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CE 100 = STATICS (Fall'00)
Required Book = Vector Mechanics For Engineers
by Beer & Johnston
(6-th Edition)
Teaching Assistant =

INSTRUCTOR = Dr. Duc T. Nguyen
OFFICE = 130-C, KAUF Bldg.
PHONE = 683-3761
OFFICE HRS. = see 130-C, KAUF
CLASS =

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| CHAPTERS | HOMEWORK ASSIGNMENTS | | |
|---|---|---|---|
| I. Introduction (1.1--->1.6) | Students are asked to read this chapter at his/her own time ! | | |
| II. Statics of particles (2.1--->2.15) Approx. 2.5 weeks | SET 1= 3,7,11,21<br>SET 4= 73,89 | SET 2= 36<br>SET 5= 94,99,121 | SET 3= 46,63 |
| III. Rigid Bodies (3.1--->3.6, 3.8--->3.20) Approx. 2.5 weeks (Test1) | SET 6= 5,12<br>SET 9= 72,83,96 | SET 7= 24,38<br>SET 10= 105,114,129 | SET 8= 41,48,57 |
| IV. Equil. of Rigid Bodies (4.1--->4.9) Approx. 1 week | SET 11= 3,23,45 | SET 12= 63,113,123 | |
| V. Distributed Forces (5.1--->5.6, 5.8,5.10 ) Approx. 1 week | SET 13= 5,7,33 | SET 14= 43,68,78 | |
| VI. Anal. of Structures (6.1--->6.4,6.7, 6.9,6.10,6.12) Approx. 2 weeks (Test2) | SET 15= 3,8 | SET 16= 43,55 | SET 17= 75,122 |
| VII. Forces in Beams & Cables (7.1--->7.6) Approx. 2 weeks (Test3) | SET 18= 7,8 | SET 19= 31,34,37 | SET 20= 79,85 |
| VIII. Friction (8.1--->8.4,8.10) Approx. 2 weeks | SET 21= 2 | SET 22= 9,29,136 | |
| IX. Distributed Forces: Moment of Inertia (9.1-9.6,9.8-9.10) Approx. 2 weeks (Thus, totally = 15 weeks) | SET 23= 1,31 | | |

current = 4th week

NOTES:
Homeworks will be assigned and collected
Approx. 2 or 3 tests + Final exam will be given
Course grade will be based upon:
2 or 3 tests 60% + final exam 40%
No make-up quizes,tests or final exam will be given, unless medical certificate of illness is shown (Make-up Test will be MORE DIFFICULT)
HONOR CODE is observed

Professional Review For Statics
Friday, Sept. 29'2000
2 PM → 3:30 PM, Room 239 KAUF

# Chapter 2 : Statics of Particles

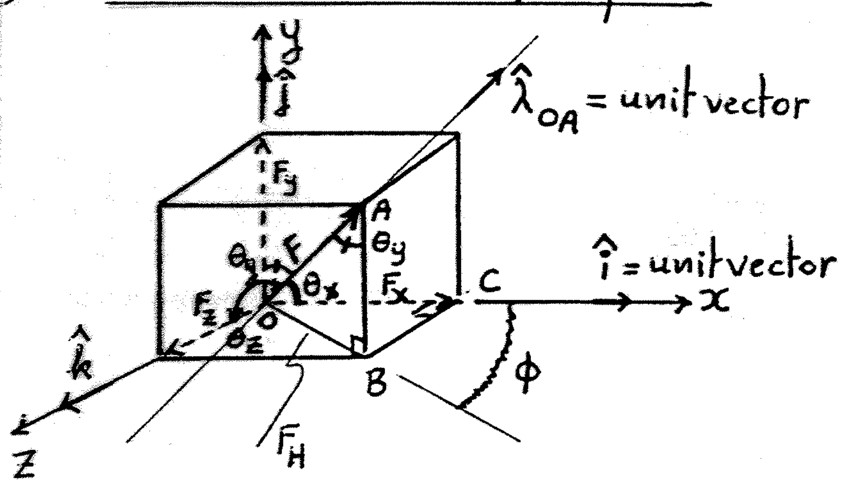
## (A) Equilibrium :

$$\Sigma \vec{F} = \vec{0}$$

or $\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$

or $R_x = R_y = R_z = 0$

## (B) Forces In (3-D) Space :




$$\vec{F} = (F_x)\hat{i} + (F_y)\hat{j} + (F_z)\hat{k}$$

where :

$$\begin{cases} F_y = F\cos\theta_y \\ F_x = (F\sin\theta_y)\cos\phi \\ F_z = (F\sin\theta_y)\sin\phi \end{cases}$$

Thus : $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

Alternatively : $\vec{F} = |F| * \hat{\lambda}_F = |F| * \hat{\lambda}_{OA}$

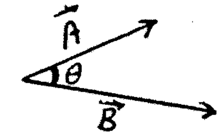
where $\hat{\lambda}_{OA} = \dfrac{\vec{OA}}{|OA|} = \dfrac{(x_A - x_o)\hat{i} + (y_A - y_o)\hat{j} + (z_A - z_o)\hat{k}}{|OA|}$

hence $\vec{F} = |F| * \left\{ \dfrac{(x_A - x_o)\hat{i} + (y_A - y_o)\hat{j} + (z_A - z_o)\hat{k}}{|OA|} \right\}$

Therefore : $F_x = \dfrac{F * (x_A - x_o)}{OA}$ ; $F_z = F * \dfrac{(z_A - z_o)}{OA}$

# Chapter 3 : Rigid Bodies

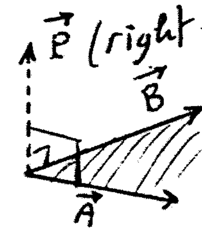
## (A) Dot Product of 2 vectors




$$\vec{A} \cdot \vec{B} = \begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix} \cdot \begin{Bmatrix} B_x \\ B_y \\ B_z \end{Bmatrix} = A_x B_x + A_y B_y + A_z B_z = \text{scalar}$$

$$\overset{\text{or}}{=} |\vec{A}| * |\vec{B}| * \cos(\vec{A}, \vec{B})$$

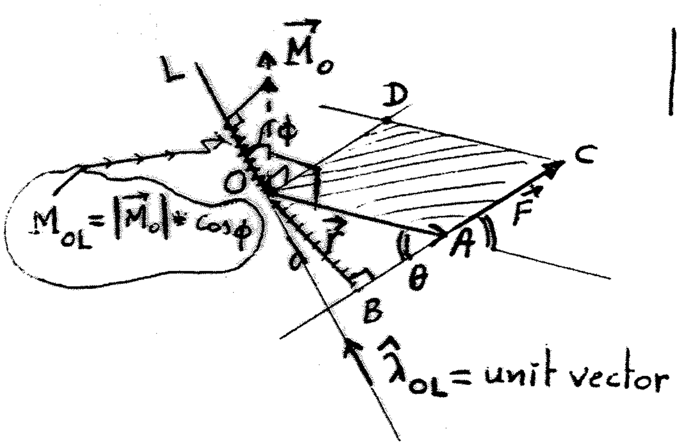
## (B) Cross Product of 2 vectors




$$\vec{A} \times \vec{B} = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z \hat{i} + A_z \hat{j} B_x + \hat{k} A_x B_y) - (B_x A_y \hat{k} + B_y A_z \hat{i} + B_z A_x \hat{j})$$

$$\overset{\text{or}}{=} \vec{P} = (P_x)\hat{i} + (P_y)\hat{j} + (P_z)\hat{k}$$

## (C) Moment @ Point O Due to Force Applied @ Point A




$$|\vec{M}_o| = |\vec{r} \times \vec{F}| = |r| * |F| * \sin\theta = F * d$$

$$M_{\text{axis } OL} = \vec{M}_o \cdot \hat{\lambda}_{OL} = \vec{M}_o \cdot \frac{\vec{OL}}{|OL|}$$

$$M_{OL} = \det \begin{vmatrix} \lambda_{OL_x} & \lambda_{OL_y} & \lambda_{OL_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

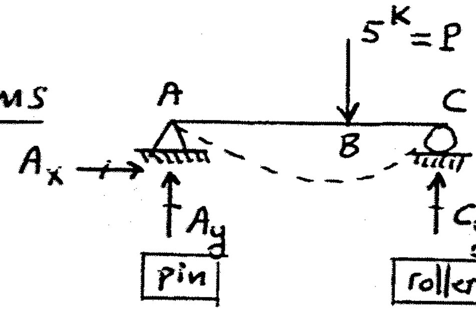
## (D) Equivalent Systems of Forces & Moments

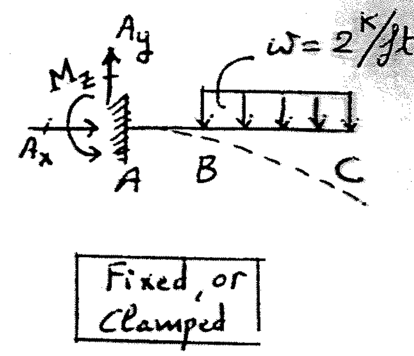
only if $(\Sigma \vec{F})_{\text{system I}} = (\Sigma \vec{F})_{\text{system II}}$ AND $(\Sigma \vec{M})_{I} = (\Sigma \vec{M})_{II}$

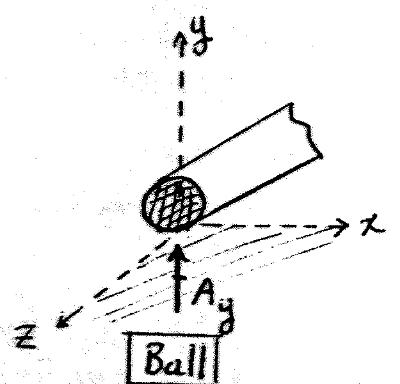
# Chapter 4 : Equilibrium of Rigid Bodies

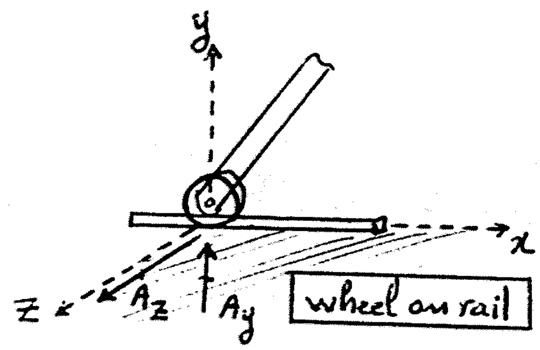
(A) Equilibrium IF : $\left.\begin{array}{l}\Sigma\vec{F} = \vec{0}\\ \Sigma\vec{M} = \vec{0}\end{array}\right\}$ In 3-D case: 6 Scalar Equations

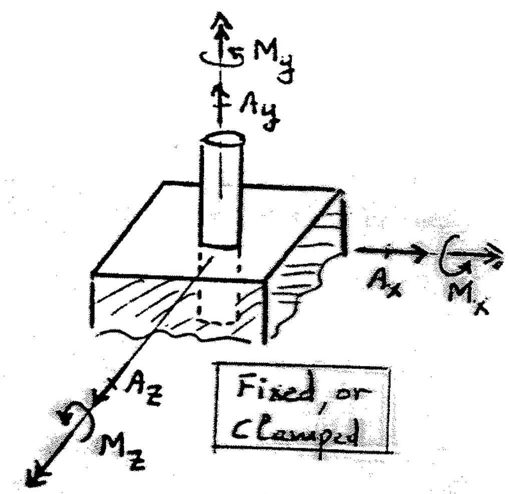
(B) Support Reactions



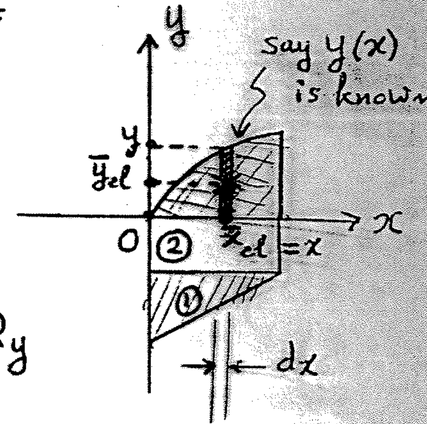



# Chapter 5 : Distributed Forces

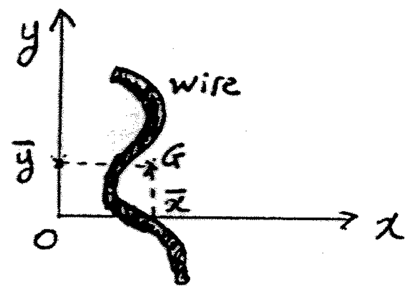



## (A) For 2-D Case

$$\bar{x} A = \sum x_i A_i \overset{\text{or}}{=} \int \underbrace{\bar{x}_{el}}_{x} \underbrace{dA}_{y\,dx} \equiv Q_y$$

$$\bar{y} A = \sum y_i A_i \overset{\text{or}}{=} \int \underbrace{\bar{y}_{el}}_{\frac{y}{2}} dA \equiv Q_x$$

## (B) For 1-D Case




$$\bar{x} L = \int x \, dL$$

$$\bar{y} L = \int y \, dL$$

## (C) For 3-D Case

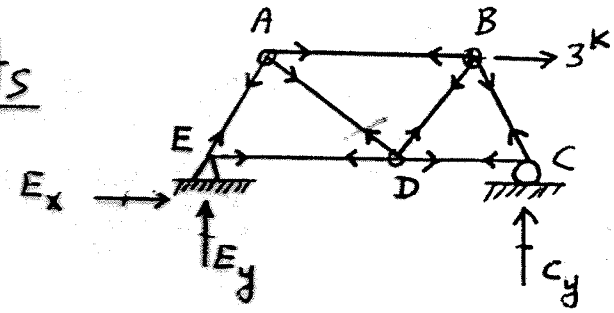
$$\bar{x} V = \sum x_i V_i \overset{\text{or}}{=} \int \bar{x}_{el} \, dV$$

$$\bar{y} V = \sum y_i V_i = \int \bar{y}_{el} \, dV$$

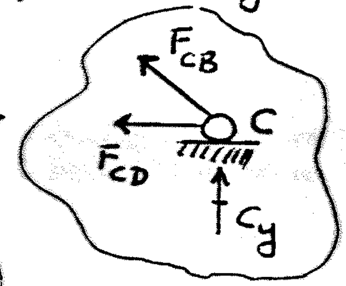
$$\bar{z} V = \sum z_i V_i = \int \bar{z}_{el} \, dV$$

