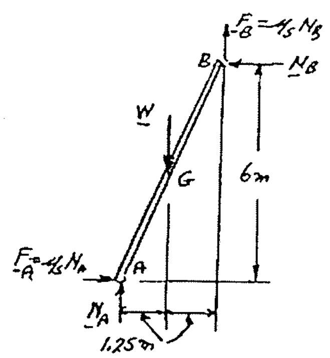
A 6.5-m ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

Free body: Ladder

Motion impending:

$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$



$$+\circlearrowleft \Sigma M_A = 0: \quad W(1.25 \text{ m}) - N_B(6 \text{ m}) - \mu_s N_B(2.5 \text{ m}) = 0$$

$$N_B = \frac{1.25W}{6 + 2.5\mu_s} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + \mu_s N_B - W = 0$$

$$N_A = W - \mu_s N_B$$
$$N_A = W - \frac{1.25\mu_s W}{6 + 2.5\mu_s} \qquad (2)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \mu_s N_A - N_B = 0$$

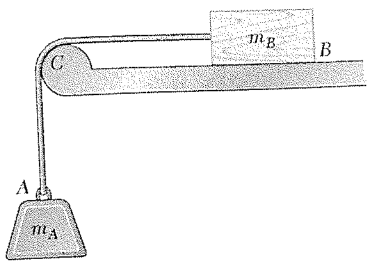
Substitute for N_A and N_B from Eqs. (1) and (2):

$$\mu_s W - \frac{1.25\mu_s^2 W}{6 + 2.5\mu_s} = \frac{1.25W}{6 + 2.5\mu_s}$$
$$6\mu_s + 2.5\mu_s^2 - 1.25\mu_s^2 = 1.25$$
$$1.25\mu_s^2 + 6\mu_s - 1.25 = 0$$
$$\mu_s = 0.2$$

and $\mu_s = -5$ (Discard)

$\mu_s = 0.200$ ◀

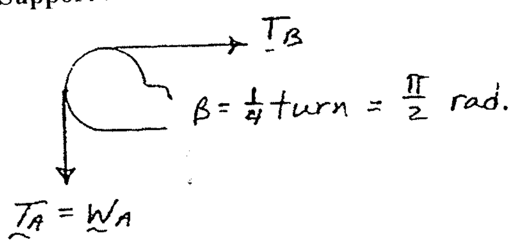
PROBLEM 8.105



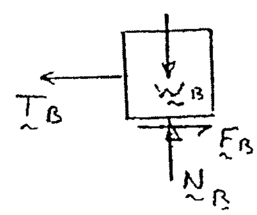
The coefficient of static friction between block B and the horizontal surface and between the rope and support C is 0.40. Knowing that $m_A = 12$ kg, determine the smallest mass of block B for which equilibrium is maintained.

SOLUTION

Support at C:



FBD block B:



$$W_A = mg = (12\text{ kg})g$$

$$\uparrow \Sigma F_y = 0: \quad N_B - W_B = 0 \quad \text{or} \quad N_B = W_B$$

Impending motion: $\quad F_B = \mu_s N_B = 0.4 N_B = 0.4 W_B$

$$\longrightarrow \Sigma F_x = 0: \quad F_B - T_B = 0 \quad \text{or} \quad T_B = F_B = 0.4 W_B$$

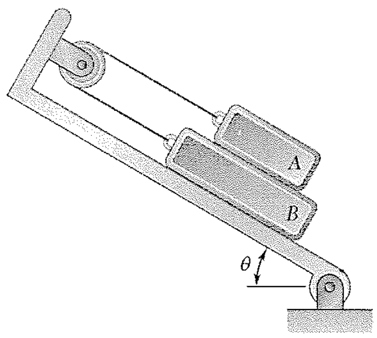
At support, for impending motion of W_A down:

$$W_A = T_B e^{\mu_s \beta}$$

so $\quad T_B = W_A e^{-\mu_s \beta} = (12\text{ kg})g^{-(0.4)\pi/2} = (6.4019\text{ kg})g$