# Chapter 6 : Analysis of Structures

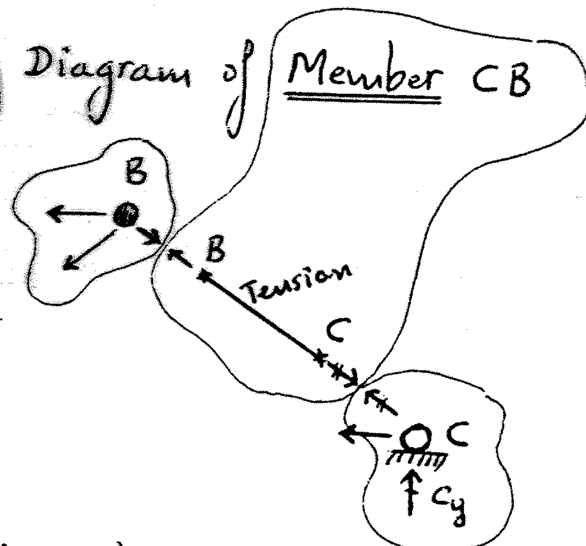
## (A) Method of Joints




* Use Equil. Equations ( $\Sigma F_x = 0 = \Sigma F_y = \Sigma M_z$ ) to find all reactions
* At each joint, we must satisfy $\Sigma F_x = 0 = \Sigma F_y$
* Free Body Diagram (FBD) of Joint C ⇒



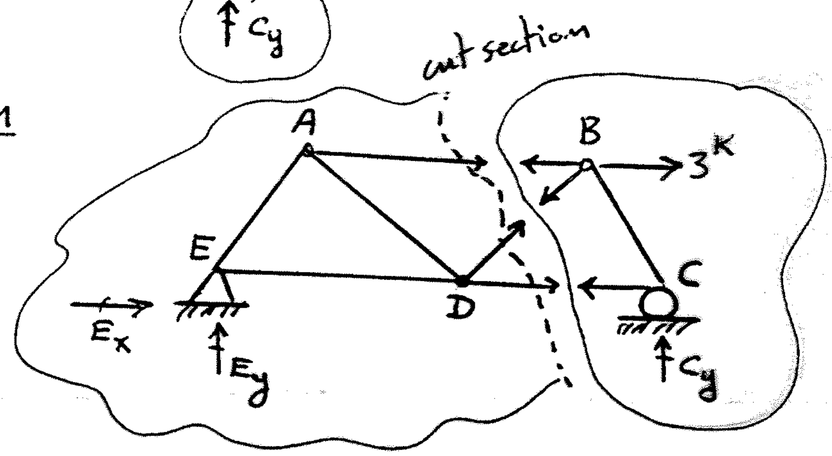

* Free Body Diagram of Member CB




## (B) Method of Section

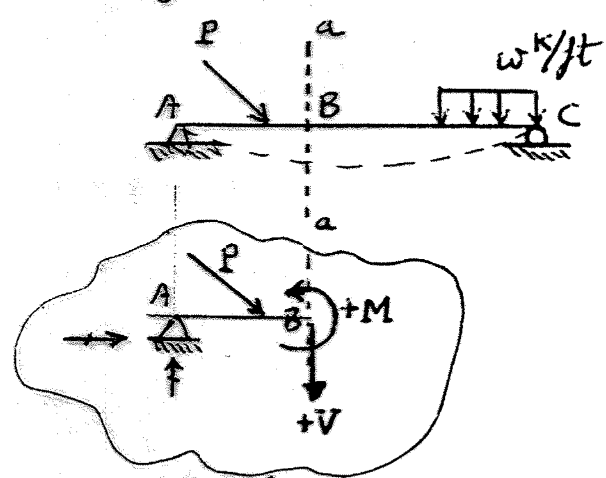
* Use either LEFT or Right FBD

Apply 3 Equil. Eqs to find 3 unknown bar forces



# Chapter 7: Forces In Beams & Cables

## (A) Sign Conventions




FBD of Beam AB

## (B) Relations Between Load, Shear & Moment

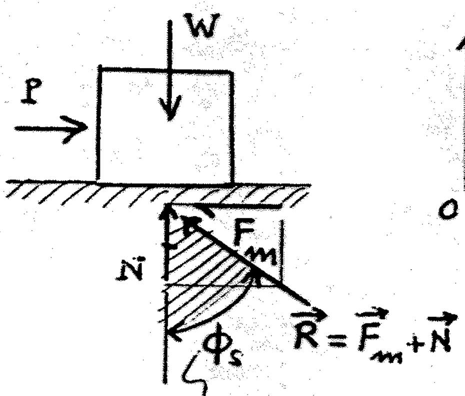
$$\frac{dV}{dx} = -w$$

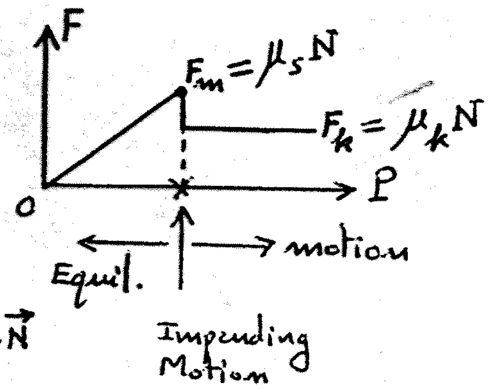
$$\frac{dM}{dx} = V \Rightarrow \int_{1}^{2} dM = \int_{1}^{2} V\,dx \Rightarrow M_{2} - M_{1} = \int_{1}^{2} V\,dx = \text{Area Under Shear Curve}$$

Slope of Moment curve

# Chapter 8 : Friction

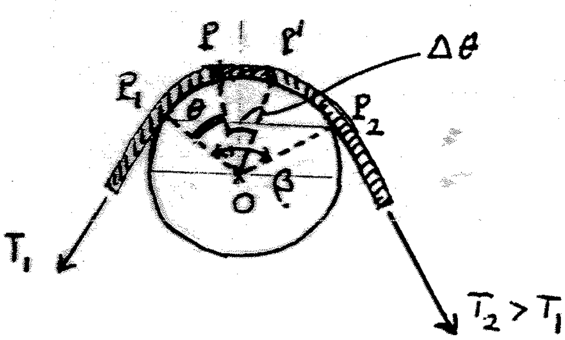
## (A) Frictional Force (opposite to the motion)




Angle of Friction $\Rightarrow \tan \phi_s = \frac{F_m}{N} = \frac{\mu_s \cancel{N}}{\cancel{N}}$ or $\tan \phi_k = \mu_k$

## (B) Belt Friction



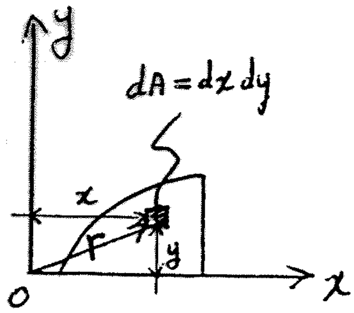

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu_s \, d\theta$$

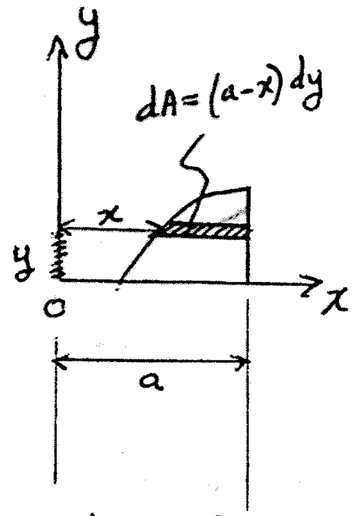
or $\ln\left(\frac{T_2}{T_1}\right) = \mu_s \beta$

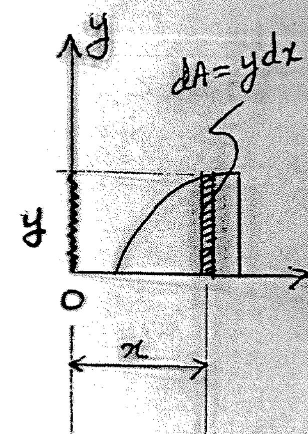
or $\frac{T_2}{T_1} = e^{\mu_s \beta}$

# Chapter 9 : Moment of Inertia

(A)




$dI_x = y^2 dA$

$dI_y = x^2 dA$

$dI_x = y^2 dA$

$dI_y = x^2 dA$

or $\boxed{I_x = \int y^2 dA \quad \text{and} \quad I_y = \int x^2 dA}$

$k_x^2 A$ $\qquad$ $k_y^2 A$

$J_o$ = Polar moment of inertia $= \int r^2 dA = I_x + I_y = k_o^2 A$

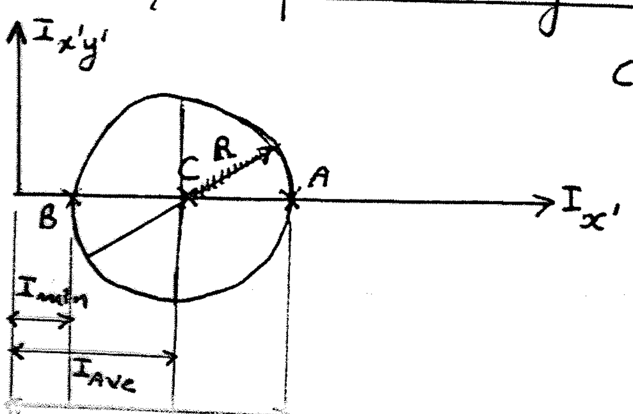
$k_x, k_y$ = radius of gyration

(C) Parallel Axis Theorem $\quad I_{\text{any axis}} = I_{\text{centroid axis}} + (A)(d)^2$

d: distance between 2 axis

(D) Product of Inertia $\quad I_{xy} = \int xy\,dA$

(E) Principal Axis, Principal Moments of Inertia, Mohr Circle



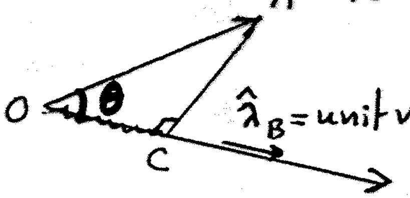

Center $\Rightarrow I_{Ave} = \dfrac{I_x + I_y}{2}$

Radius $\Rightarrow R = \sqrt{\left(\dfrac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$

# Practice Problems' Solutions [based on Previous/Sample PE/EIT Exams, See pages 38 → 47]

#*8.1

$\vec{A} = 15\hat{i} - 9\hat{j} + 15\hat{k}$




$\vec{B} = 1\hat{i} - 2\hat{j} - 2\hat{k}$

Component of vector $\vec{A}$ along direction $\vec{B}$ is

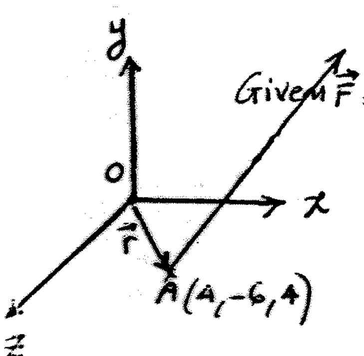
$$OC = \vec{A} \cdot \hat{\lambda}_B = |A| * |1| * \cos\theta \quad \text{where } \hat{\lambda}_B = \frac{\vec{B}}{|\vec{B}|} = \frac{(1,-2,-2)}{3} = \left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right)$$

$$OC = \begin{Bmatrix} 15 \\ -9 \\ 15 \end{Bmatrix} \cdot \begin{Bmatrix} 1/3 \\ -2/3 \\ -2/3 \end{Bmatrix} = 1$$

#*8.3

$$\vec{M}_o = \vec{r} \times \vec{F} = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -6 & 4 \\ 200 & 400 & 0 \end{vmatrix} = \left(800\hat{j} + 1600\hat{k}\right) - \left(-1200\hat{k} + 160\ldots\right)$$

$\vec{r}$ → $\vec{OA}$



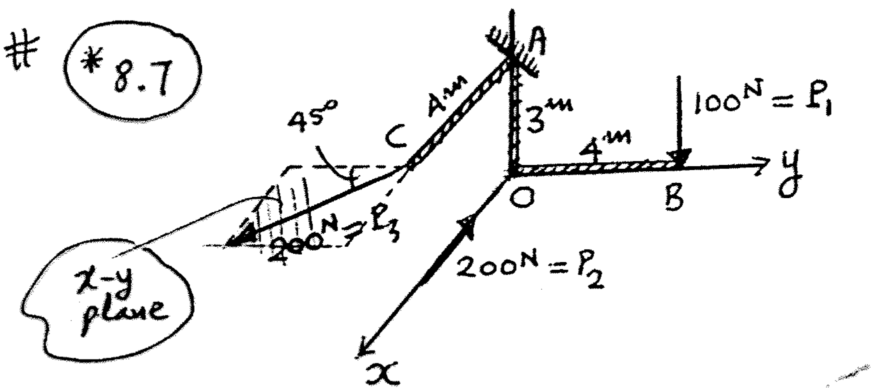

Given $\vec{F} = (200\hat{i} + 400\hat{j})$

$$\vec{M}_o = (-1600)\hat{i} + (800)\hat{j} + (2800)\hat{k}$$

Hence: $M_{y\,axis} \equiv M_{axis\,Oy} = \vec{M}_o \cdot \hat{j} = \begin{Bmatrix} -1600 \\ 800 \\ 2800 \end{Bmatrix} \cdot \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$

$$M_{y\,axis} = 800$$

# *8.7




Note: AC // x-axis

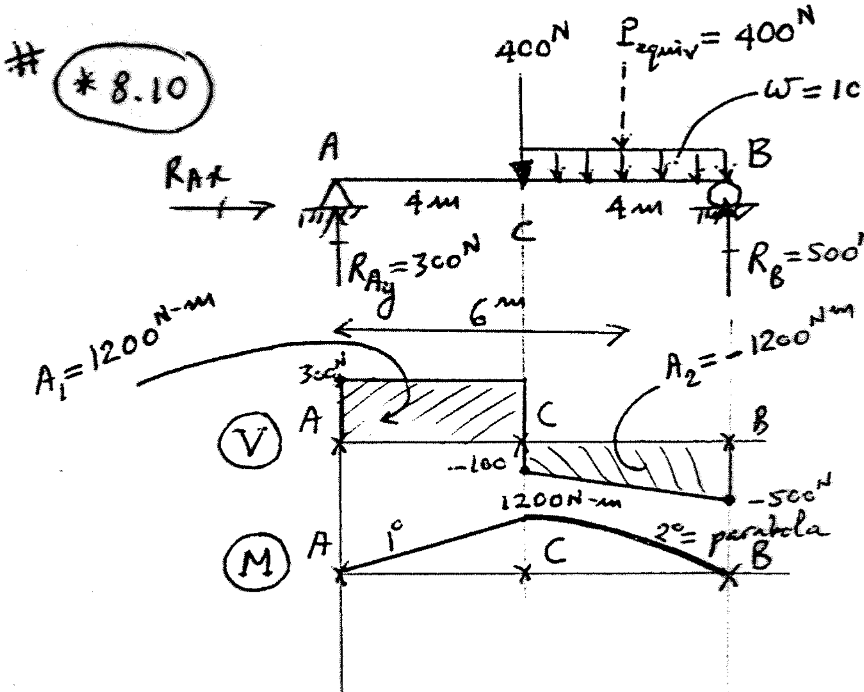
Moment @ A due to forces $\vec{P}_1$, $\vec{P}_2$ & $\vec{P}_3$ is

$$\vec{M}_A = \sum \vec{r} \times \vec{F} = \left(\vec{AB} \times \vec{P}_1\right) + \left(\vec{AO} \times \vec{P}_2\right) + \left(\vec{AC} \times \vec{P}_3\right)$$

$$\vec{M}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & -3^m \\ 0 & 0 & -100^N \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -3^m \\ -200^N & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4^m & 0 & 0 \\ 141^N & -141 & 0 \end{vmatrix} = 400\hat{i} - 600\hat{j} + 564\hat{k}$$

$141^N$: $200^N * \cos 45^\circ$

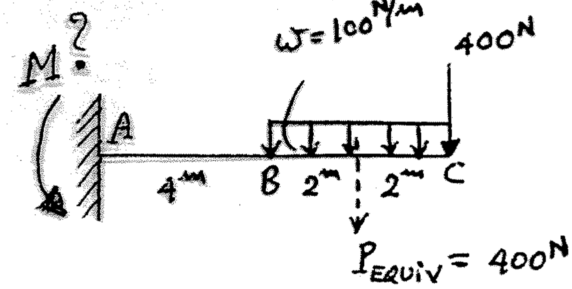
# *8.10




$$\overset{+\circlearrowleft}{\sum M_A} = 0 = -(400^N)(4^m) - (400^N)(6^m) + 8R_B$$

Solve $R_B = 500^N \Rightarrow R_{Ay} = 300^N$

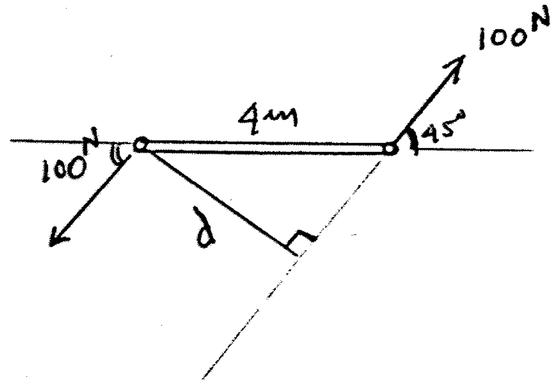
#(8.11)




$$\overset{+\circlearrowleft}{\Sigma M_A} = 0 = M - (400^N)(6^m) - (400^N)(8^m)$$

solve $\boxed{M = 5600^{N\text{-}m}}$

#(8.14)

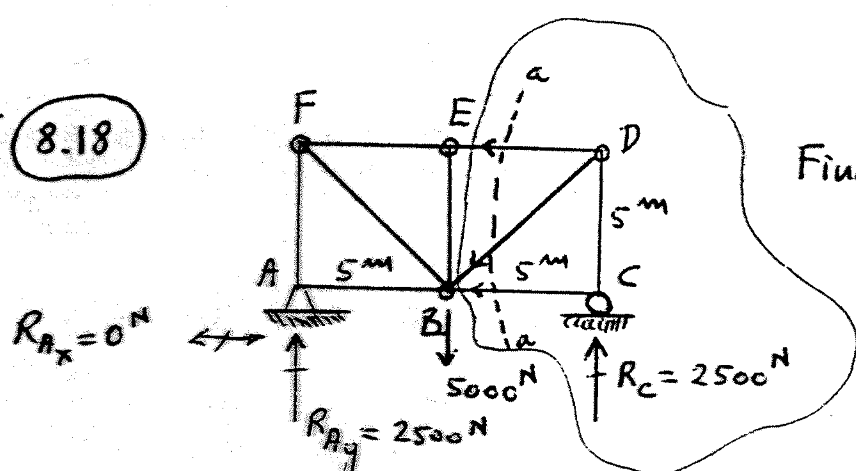



$d = 4^m \sin(45°) = 2.828^m$

Couple $= (100^N)(d = 2.828^m)$

Couple $= 282.8 \approx \boxed{283^{N\text{-}m}}$ for equilibrium

#(8.18)



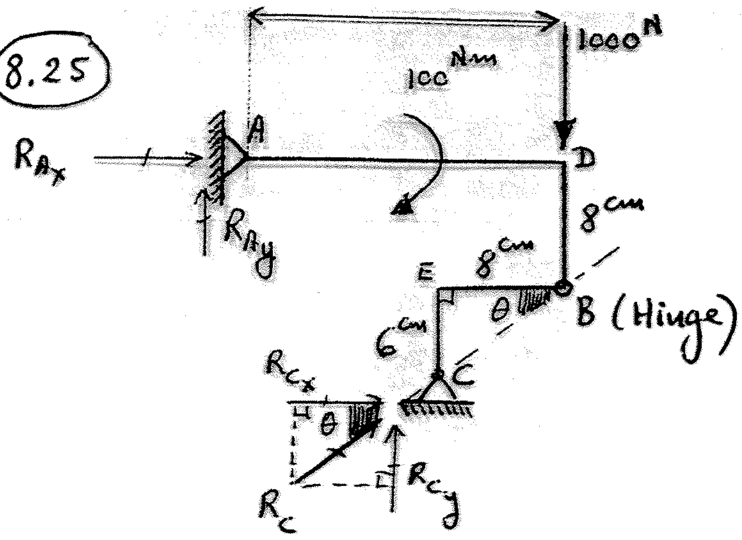

Find $F_{DE}$ ?

Use the right FBD (= Free Body Diagram), and apply Equil. Eqs.

$$\Sigma M_B = 0 \overset{+\circlearrowleft}{=} +(F_{DE})(\cancel{5^m}) + (R_C = 2500)(\cancel{5^m}) \Rightarrow \boxed{F_{DE} = -2500^N}$$

(Compression)

#(8.25)




Note: Link BEC is a 2-force member

$$R_{c_x} = R_c(\cos\theta) = R_c\left(\frac{8^{cm}}{10^{cm}}\right)$$

$$R_{c_x} = 0.8\,R_c$$

$$R_{c_y} = R_c(\sin\theta) = 0.6\,R_c$$

$$\overset{+\circlearrowleft}{\Sigma M_A} = 0 = -100^{Nm} - (1000^N)(0.2^m) + \underbrace{(R_{c_y})}_{0.6R_c}(0.2^m - 0.08^m) + \underbrace{(R_{c_x})}_{0.8R_c}(0.08^m + 0.06^m)$$

solve $R_c = +1630.43^N$

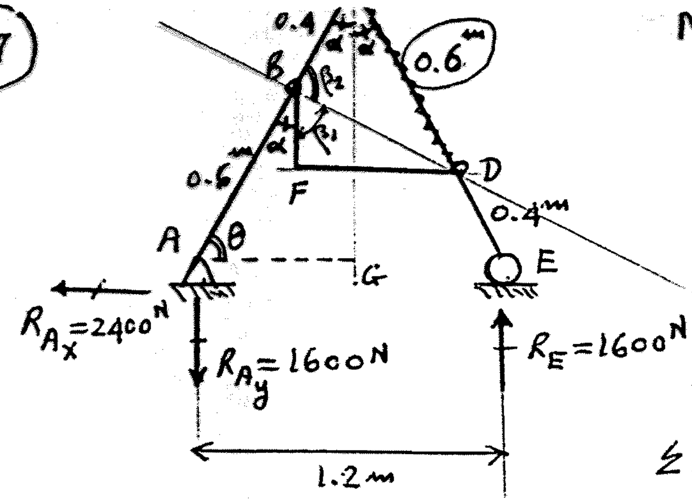
$$\Sigma F_x = 0 \overset{+\rightarrow}{=} R_{A_x} + R_{c_x} = R_{A_x} + (0.8)(R_c = 1630.43^N)$$

Hence: $R_{A_x} = -1304.35^N$

$$\overset{\uparrow +}{\Sigma F_y} = 0 = R_{A_y} - 1000^N + (R_{c_y} = 0.6 * \overbrace{R_c}^{+1630.43^N}) \Rightarrow R_{A_y} = +21.74^N$$

$$R_A = \sqrt{R_{A_x}^2 + R_{A_y}^2} = \sqrt{(-1304.35)^2 + (21.74)^2} = 1304.53^N$$

# (8.27)




Member BFD is a 2-force member

Find $|R_A|$ ?

From geometry $\Rightarrow CG = 0.8^m$

$\cos\theta = \dfrac{0.6^m}{1^m} = 0.6 \Rightarrow \theta = 53.13$

$$\overset{+\circlearrowleft}{\sum M_A} = 0 = -(2400^N)(0.8^m) + (R_E)(1.2^m)$$

Hence, solve $R_E = +1600^N \Rightarrow R_{A_y} = +1600^N$

$\overset{+\rightarrow}{\sum F_x} = 0 \Rightarrow 2400^N - R_{A_x} = 0 \Rightarrow R_{A_x} = 2400^N$

$\Rightarrow |R_A| = \sqrt{R_{A_x}^2 + R_{A_y}^2}$

$|R_A| = 2884.44^N$

# (8.28) Find $F_{BD}$ in problem #(8.27) ??

since $\theta = 53.13°$ (see problem #8.27), hence

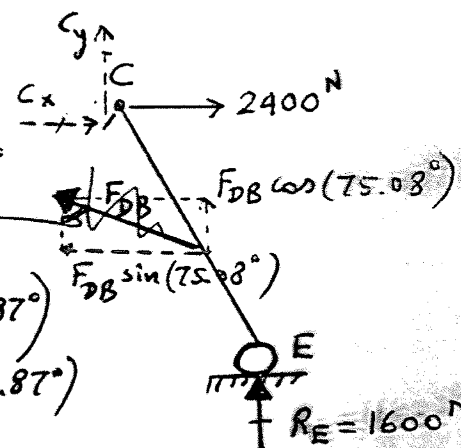
$\alpha = 36.87° \Rightarrow 2\alpha = 73.74°$

From triangle CBD $\Rightarrow BD^2 = (0.4)^2 + (0.6)^2 - \overbrace{2(0.4)(0.6)\cos(73.74°)}^{\equiv 0.1344}$

or $BD = 0.621^m$

Also: $\dfrac{\sin(73.74°)}{0.621^m} = \dfrac{\sin(\beta_2)}{0.6^m} \Rightarrow \beta_2 = 68.05° \Rightarrow \beta_1 = 75.08°$

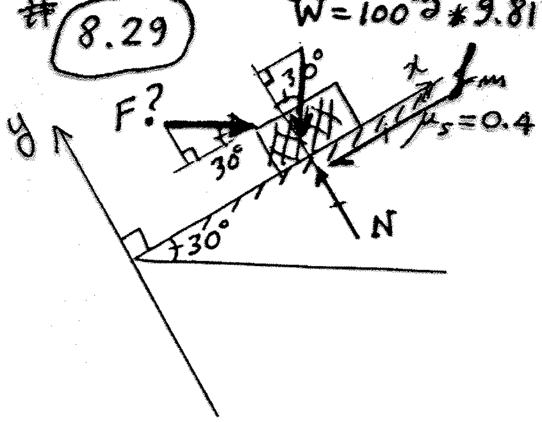
Draw FBD of member EDC:




$$\overset{+\circlearrowleft}{\sum M_C} = 0 = F_{DB}\underbrace{\cancel{\cos(75.08°)}}_{0.2575} * (0.6^m \sin 36.87°) - F_{DB}\underbrace{\cancel{\sin(75.08°)}}_{0.9663} * (0.6^m \cos 36.87°) + (1600^N)(0.6m)$$

$0 = 0.093 F_{DB} - 0.4638 F_{DB} + 960 \Rightarrow F_{DB} = +2589^{lbs}$

**#(8.29)** $W = 100^{kg} * 9.81 \frac{m}{sec^2} = 981^N$




Find F ? for upward impending motion

Solution

$$\sum F_x = 0 \overset{+\nearrow}{=} F\cos(30°) - f_m - W\sin 30°$$

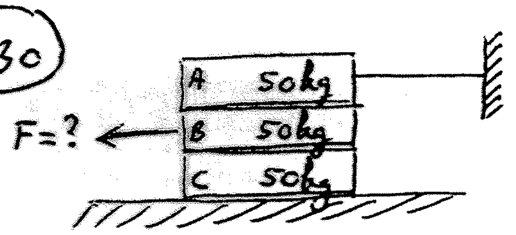
$$\boxed{0 = 0.866F - \mu_s N - (981^N)(0.5)}$$

$$\sum F_y = 0 \overset{\nwarrow +}{=} N - F\sin(30°) - (981^N)\cos(30°)$$

$$\boxed{0 = N - 0.5F - 849.6^N}$$

solve for $F = 1246.8^N$

**#(8.30)**



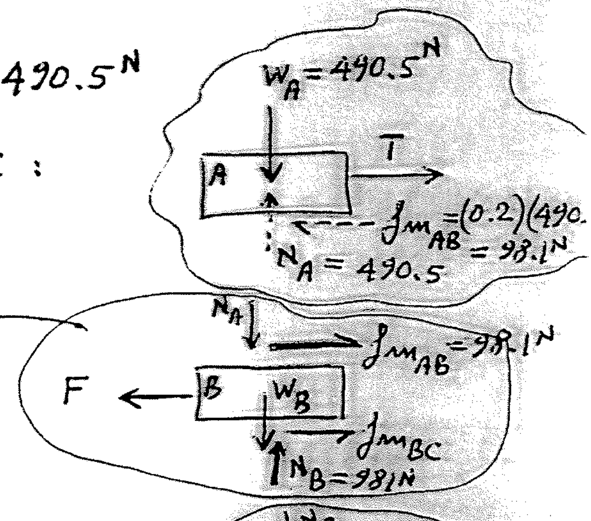

All $\mu_s = 0.2$

Find F without causing motion to impend!