Now $\quad W_B = \dfrac{T_B}{0.4} = \left(\dfrac{6.4019\text{ kg}}{0.4}\right) g = 16.0048 g$

so that $\quad m_B = \dfrac{W_B}{g} = \dfrac{16.0048 g}{g}$ $\qquad m_B = 16.00\text{ kg}$ ◀



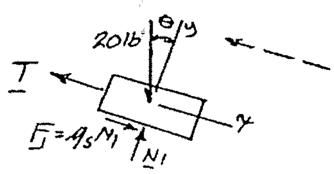
PROBLEM 8.133

The 20-lb block A and the 30-lb block B are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ at all surfaces.

Free body: Block A



Impending motion:

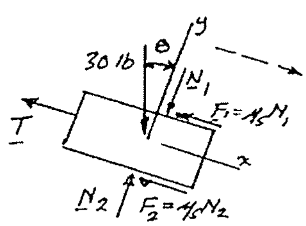
$$\Sigma F_y = 0: \quad N_1 = 20\cos\theta$$

$$\overset{+}{\nwarrow} \Sigma F_x = 0: \quad T - 20\sin\theta - \mu_s N_1 = 0$$

$$T = 20\sin\theta + 0.15(20\cos\theta)$$

$$T = 20\sin\theta + 3\cos\theta \qquad (1)$$

Free body: Block B



Impending motion:

$$+\!\nearrow \Sigma F_y = 0: \quad N_2 - 30\cos\theta - N_1 = 0$$

$$N_2 = 30\cos\theta + 20\cos\theta = 50\cos\theta$$

$$F_2 = \mu_s N_2 = 0.15(50\cos\theta) = 6\cos\theta$$

$$\overset{+}{\nwarrow} \Sigma F_x = 0: \quad T - 30\sin\theta + \mu_s N_1 + \mu_s N_2 = 0$$

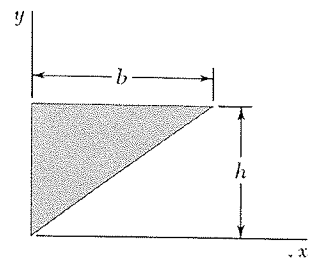
$$T = 30\sin\theta - 0.15(20\cos\theta) - 0.15(50\cos\theta)$$

$$T = 30\sin\theta - 3\cos\theta - 7.5\cos\theta \qquad (2)$$

Eq. (1) subtracted by Eq. (2): $\quad 20\sin\theta + 3\cos\theta - 30\sin\theta + 3\cos\theta + 7.5\cos\theta = 0$