Solution

$W_A = W_B = W_C = 50kg * 9.81 \frac{m}{sec^2} = 490.5^N$

Draw FBD of blocks A, B and C :

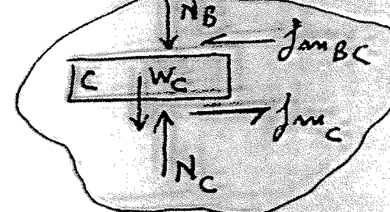



$$\sum F_y = 0 \overset{\uparrow +}{=} -N_A - W_B + N_B$$

$$0 = -490.5 - 490.5 + N_B$$

Solve $N_B = 981^N$

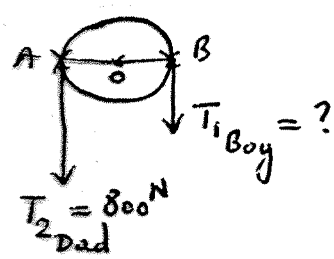
Hence $f_{m_{BC}} = (0.2)(981^N) = 196.2^N$




$$\sum F_x = 0 \overset{+}{\rightleftharpoons} -F + (f_{m_{AB}} = 98.1) + (f_{m_{BC}} = 196.2)$$

Solve $F = 294.3^N$

# (8.35)



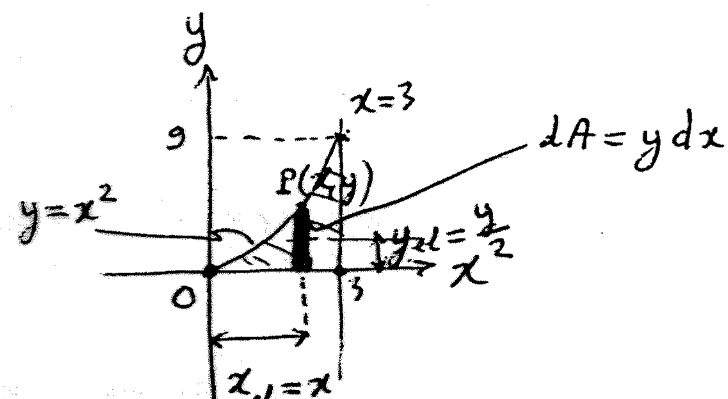

Assuming $T_{2_{Dad}} > T_{1_{Boy}}$

Then:

$$\frac{\left(T_{2_{Dad}} = 800^N\right)}{T_{1_{Boy}}} = e^{(\mu_s = 0.5)(\pi = \beta)}$$

Solve for $T_{1_{Boy}} = 166.32^N$

# (8.38)

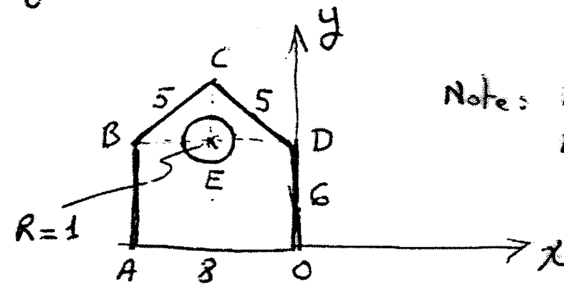



$$\bar{x} A = \int x_{el}\, dA \quad \text{where} \quad A = \int_{x=0}^{3} x^2\, dx = \left[\frac{x^3}{3}\right]_0^3 = 9$$

So $$\bar{x}(9) = \int_0^3 x\,(y\,dx) = \int_0^3 x^3\, dx = \left[\frac{x^4}{4}\right]_0^3 = 20.25$$

Hence $\bar{x} = 2.25$

# (8.42) Find $\bar{y}_c$ ?



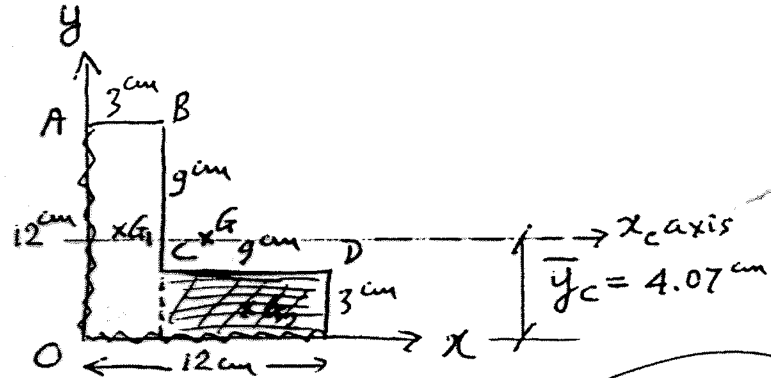

Note: $ED = 4$
$EC = \sqrt{(5)^2 - (4)^2} = 3$

| Components | Area | $y_i$ | Area * $y_i$ |
|---|---|---|---|
| BCD | +12 | 7 | +84 |
| OABD | +48 | 3 | +144 |
| Circular hole | $-\pi R^2 = -3.14$ | 6 | $-18.85$ |
| | $A_T = 56.86$ | | $\Sigma A_i y_i = 209.15$ |

$\Rightarrow \bar{y}_c = \dfrac{209.15}{56.86}$

$\bar{y}_c = 3.68$

For Problems # $\boxed{8.57 \rightarrow 8.59}$




# $\boxed{8.57}$ Find $I_x^{?} = I_{x_{1}} + I_{x_{2}} = \frac{(3)(12)^3}{3} + \frac{(9)(3)^3}{3} = \boxed{1809\ cm^4}$

# $\boxed{8.58}$ Find $\bar{y}_c$ ??

| Components | $A_i$ | $y_i$ | $A_i y_i$ |
|---|---|---|---|
| 1 | 36 | 6 | 216 |
| 2 | 27 | 1.5 | 40.5 |
| | $A_T = 63$ | | $\Sigma A_i y_i = 256.5$ |

So $\bar{y}_c = \frac{256.5}{63} = \boxed{4.07\ cm}$

# $\boxed{8.59}$ Find $I_{x_c} = ? = \underbrace{\left(I_{x_{G_1}} + A_1 d_1^2\right)}_{I_{x_{c_1}}} + \underbrace{\left(I_{x_{G_2}} + A_2 d_2^2\right)}_{I_{x_{c_2}}}$

$566.10 \equiv I_{x_{c_1}} = \frac{(3)(12)^3}{12} + (3*12)(d_1 = 6 - 4.07)^2$

$198.58 \equiv I_{x_{c_2}} = \frac{(9)(3)^3}{12} + (9*3)(d_2 = 4.07 - 1.5)^2$

So: $I_{x_c} = 566.10 + 198.58 = \boxed{764.68\ cm^4}$

individual forces) is the same at all points, but the resultant moment will vary (in magnitude and direction) from point to point.

The resultant of a pair of equal but oppositely directed parallel forces, known as a *couple*, is simply a moment having a magnitude $F \times d$ where $F$ is the magnitude of the forces and $d$ the perpendicular distance between their lines of action. Figure 8.2 illustrates equivalent couples and the curly symbol often used.

A pair of equal but oppositely directed parallel forces is known as a *couple*.

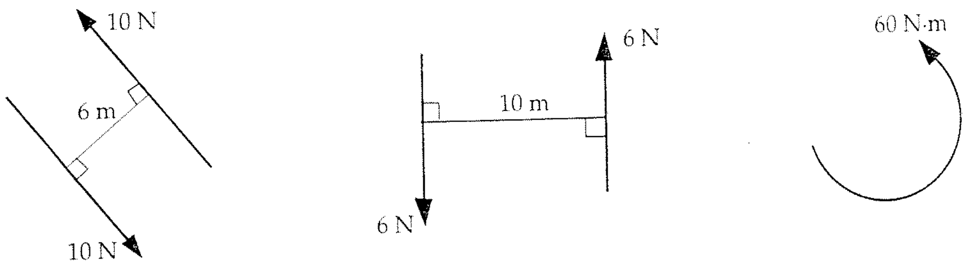
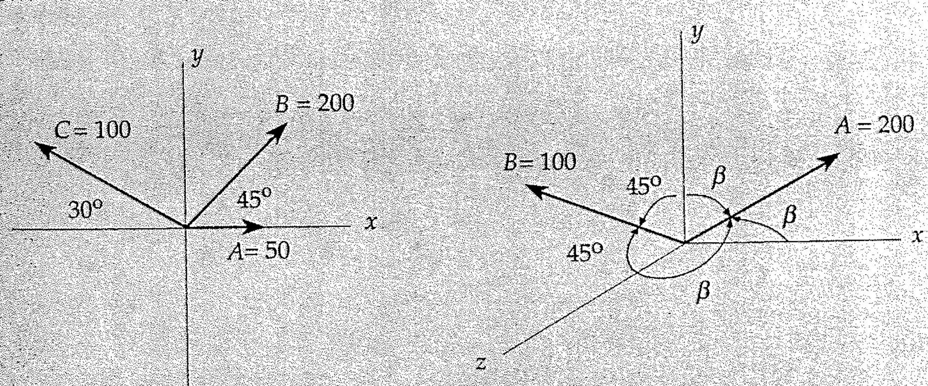



**Figure 8.2 Equivalent couples.**

**Example 8.1**

Determine the resultant force for the (a) plane, and (b) space concurrent systems shown.




(a) (b)

**Solution.** (a) $\mathbf{A} = 50\mathbf{i}$, $\mathbf{B} = 141.4\mathbf{i} + 141.4\mathbf{j}$ $\mathbf{C} = -86.6\mathbf{i} + 50\mathbf{j}$

$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} = 104.8\mathbf{i} + 191.4\mathbf{j}$$

(b) $\mathbf{A} = 200(\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{3}$, $\mathbf{B} = 70.7\mathbf{j} + 70.7\mathbf{k}$

$$\mathbf{R} = 115.47\mathbf{i} + 186.17\mathbf{j} + 186.17\mathbf{k}$$

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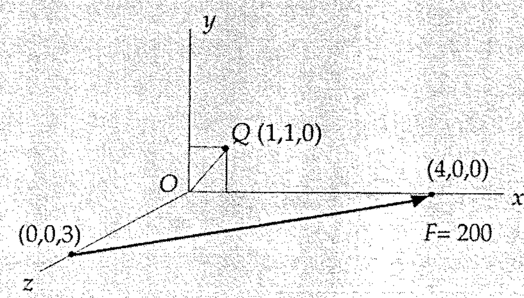
**Example 8.2**

Determine the moment of force **F**

(a) with respect to the origin. (b) with respect to point $Q$. (c) about the axis $OQ$.
(d) about the $x$-axis. (e) about the $y$-axis.




**Solution.** $\mathbf{F} = 0.8(200)\mathbf{i} - 0.6(200)\mathbf{k} = 160\mathbf{i} - 120\mathbf{k}$.

(a) $\mathbf{M}_O = 4\mathbf{i} \times (160\mathbf{i} - 120\mathbf{k}) = 3\mathbf{k} \times (160\mathbf{i} - 120\mathbf{k}) = 480\mathbf{j}$.

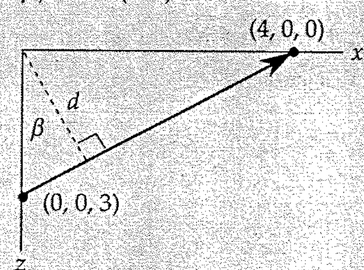
(b) $\mathbf{M}_Q = (3\mathbf{i} - \mathbf{j}) \times (160\mathbf{i} - 120\mathbf{k}) = (-\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \times (160\mathbf{i} - 120\mathbf{k}) = 120\mathbf{i} + 360\mathbf{j} + 160\mathbf{k}$.

(c) $M_{OQ} = (120\mathbf{i} + 360\mathbf{j} + 160\mathbf{k}) \cdot (\mathbf{i} + \mathbf{j})/\sqrt{2}$

$= (120 + 360)/\sqrt{2} = 480/\sqrt{2} = 339.5$.

(d) $M_x = 0$ (force **F** intersects the $x$-axis).

(e) $M_y = Fd = 200(3\cos\beta) = 200(2.4) = 480$ N·m.