$$13.5\cos\theta = 10\sin\theta, \quad \tan\theta = \frac{13.5°}{10}$$

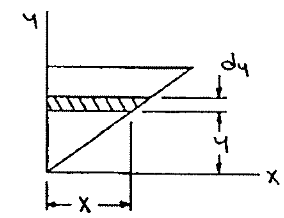
$\theta = 53.5°$ ◀



PROBLEM 9.5

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



By observation $\quad y = \frac{h}{b}x$

or $\quad x = \frac{b}{h}y$

Now

$$dI_x = y^2 dA = y^2(x\,dy)$$

$$= y^2\left(\frac{b}{h}y\,dy\right)$$

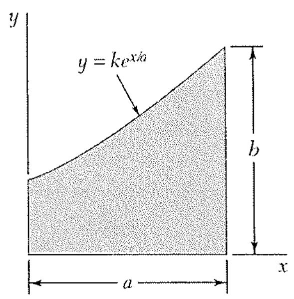
$$= \frac{b}{h}y^3 dy$$

Then

$$I_x = \int dI_x = \int_0^h \frac{b}{h}y^3 dy$$

$$= \frac{b}{h}\left[\frac{1}{4}y^4\right]_0^h$$

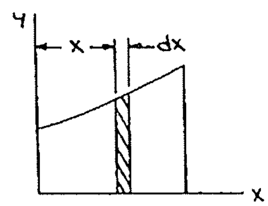
or $\quad I_x = \frac{1}{4}bh^3$ ◀



PROBLEM 9.10

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



At $x = a, y = b$: $$b = ke^{a/a} \quad \text{or} \quad k = \frac{b}{e}$$

Then $$y = \frac{b}{e}e^{x/a} = be^{x/a-1}$$

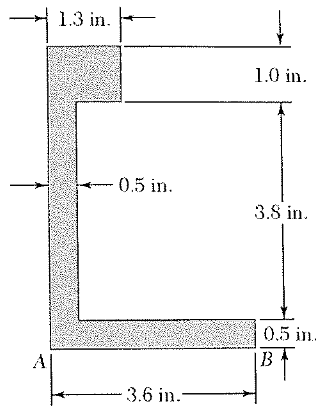
Now $$dI_x = \frac{1}{3}y^3 dx = \frac{1}{3}(be^{x/a-1})^3 dx$$

$$= \frac{1}{3}b^3 e^{3(x/a-1)} dx$$

Then $$I_x = \int dI_x = \int_0^a \frac{1}{3}b^3 e^{3(x/a-1)} dx = \frac{b^3}{3}\left[\frac{a}{3}e^{3(x/a-1)}\right]_0^a$$

$$= \frac{1}{9}ab^3(1 - e^{-3})$$

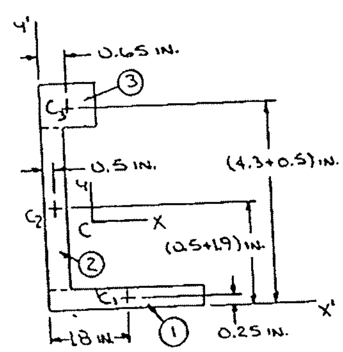
or $I_x = 0.1056ab^3$ ◀



PROBLEM 9.44

Determine the moments of inertia $\overline{I}_x$ and $\overline{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side AB.

SOLUTION



First locate centroid C of the area.

	A, in.^{2}	$\overline{x}$, in.	$\overline{y}$, in.	$\overline{x}A$, in.^{3}	$\overline{y}A$, in.^{3}
1	$3.6\times 0.5 = 1.8$	1.8	0.25	3.24	0.45
2	$0.5\times 3.8 = 1.9$	0.25	2.4	0.475	4.56
3	$1.3\times 1 = 1.3$	0.65	4.8	0.845	6.24
Σ	5.0			4.560	11.25

Then $\quad \overline{X}\Sigma A = \Sigma\overline{x}A: \quad \overline{X}(5\text{ in.}^2) = 4.560\text{ in.}^3$

or $\quad \overline{X} = 0.912\text{ in.}$

and $\quad \overline{Y}\Sigma A = \Sigma\overline{y}A: \quad \overline{Y}(5\text{ in.}^2) = 11.25\text{ in.}^3$

or $\quad \overline{Y} = 2.25\text{ in.}$

PROBLEM 9.44 (Continued)

Now $\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

where $(I_x)_1 = \frac{1}{12}(3.6\text{ in.})(0.5\text{ in.})^3 + (1.8\text{ in.}^2)[(2.25 - 0.25)\text{in.}]^2$

$= (0.0375 + 7.20)\text{in.}^4 = 7.2375\text{ in.}^4$

$(I_x)_2 = \frac{1}{12}(0.5\text{ in.})(3.8\text{ in.})^3 + (1.9\text{ in.}^2)[(2.4 - 2.25)\text{in.}]^2$

$= (2.2863 + 0.0428)\text{in.}^4 = 2.3291\text{ in.}^4$

$(I_x)_3 = \frac{1}{12}(1.3\text{ in.})(1\text{ in.})^3 + (1.3\text{ in.}^2)[(4.8 - 2.25\text{ in.})]^2$