## 8.2 Equilibrium

If the system of forces acting on a body is one whose resultant is absolutely zero (vector sum of all forces is zero, and the resultant moment of the forces about every point is zero) the body is in *equilibrium*. Mathematically, equilibrium requires the equations

$$\sum \mathbf{F} = 0, \qquad \sum \mathbf{M}_A = 0 \tag{8.2.1}$$

to be simultaneously satisfied, with $A$ arbitrary. These two vector equations are equivalent to the six scalar equations:

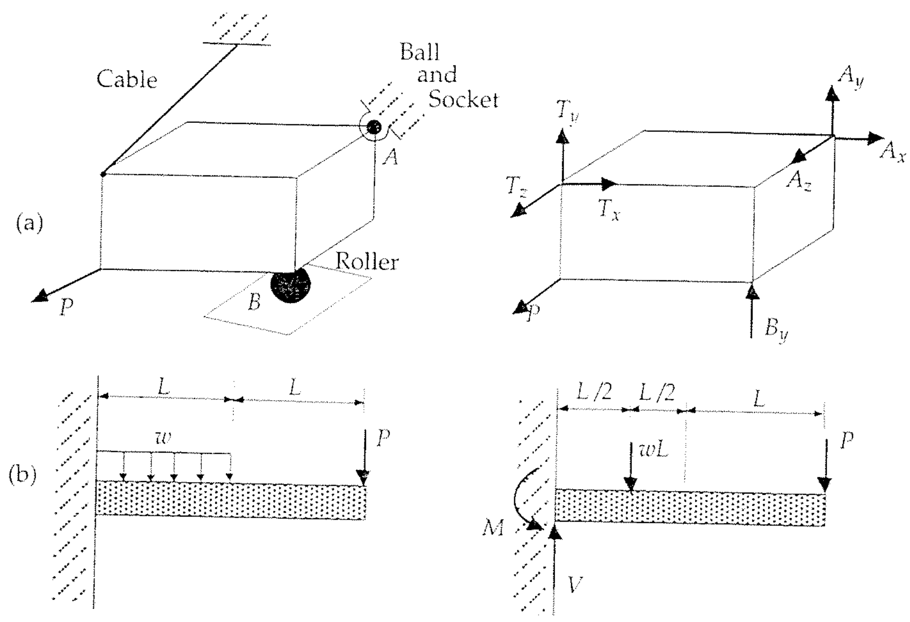
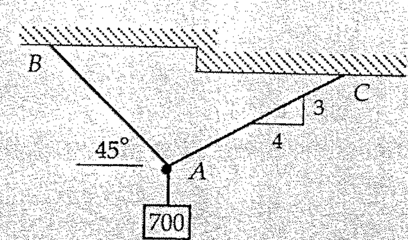



Figure 8.4 Free body diagrams.

**Example 8.3**

Determine the tension in the two cables supporting the 700 N block.

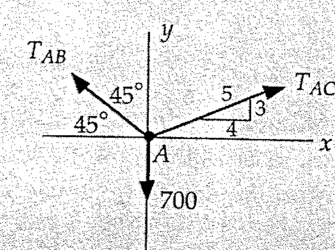



**Solution.** Construct the FBD of junction $A$ of the cables. Sum forces in $x$ and $y$ directions:

$$\sum F_x = -0.707T_{AB} + 0.8T_{AC} = 0$$
$$\sum F_y = 0.707T_{AB} + 0.6T_{AC} - 700 = 0$$

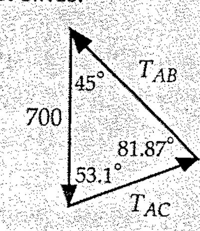
Solve, simultaneously, and find

$$T_{AC} = 700/1.4 = 500 \text{ N}$$
$$T_{AB} = 0.8(500)/0.707 = 565.8 \text{ N}$$




Alternative solution. Draw the force polygon (vector sum of forces) which must close for equilibrium. Determine angles, and use law of sines:

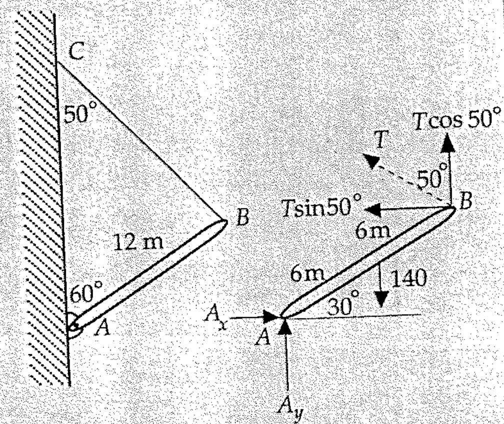
$$\frac{700}{\sin 81.87^\circ} = \frac{T_{AB}}{\sin 53.1^\circ} = \frac{T_{AC}}{\sin 45^\circ}$$
$$T_{AC} = \frac{700(0.707)}{(0.99)} = 500 \text{ N}$$
$$T_{AB} = \frac{700(0.8)}{(0.99)} = 565.8 \text{ N}$$



**Example 8.4**

A 12-m bar weighing 140 N is hinged to a vertical wall at *A*, and supported by the cable *BC*. Determine the tension in the cable together with the horizontal and vertical components of the force reaction at *A*.

**Solution.** Construct the FBD showing force components at *A* and *B*. Write the equilibrium equations and solve:




$$\sum M_A = 6T\sin 50^\circ + 6\sqrt{3}T\cos 50^\circ - 140(6)\sqrt{3}/2 = 0$$

$$\therefore T = 64.5 \text{ N}$$

$$\sum F_x = A_x - T\sin 50^\circ = A_x - 64.5(0.766) = 0$$

$$\therefore A_x = 49.4 \text{ N}$$

$$\sum F_y = A_y + T\cos 50^\circ - 140 = A_y + 64.5(0.643) - 140 = 0$$

$$\therefore A_y = 98.5 \text{ N}$$

All members are assumed to be two-force members and are therefore in simple (axial) tension or compression.

## 8.3 Trusses and Frames

Simple pin-connected trusses and plane frames provide us with elementary examples of structures that may be solved by the equilibrium concepts of statics.

The classic truss problem resembles the one-lane country bridge, as shown schematically in Fig. 8.5a. All members are assumed to be two-force members and are therefore in simple (axial) tension or compression. All loads are assumed to act at the joints (labeled *A*, *B*, *C*, etc.) where the members are pinned together. External reactions such as $A_x$, $A_y$ and $E_y$ may be determined as a non-concurrent force problem from a FBD of the entire truss. Following that, the internal forces in the members themselves may be determined from a FBD of each joint in turn

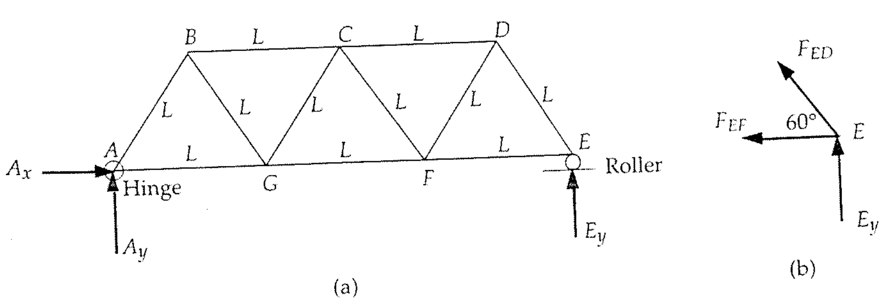



Figure 8.5 Simple truss.

(method of joints) starting, for example, with joint $E$ of the truss as shown in 8.5b. Thus, we solve a sequence of concurrent force problems at successive jc having only two unknowns. As noted by Fig. 8.5b we may assume the unkn internal forces such as $F_{ED}$, $F_{EF}$, etc., to be tension. A negative result indic compression.

**Example 8.5**

Using the right sub-truss of Fig 8.6, determine the forces in members *CD*,*CF*and*FG*

**Solution.** Summing moments about pin *F* of the FBD of the right sub-truss,

$$\sum M_F = F_{DC}(\sqrt{3}L/2)+10P(L)-6P(L/2)=0$$

$$\therefore F_{DC} = -8.085P \text{ (comp)}$$

Summing vertical forces on the sub-truss,

$$\sum F_y = 10P - 3P - 6P + 0.866F_{FC} = 0$$

$$\therefore F_{FC} = -1.156P \text{ (comp)}$$

Summing horizontal forces,

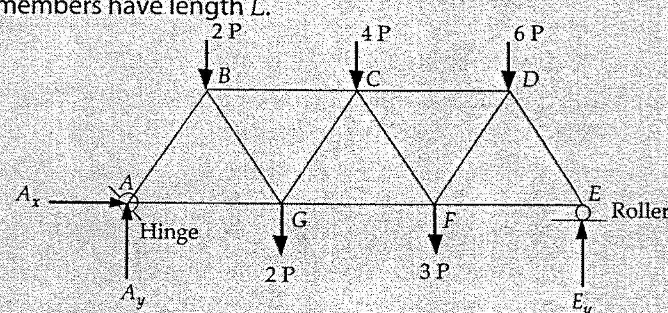
$$\sum F_x = -F_{FG} + 8.085P + 1.156P/2 = 0$$

$$\therefore F_{FG} = 8.663P \text{ (tens)}$$

Note: These results agree with those determined by the method of joints in Exam ple 8.6.

**Example 8.6**

Determine the forces in the members of the pin-connected truss loaded as show below. All members have length $L$.




**Solution.** Summing moments about pin *A* we solve for the reaction at roller *E* (counterclockwise moments are positive):

$$\sum M_A = 3LE_y - 2PL - 3P(2L) - 2P(L/2) - 4P(3L/2) - 6P(5L/2) = 0$$

$$\therefore E_y = 10P$$

Also,

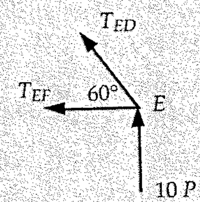
$$\sum M_E = -3LA_y + 3P(L) + 2P(2L) + 6P(L/2) + 4P(3L/2) + 2P(5L/2) = 0$$

$$\therefore A_y = 7P$$

Summing Horizontal forces,

$$\sum F_x = A_x = 0$$

Consider joint $E$ (see FBD at right),



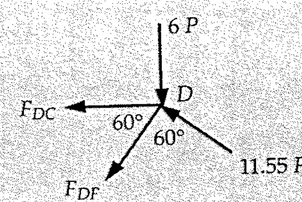

$$\sum F_y = T_{ED}\sin 60^\circ + 10P = 0$$

$$\therefore T_{ED} = -11.55P \text{ (comp)}$$

$$\sum F_x = -T_{EF} - T_{ED}\cos 60^\circ = 0$$

$$\therefore T_{EF} = 5.775P \text{ (tens)}$$

Consider next joint $D$ (see FBD at right),



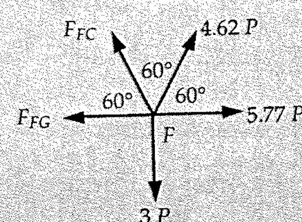

$$\sum F_y = -6P - 0.866F_{DF} + 0.866(11.55P) = 0$$

$$\therefore F_{DF} = 4.62P \text{ (tens)}$$

$$\sum F_x = -F_{DC} - 0.5(11.55P) - 0.5(4.62P) = 0$$

$$\therefore F_{DC} = -8.085P \text{ (comp)}$$

Consider next joint $F$ (see FBD at right),




$$\sum F_y = -3P + 0.866(4.62P) + 0.866F_{FC} = 0$$

$$\therefore F_{FC} = -1.156P \text{ (comp)}$$

$$\sum F_x = -F_{FG} - 0.5(-1.156P) + 0.5(4.62P) + 5.775P = 0$$

$$\therefore F_{FG} = 8.663P \text{ (tens)}$$

Note: This example should be completed by considering joint $C$ next, then joint $G$, and so on.

If the internal forces in only a few selected members are required, the *method of sections* may be used. For example, to obtain only the forces in members $CD$, $CF$ and $GF$ of the above truss, we "section" it into two portions by cutting across those members as shown in Fig. 8.6. Each portion becomes a sub-truss, and the internal forces of the sectioned members become external reactions of the two sub-trusses. Both force $\left(\sum F = 0\right)$ and moment $\left(\sum M_A = 0\right)$ equations are useful in this method.

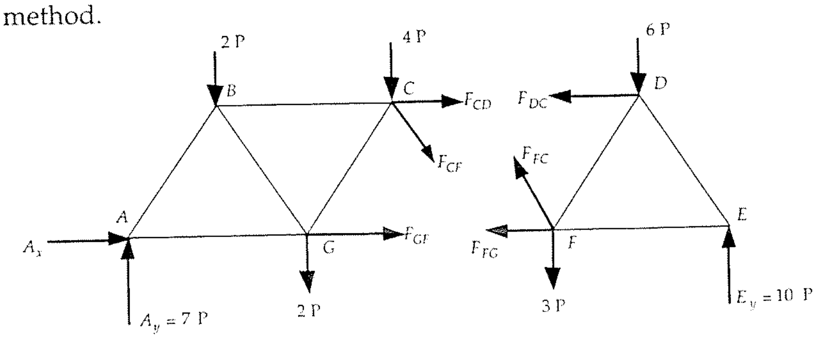



Figure 8.6 Sectioned truss.

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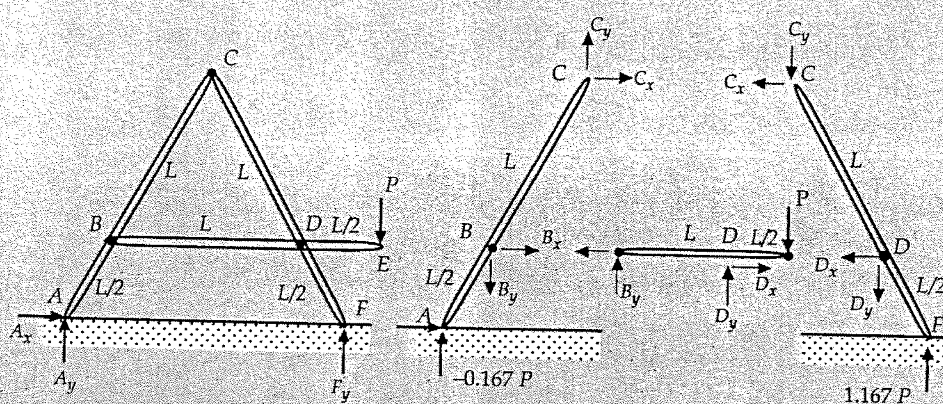
A plane frame is a structure that consists of both two-force and three-forc members, or even four-force members, etc. Loads may act at any location on th frame. The problem is to determine the components of the reactions at all pins c the frame. This usually requires not only a FBD of the entire frame, but also FBD of each member. We illustrate by the following example.

**Example 8.7**

For the frame shown, determine the horizontal and vertical components of the reactions at all pins.