$= (0.1083) + 8.4533)\text{in.}^4 = 8.5616\text{ in.}^4$

Then $\bar{I}_x = (7.2375 + 2.3291 + 8.5616)\text{ in.}^4 = 18.1282\text{ in.}^4$

or $\bar{I}_x = 18.13\text{ in.}^4$ ◀

Also $\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3$

where $(I_y)_1 = \frac{1}{12}(0.5\text{ in.})(3.6\text{ in.})^3 + (1.8\text{ in.}^2)[(1.8 - 0.912)\text{in.}]^2$

$= (1.9440 + 1.4194)\text{in.}^4 = 3.3634\text{ in.}^4$

$(I_y)_2 = \frac{1}{12}(3.8\text{ in.})(0.5\text{ in.})^3 + (1.9\text{ in.}^2)[(0.912 - 0.25)\text{ in.}]^2$

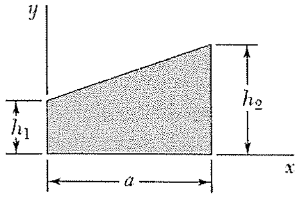
$= (0.0396 + 0.8327)\text{ in.}^4 = 0.8723\text{ in.}^4$

$(I_y)_3 = \frac{1}{12}(1\text{ in.})(1.3\text{ in.})^3 + (1.3\text{ in.}^2)[(0.912 - 0.65)\text{ in.}]^2$

$= (0.1831 + 0.0892)\text{ in.}^4 = 0.2723\text{ in.}^4$

Then $\bar{I}_y = (3.3634 + 0.8723 + 0.2723)\text{in.}^4 = 4.5080\text{ in.}^4$

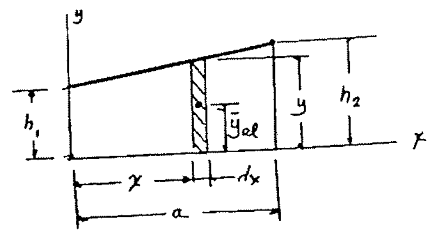
or $\bar{I}_y = 4.51\text{ in.}^4$ ◀



PROBLEM 9.70

Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION



$$y = h_1 + (h_2 - h_1)\frac{x}{a}$$

$$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

$$d\bar{I}_{x'y'} = 0 \quad \text{(by symmetry)}$$

$$\bar{x}_{EL} = x \qquad \bar{y}_{EL} = \frac{1}{2}y \qquad dA = y\,dx$$

$$I_{xy} = \int dI_{xy} - \int_0^a x\left(\frac{1}{2}y\right)y\,dx = \frac{1}{2}\int_0^a xy^2\,dx$$

$$= \frac{1}{2}\int_0^a x\left[h_1 + (h_2 - h_1)\frac{x}{a}\right]^2 dx$$

$$= \frac{1}{2}\int_0^a \left[h_1^2 x + 2h_1(h_2 - h_1)\frac{x^2}{a} + (h_2 - h_1)^2\frac{x^3}{a^2}\right]dx$$

$$= \frac{1}{2}\left[h_1^2\frac{a^2}{2} + 2h_1(h_2 - h_1)\frac{a^3}{3a} + (h_2 - h_1)^2\frac{a^4}{4a^2}\right]$$

$$= \frac{1}{2}\left[h_1^2\frac{a^2}{2} + \frac{2}{3}h_1h_2a^2 - \frac{2}{3}h_1^2a^2 + \frac{1}{4}h_2^2a^2 - \frac{1}{2}h_2h_1a + \frac{1}{4}h_1^2a^2\right]$$

$$= \frac{a^2}{2}\left[h_1^2\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) + h_1h_2\left(\frac{2}{3} - \frac{1}{2}\right) + h_2^2\left(\frac{1}{4}\right)\right]$$

$$= \frac{a^2}{2}\left[h_1^2\left(\frac{1}{12}\right) + h_1h_2\left(\frac{1}{6}\right) + h_2^2\left(\frac{1}{4}\right)\right]$$

$$I_{xy} = \frac{a^2}{24}\left(h_1^2 + 2h_1h_2 + 3h_2^2\right) \blacktriangleleft$$