**Solution.** From a FBD of entire frame,

$$\sum F_x = A_x = 0 \qquad \therefore A_x = 0$$
$$\sum M_A = (1.5L)F_y - (1.75L)P = 0 \qquad \therefore F_y = 1.167P$$
$$\sum F_y = A_y + 1.167P - P = 0 \qquad \therefore A_y = -0.167P$$

From a FBD of member *BDE,*

$$\sum M_D = -(L)B_y - (L/2)P = 0 \qquad \therefore B_y = -0.5P$$
$$\sum F_y = -0.5P + D_y - P = 0 \qquad \therefore D_y = 1.5P$$

Transfer the vertical components at *B* and *D* of members *BDE* to members *ABC* and *CDF* by changing directions (action and reaction principle). From a FBD of member *ABC,*

$$\sum F_y = -0.167P + 0.5P + C_y = 0 \qquad \therefore C_y = -0.333P$$
$$\sum M_B = -(\sqrt{3}L/2)C_x - (L/2)0.333P + (L/4)0.167P = 0 \qquad \therefore C_x = -0.144P$$
$$\sum F_x = B_x - 0.144P = 0 \qquad \therefore B_x = 0.144P$$

Transfer horizontal components $B_x$ and $C_x$ to members *BDE* and *CDF.* From a FBD of member *BDE,*

$$\sum F_x = -0.144P + D_x = 0 \qquad \therefore D_x = 0.144P$$

From a FBD of member *CDF,*

$$\sum F_y = 0.333P + D_y + 1.167P = 0 \qquad \therefore D_y = -1.5P$$

Completed FBDs of all members are shown on the next page.

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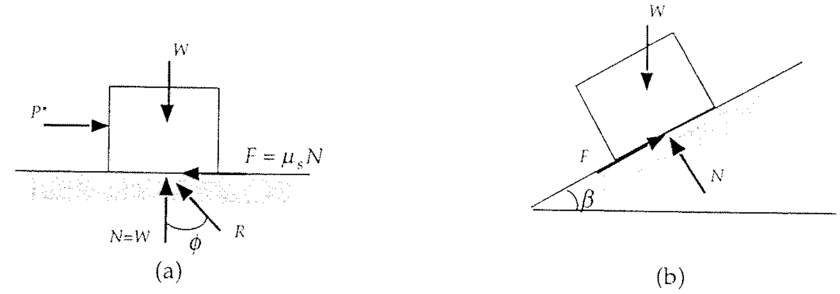
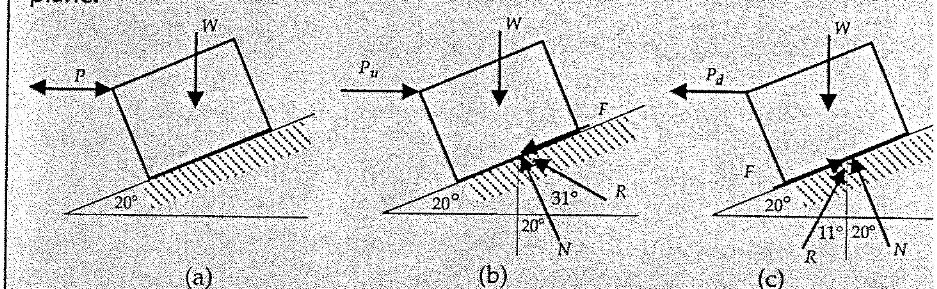



Figure 8.8 Friction angle.

**Example 8.8**

Determine the horizontal force $P$ required to cause impending motion of the 50 block, (a) up the plane, (b) down the plane, if $\mu_s = 0.6$ between the block and plane.




**Solution.** Note that, $\phi = \tan^{-1}(0.6) = 31°$ so that the block would remain in plac if undisturbed.

(a) At impending motion up the plane, Fig. b, the resultant $R$ of the friction force and normal force $N$ makes an angle of 51° to the right of the vertical and from th FBD of the block,

$$\sum F_x = P_u - R\sin 51^\circ = 0$$

$$\sum F_y = R\cos 51^\circ - 50 \times 9.81 = 0$$

Solving these equations,

$$R = 779 \text{ N}, \qquad P_u = 606 \text{ N}$$

(b) At impending motion down the plane, $R$ makes an angle of 11° to the left of the vertical, Fig. c, so that now

$$\sum F_x = -P_d + R\sin 11^\circ = 0$$

$$\sum F_y = R\cos 11^\circ - 50 \times 9.81 = 0$$

Therefore

$$R = 499 \text{ N}, \qquad P_d = 95.2 \text{ N}$$

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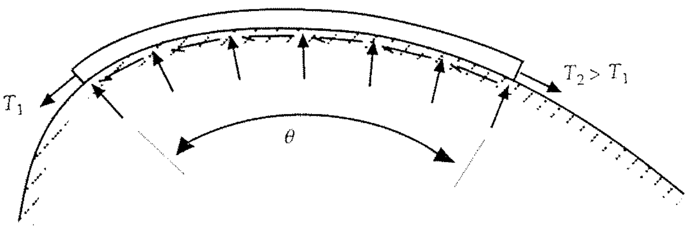



Figure 8.9 A belt with friction.

If a belt, or rope, is pressed firmly against some portion of a rough stationary curved surface, and pulled in one direction or the other, the tension in the belt will increase in the direction of pull due to the frictional resistance between the belt and surface, as shown in Fig. 8.9. It may be shown that

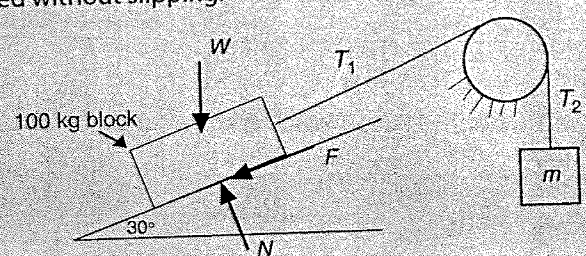
$$T_2 = T_1 e^{\mu_s \theta} \qquad T_2 > T_1 \tag{8.4.4}$$

where $\theta$ is the angle of contact, in radians, and $\mu_s$ is the static coefficient of friction.

Tension in a belt will increase in the direction of pull due to the frictional resistance.

**Example 8.9**

A 100 kg block rests on a 30° rough inclined plane ($\mu_s = 0.4$) and is attached by a rope to a mass $m$ in the arrangement shown. If the static coefficient of friction between the rope and the circular support is 0.25, determine the maximum $m$ that can be supported without slipping.




**Solution.** Summing forces perpendicular to the plane we determine

$$N = 981\cos 30° = 849.5 \text{ N}$$

Thus,

$$F = 0.4(849.5) = 339.8 \text{ N}$$

at impending motion, and by summing forces along the plane

$$T_1 = 339.8 + 981\sin 30° = 830.3 \text{ N}$$

For the circular support we have,

$$T_2 = T_1 e^{(0.25)(120\times\pi/180)} = 830.3(1.69) = 1403 \text{ N}$$

and

$$m = 1403/9.81 = 143 \text{ kg}$$

30

which normally must be evaluated by a double integration over $A$. If either one r both) of the reference axes is an axis of symmetry, the product of inertia is ero relative to that pair of axes.

The *transfer theorem* establishes a relationship between the moment of inertia about an arbitrary axis and the moment of inertia about a parallel axis passing through the centroid $C$. Thus

$$I_P = I_C + Ad^2, \qquad J_P = J_C + Ad^2 \tag{8.5.7}$$

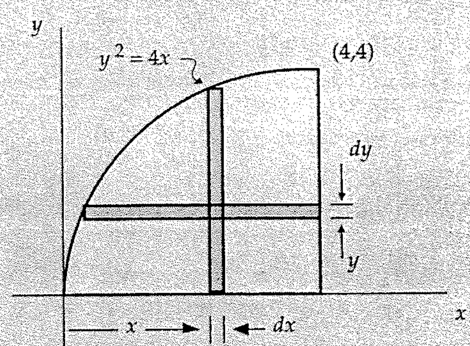
where the subscript $C$ indicates the centroidal moment of inertia, subscript $P$ indicates the moment of inertia about the parallel axis, $A$ is the area, and $d$ the distance separating the two axes. Similarly, for products of inertia,

$$I_{x_P y_P} = I_{x_C y_C} + Ax_1 y_1 \tag{8.5.8}$$

where $x_1$ and $y_1$ are the distances between axes $x$ and $x_C$, and $y$ and $y_C$, respectively.

**Example 8.10**

For the shaded area shown by the sketch, determine $\bar{x}$, $\bar{y}$, $I_x$, $I_y$, $J_O$, $I_{xy}$, $(I_x)_C$, $(I_y)_C$, $J_C$ and $I_{x_C y_C}$. Units are mm.




**Solution.** First we calculate $A$ using the vertical strip, $dA = ydx = 2\sqrt{x}dx$:

$$A = \int_0^4 2\sqrt{x}dx = 32/3 \text{ mm}^2$$

Thus, from Eq. 8.5.1, with $dA = 2\sqrt{x}dx$,

$$\bar{x} = \frac{3}{32}\int_0^4 x(2\sqrt{x})dx = 2.4 \text{ mm}$$

and, using the horizontal strip $dA = (4 - y^2/4)dy$,

$$\bar{y} = \frac{3}{32}\int_0^4 y(4 - y^2/4)dy = 1.5 \text{ mm}$$

From Eq. 8.5.3, with $dA = (4 - y^2/4)dy$,

$$I_x = \int_0^4 y^2(4 - y^2/4)dy = 34.13 \text{ mm}^4$$

and, using $dA = 2\sqrt{x}dx$,

32

$$I_y = \int_0^4 x^2(2\sqrt{x})dx = 73.14 \text{ mm}^4$$

From Eq. 8.5.4

$$J_O = I_x + I_y = 34.13 + 73.14 = 107.27 \text{ mm}^4$$

From Eq. 8.5.6, with $dA = dxdy$,

$$I_{xy} = \int_0^4 \int_0^{2\sqrt{x}} xy\,dydx = \int_0^4 \left[\frac{xy^2}{2}\right]^{2\sqrt{x}} dx = \int_0^4 2x^2 dx = 42.67 \text{ mm}^4$$

From Eq. 8.5.7,

$$(I_x)_C = I_x - Ad^2 = 34.13 - 10.67(1.5)^2 = 10.13 \text{ mm}^4$$

$$(I_y)_C = I_y - Ad^2 = 73.14 - 10.67(2.4)^2 = 11.68 \text{ mm}^4$$

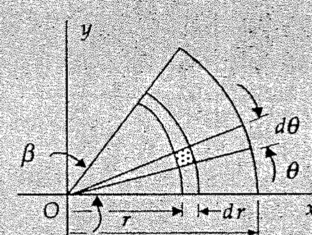
$$J_C = J_O - Ad^2 = 107.27 - 10.67\left[(1.5)^2 + (2.4)^2\right] = 21.80 \text{ mm}^4$$

Note that moments of inertia are always minimum about a centroidal axis. Finally, from Eq. 8.5.8,

$$I_{x_C y_C} = I_{xy} - A(-1.5)(-2.4) = 42.67 - 38.40 = 4.27 \text{ mm}^4$$

**Example 8.11**

Determine $I_x$ and $J_O$ for the circular sector shown below.




**Solution.** Using polar coordinates with $dA = rdrd\theta$ and $y = r\sin\theta$ in Eq. 8.5.3:

$$I_x = \int_0^\beta \int_0^a (r\sin\theta)^2 rdrd\theta = \frac{a^4}{4}\left[\frac{\beta}{2} - \frac{\sin 2\beta}{4}\right]$$

When $\beta = \pi/2$,

$$I_x = \pi a^4/16$$

From Eq. 8.5.4,

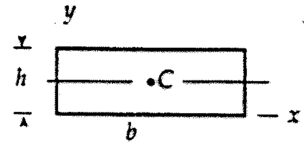
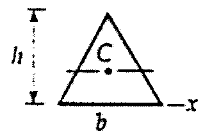
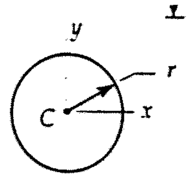
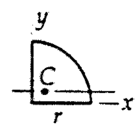
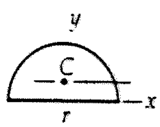
$$J_O = \int_0^\beta \int_0^a r^3 drd\theta = \frac{a^4\beta}{4}$$

When $\beta = \pi/2$,

$$J_O = \pi a^4/8$$

33

**TABLE 8.1. Properties of Areas**

| Shape | Dimensions | Centroid | Inertia |
|---|---|---|---|
| Rectangle |  | $\bar{x} = b/2$<br>$\bar{y} = h/2$ | $I_C = bh^3/12$<br>$I_x = bh^3/3$<br>$I_y = hb^3/3$ |
| Triangle |  | $\bar{y} = h/3$ | $I_C = bh^3/36$<br>$I_x = bh^3/12$ |
| Circle |  | $\bar{x} = 0$<br>$\bar{y} = 0$ | $I_x = \pi r^4/4$<br>$J = \pi d^4/32$ |
| Quarter Circle |  | $\bar{y} = 4r/3\pi$ | $I_x = \pi r^4/16$ |
| Half Circle |  | $\bar{y} = 4r/3\pi$ | $I_x = \pi r^4/8$ |

The properties of common areas may be determined by integration. A brief list is given in Table 8.1. Using data from Table 8.1, we may calculate centroids and moments of inertia of *composite areas* made up of combinations of two or more (including cutouts) of the common areas. Thus,

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i A_i}{\sum_{i=1}^{N} A_i}, \qquad \bar{y} = \frac{\sum_{i=1}^{N} y_i A_i}{\sum_{i=1}^{N} A_i} \tag{8.5.9}$$

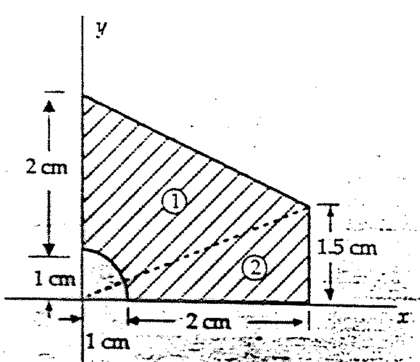
where $N$ is the number of areas, and $x_i$ is the centroidal distance for area $A_i$. Likewise, for moments of inertia

$$I = \sum_{i=1}^{N} I_i = I_1 + I_2 + \ldots + I_N \tag{8.5.10}$$

An example illustrates.

**Example 8.12**

Determine the centroidal coordinates, and $I_x$ and $I_y$ for the composite area shown.




**Solution.** Decompose the composite into two triangular areas 1 and 2, and the negative quarter circular area 3:

$$A = A_1 + A_2 + A_3 = 4.5 + 2.25 - \pi/4 = 5.97\ \text{cm}^2$$
$$A\bar{x} = x_1 A_1 + x_2 A_2 + x_3 A_3$$
$$= (1)(4.5) + (2)(2.25) + (4/3\pi)(-\pi/4) = 8.67$$
$$\therefore \bar{x} = 8.67/5.97 = 1.45\ \text{cm}$$
$$A\bar{y} = y_1 A_1 + y_2 A_2 + y_3 A_3$$
$$= (1.5)(4.5) + (0.5)(2.25) + (4/3\pi)(-\pi/4) = 7.54$$
$$\therefore \bar{y} = 7.54/5.97 = 1.26\ \text{cm}$$
$$I_x = I_{1x} + I_{2x} + I_{3x}$$
$$= \left\{2\left[3(1.5)^3/36\right] + (4.5)(1.5)^2\right\} + 3(1.5)^3/12 - \pi/16 = 11.33\ \text{cm}^4$$

where we have used the parallel-axis theorem to obtain $I_{1x}$. Finally,

$$I_y = I_{1y} + I_{2y} + I_{3y}$$
$$= 3(3)^3/12 + 1.5(3)^3/4 - \pi/16 = 16.68\ \text{cm}^4$$

## 8.6 Properties of Masses and Volumes

The coordinates of the *center of gravity* $G$ of an arbitrary mass $m$ occupying a vo-
ume $V$ of space are defined by

$$x_G = \frac{\int_V x\rho dV}{\int_V \rho dV}, \qquad y_G = \frac{\int_V y\rho dV}{\int_V \rho dV}, \qquad z_G = \frac{\int_V z\rho dV}{\int_V \rho dV} \qquad \textbf{(8.6.}$$

where $\rho$ is the mass density, $dV$ is the differential element of volume, and $x$, $y$ ar
$z$ are the coordinates of $dV$, as shown in Fig. 8.11.

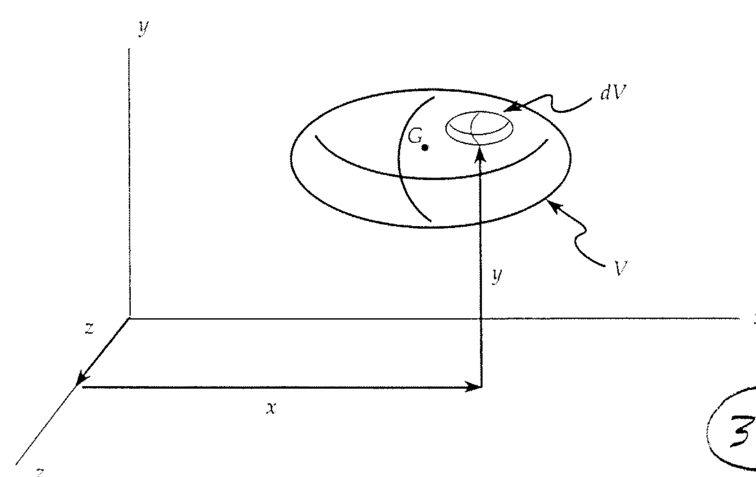



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Figure 8.11 Center of gravity of an arbitrary mass.

# Practice Problems

*(If you choose to work only a few problems, select those with a star.)*

L = Lectures

## General

*8.1 Find the component of the vector $\mathbf{A} = 15\mathbf{i} - 9\mathbf{j} + 15\mathbf{k}$ in the direction of $\mathbf{B} = \mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$.

a) 1 b) 3 c) 5 d) 7

8.2 Find the magnitude of the resultant of $\mathbf{A} = 2\mathbf{i} + 5\mathbf{j}$, $\mathbf{B} = 6\mathbf{i} - 7\mathbf{k}$, and $\mathbf{C} = 2\mathbf{i} - 6\mathbf{j} + 10\mathbf{k}$.

a) 8.2 b) 9.3 c) 10.5 d) 11.7

L

*8.3 Determine the moment about the $y$-axis of the force $\mathbf{F} = 200\mathbf{i} + 400\mathbf{j}$ acting at (4, –6, 4).

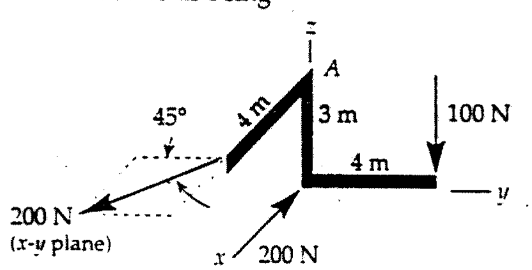
a) 0 b) 200 c) 400 d) 800

8.4 What total moment do the two forces $\mathbf{F}_1 = 50\mathbf{i} - 40\mathbf{k}$ and $\mathbf{F}_2 = 60\mathbf{j} + 80\mathbf{k}$ acting at (2, 0, –4) and (–4, 2, 0), respectively, produce about the $x$-axis?

a) 0 b) 80 c) 160 d) 240

HW *8.5 The force system shown may be referred to as being

a) non-concurrent, non-coplanar
b) coplanar
c) parallel
d) two-dimensional




## Equilibrium

HW 8.6 If equilibrium exists due to a rigid support at $A$ in the figure of Prob. 8.5, what reactive force must exist at $A$?

a) $-59\mathbf{i} - 141\mathbf{j} + 10\mathbf{k}$
b) $59\mathbf{i} + 141\mathbf{j} + 100\mathbf{k}$
c) $341\mathbf{i} - 141\mathbf{j} - 100\mathbf{k}$
d) $341\mathbf{i} + 141\mathbf{j} - 100\mathbf{k}$

L

*8.7 If equilibrium exists on the object in Prob. 8.5, what reactive moment must exist at the rigid support $A$?

a) $600\mathbf{i} + 400\mathbf{j} + 564\mathbf{k}$
b) $400\mathbf{i} + 564\mathbf{k}$
c) $400\mathbf{i} - 600\mathbf{j} + 564\mathbf{k}$
d) $400\mathbf{i} - 600\mathbf{j}$

*8.8 If three nonparallel forces hold a rigid body in equilibrium, they must

a) be equal in magnitude.
b) be concurrent.
c) be non-concurrent.
d) form an equilateral triangle.

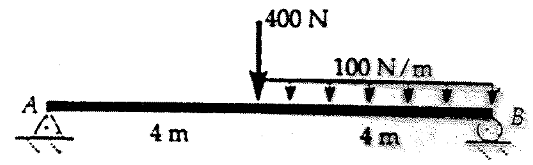
HW *8.9 A truss member

a) is a two-force body.
b) is a three-force body.
c) resists forces in compression only.
d) may resist three concurrent forces.

L

*8.10 Find the magnitude of the reaction at support *B*.

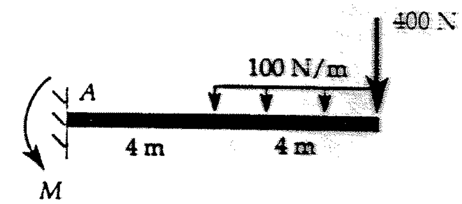
a) 400
b) 500
c) 600
d) 700




L

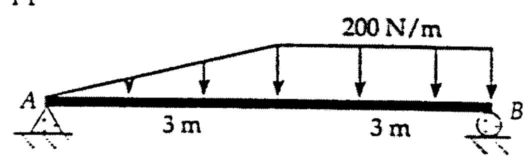
8.11 What moment *M* exists at support *A*?

a) 5600
b) 5000
c) 4400
d) 4000



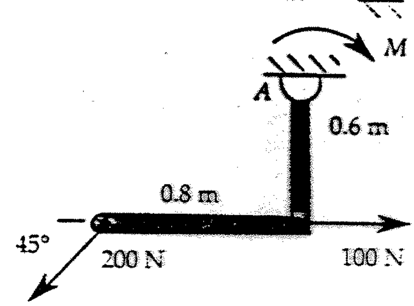

8.12 Calculate the reactive force at support *A*.

a) 250
b) 350
c) 450
d) 550




8.13 Find the support moment at *A*.

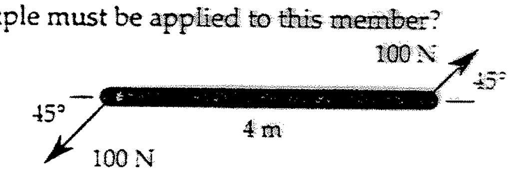
a) 66
b) 77
c) 88
d) 99




L

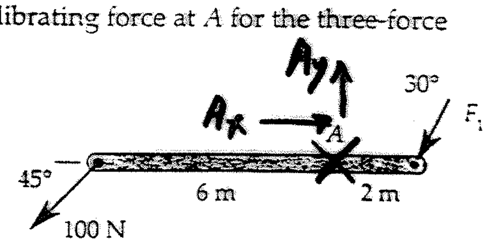
*8.14 To ensure equilibrium, what couple must be applied to this member?

~~a) 283 cw~~
b) 283 ccw
c) 400 cw
d) 400 ccw




HW *8.15 Calculate the magnitude of the equilibrating force at *A* for the three-force body shown.

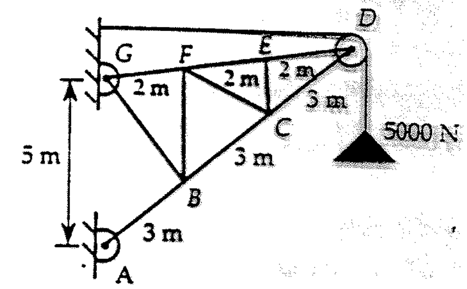
a) 217
b) 287
c) 343
d) 385



8.23 Find the force in member $FC$.

a) 5320 c) 2560

b) 3420 d) 0



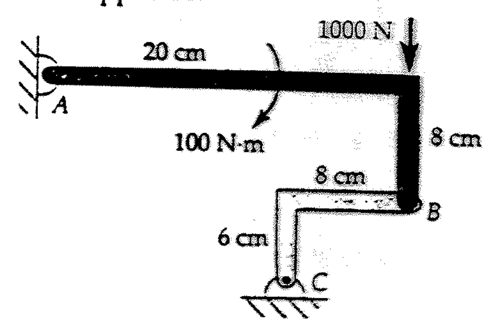

8.24 Determine the force in member $BC$ in the truss of Prob. 8.23.

a) 3560 b) 4230 c) 5820 d) 6430

*8.25 Find the magnitude of the reactive force at support $A$.

a) 1400 c) 1200

~~b)~~ 1300 d) 1100

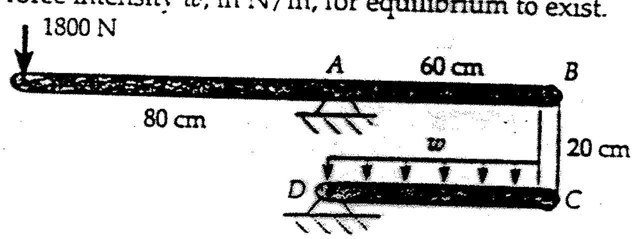



HW

8.26 Determine the distributed force intensity $w$, in N/m, for equilibrium to exist.

a) 2000 c) 6000

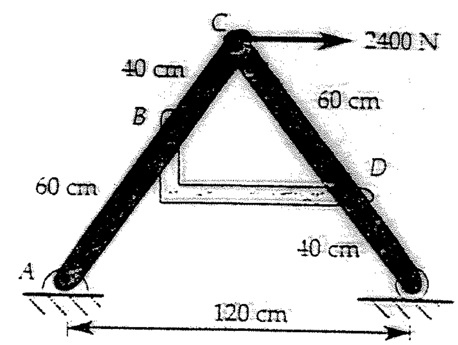
b) 4000 d) 8000




*8.27 Find the magnitude of the reactive force at support $A$.

a) 2580 c) 2790

b) 2670 ~~d)~~ 2880

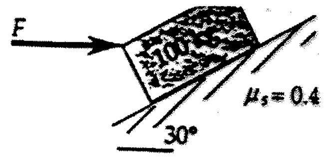



8.28 Calculate the magnitude of the force in member $BD$ of Prob. 8.27.

~~a)~~ 2590 b) 2670 c) 2790 d) 2880

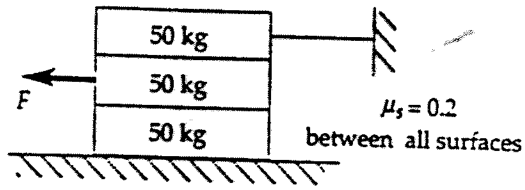
*8.29 What force, in newtons, will cause impending motion up the plane?

a) 731 c) 973
b) 821 d) 1245




*8.30 What is the maximum force $F$, in newtons, that can be applied without causing motion to impend?

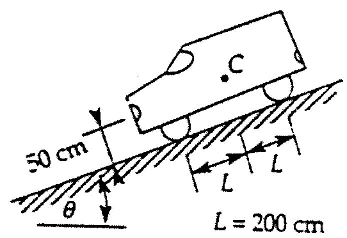
a) 184 c) 316
~~b)~~ 294 d) 346




HW

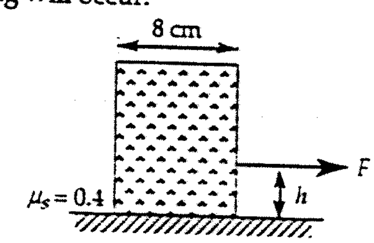
*8.31 Only the rear wheels provide braking. At what angle $\theta$ will the car slide if $\mu_s = 0.6$?

a) 10 c) 16
b) 12 d) 20




8.32 Find the minimum $h$ value at which tipping will occur.

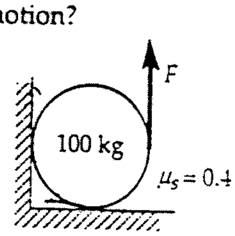
a) 8 cm c) 12 cm
b) 10 cm d) 14 cm




HW

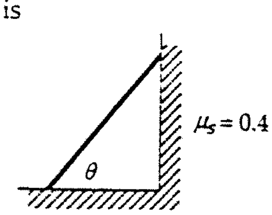
*8.33 What force $F$, in newtons, will cause impending motion?

a) 240 c) 280
b) 260 d) 320




8.34 The angle $\theta$ at which the ladder is about to slip is

a) 50 c) 42
b) 46 d) 38

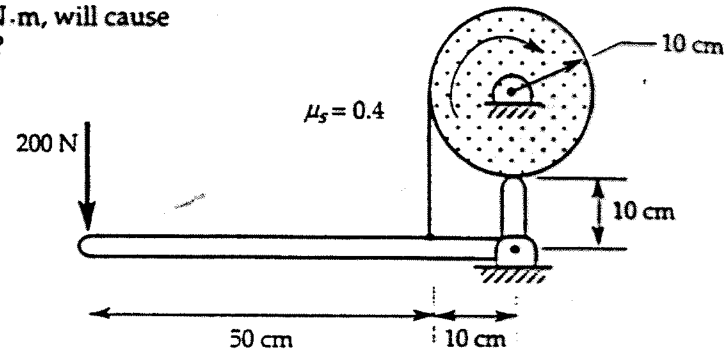


*8.35 A boy and his dad put a rope around a tree and stand side by side. What force by the boy can resist a force of 800 N by his dad? Use $\mu_s = 0.5$.

a) 166 N  b) 192 N  c) 231 N  d) 246 N

8.36 What moment, in N·m, will cause impending motion?

a) 88
b) 99
c) 110
d) 121




8.37 A 12-m-long rope is draped over a horizontal cylinder of 1.2-m-diameter so that both ends hang free. What is the length of the longer end at impending motion? Use $\mu_s = 0.5$.

a) 6.98 m  b) 7.65 m  c) 7.92 m  d) 8.37 m

## Centroids and Moments of Inertia

*8.38 Find the $x$-coordinate of the centroid of the area bounded by the $x$-axis, the line $x = 3$, and the parabola $y = x^2$.

a) 2.0  b) 2.15  c) 2.20  d) 2.25

8.39 What is the $y$-coordinate of the centroid of the area of Prob. 8.38?

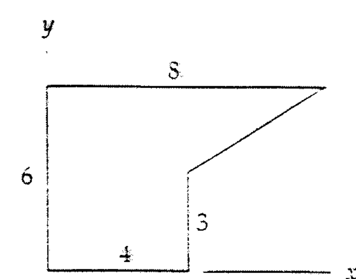
a) 2.70  b) 2.65  c) 2.60  d) 2.55

8.40 Calculate the $x$-coordinate of the centroid of the area enclosed by the parabolas $y = x^2$ and $x = y^2$.

a) 0.43  b) 0.44  c) 0.45  d) 0.46

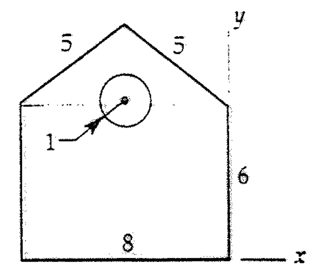
8.41 Find the $y$-component of the centroid of the area shown.

a) 3.35  c) 3.45
b) 3.40  d) 3.50

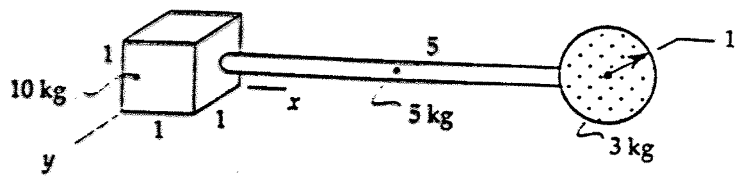



*8.42 Calculate the $y$-component of the centroid of the area shown.

a) 3.52  c) 3.60
b) 3.56  d) 3.68



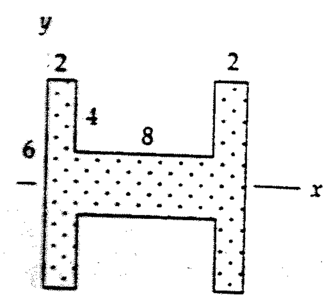
b) 2.42
c) 2.84
d) 3.22




*8.44 Calculate the moment of inertia about the $x$-axis of the area of Prob. 8.38.
a) 94
b) 104
c) 112
d) 124

8.45 What is $I_x$ for the area of Prob. 8.42?
a) 736
b) 842
c) 936
d) 1056

*8.46 Find $I_y$ for the symmetrical area shown.
a) 4267
b) 4036
c) 3827
d) 3652




8.47 Determine the mass moment of inertia of a cube with edges of length $b$, about an axis passing through an edge.

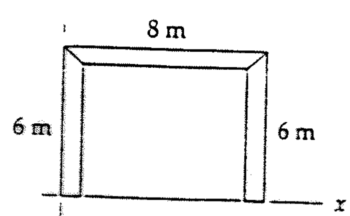
a) $2mb^2/3$

b) $mb^2/6$

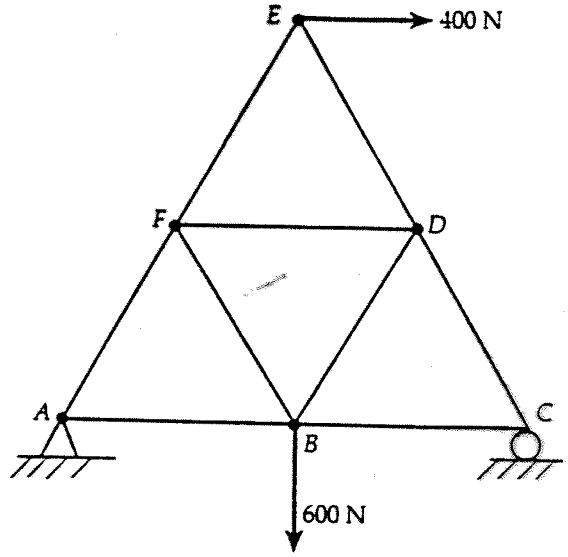
c) $3mb^2/2$

d) $mb^2/2$

*8.48 Find the mass moment of inertia about the $x$-axis if the mass of the rods per unit length is 1.0 kg/m.
a) 224
b) 268
c) 336
d) 432



**Questions 8.49 – 8.51** relate to the truss shown. All angles are equal.




8.49 The magnitude of the reaction at $A$ is

a) 403 N
b) 527
c) 672 N
d) 748 N

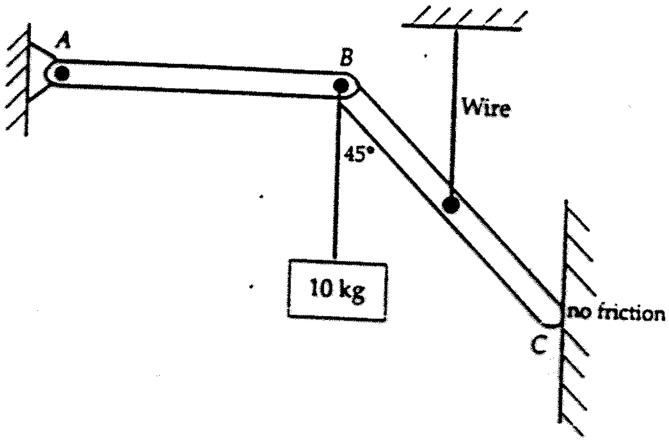
8.50 The force in link $BC$ is

a) 169 N
b) 373 N
c) 532 N
d) 746 N

8.51 The force in link $EF$ is

a) 100 N
b) 200 N
c) 300 N
d) 400 N

Questions 8.52 – 8.54 relate to the frame shown. Each link is 80 cm long and the wire is in the middle of the link.




8.52 The force in the wire is

a) 137 N  b) 98 N  c) 49 N  d) 10 N

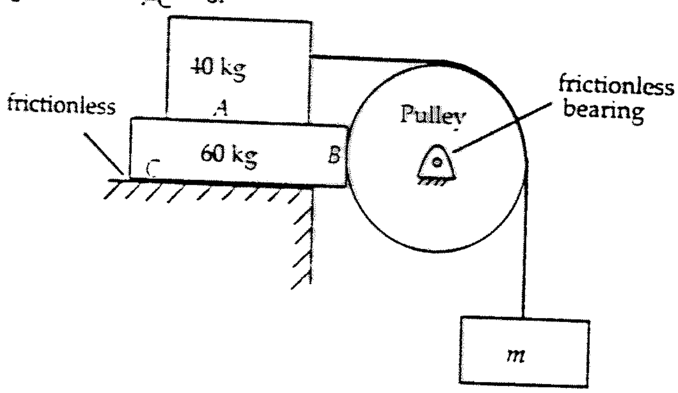
8.53 The magnitude of the force at $A$ is

a) 137 N  b) 98 N  c) 49 N  d) 10 N

8.54 If the wire were cut, the force at $C$ would

a) increase
b) stay the same
c) decrease
d) can't say, need more information

Questions 8.55 – 8.56 relate to the figure shown. Motion is impending and $\mu_A = 0.3$, $\mu_B = 0.5$ and $\mu_C = 0$.



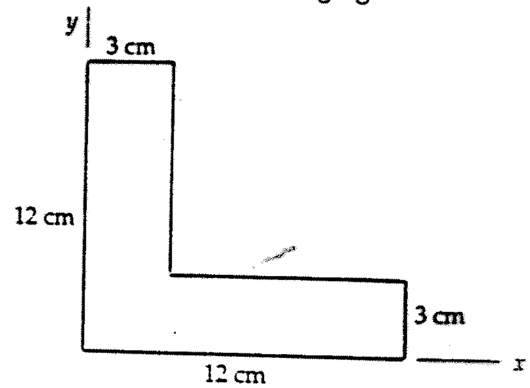

8.55 The tension in the rope where it attaches to the 40-kg mass is nearest

a) 90 N  b) 120 N  c) 150 N  d) 180 N

8.56 The mass $m$ for impending motion is nearest

a) 18 kg  b) 22 kg  c) 28 kg  d) 40 kg

Questions 8.57 – 8.59 relates to the following figure.




8.57 Find $I_x$.

a) 2372 $cm^4$  b) 2185 $cm^4$  c) 1807 $cm^4$  d 1567 $cm^4$

8.58 Find the $y$-coordinate of the centroid.

a) 3.21 cm  b) 4.07 cm  c) 5.12 cm  d) 6.32 cm

8.59 Find the second moment of the area about the centroidal axis, i.e., $I_{x_c}$.

a) 475 $cm^4$  b) 525 $cm^4$  c) 685 $cm^4$  d) 765 $cm^4$

# Solutions to Practice Problems

8.1 a) $$\mathbf{i}_B = \frac{\mathbf{i}-2\mathbf{j}-2\mathbf{k}}{\sqrt{1+4+4}} = \frac{1}{3}(\mathbf{i}-2\mathbf{j}-2\mathbf{k})$$

$$\mathbf{A}\cdot\mathbf{i}_B = (15\mathbf{i}-9\mathbf{j}+15\mathbf{k})\cdot\frac{1}{3}(\mathbf{i}-2\mathbf{j}-2\mathbf{k}) = 5+6-10 = 1$$

8.2 c) $$\mathbf{A}+\mathbf{B}+\mathbf{C} = (2\mathbf{i}+5\mathbf{j})+(6\mathbf{i}-7\mathbf{k})+(2\mathbf{i}-6\mathbf{j}+10\mathbf{k}) = 10\mathbf{i}-\mathbf{j}+3\mathbf{k}$$

$$\text{magnitude} = \sqrt{10^2+1^2+3^2} = 10.49$$

8.3 d) $\mathbf{M} = \mathbf{r}\times\mathbf{F} = (4\mathbf{i}-6\mathbf{j}+4\mathbf{k})\times(200\mathbf{i}+400\mathbf{j})$. $M_y = 4\times 200 = 800$ since $\mathbf{k}\times\mathbf{i} = \mathbf{j}$

8.4 c) $\mathbf{M} = \mathbf{r}_1\times\mathbf{F}_1+\mathbf{r}_2\times\mathbf{F}_2 = (2\mathbf{i}-4\mathbf{k})\times(50\mathbf{i}-40\mathbf{k})+(-4\mathbf{i}+2\mathbf{j})\times(60\mathbf{j}+80\mathbf{k})$

$M_x = 2\times 80 = 160$ since $\mathbf{j}\times\mathbf{k} = \mathbf{i}$

8.5 a) Concurrent $\Rightarrow$ all pass through a point.

Coplanar $\Rightarrow$ all in the same plane.

The forces are three - dimensional.

8.6 b) $\sum\mathbf{F} = 0$. $\therefore \mathbf{R}+141\mathbf{i}-141\mathbf{j}-200\mathbf{i}-100\mathbf{k} = 0$

$\therefore \mathbf{R} = 59\mathbf{i}+141\mathbf{j}+100\mathbf{k}$

8.7 c) $\sum\mathbf{M} = 0$. $\therefore \mathbf{M}_A+(4\mathbf{j}-3\mathbf{k})\times(-100\mathbf{k})-3\mathbf{k}\times(-200\mathbf{i})+4\mathbf{i}\times(141\mathbf{i}-141\mathbf{j}) = 0$

$\therefore \mathbf{M}_A = 400\mathbf{i}-600\mathbf{j}+564\mathbf{k}$

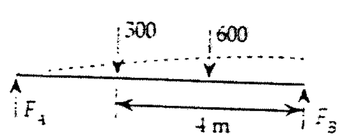
8.8 b) They must be concurrent, otherwise a resultant moment would occur.

8.9 a) It is a two-force body.

8.10 b) $\sum M_A = 0$. $F_B\times 8 = 400\times 4+400\times 6$. $\therefore F_B = 500\text{ N}$

8.11 a) $M_A = 400\times 8+400\times 6 = 5600\text{ N}\cdot\text{m}$

8.12 b) $\sum M_B = 0$. $6F_A = 4\times 300-600\times 3/2$. $\therefore F_A = 350\text{ N}$




8.13 c) $M_A = 0.6\times 100-141\times 0.6+141\times 0.8 = 88.2$

8.14 a) $M = 100\sin 45^\circ\times 4 = 282.8$ cw

8.15 c) $\sum M_A = 0. \quad \therefore 6\times 70.7 = 2\times 0.866F_1. \quad \therefore F_1 = 245$

$\sum F_x = 0. \quad \therefore -70.7 - 245\times 0.5 + F_{Ax} = 0. \quad \therefore F_{Ax} = 193$

$\sum F_y = 0. \quad \therefore -70.7 - 245\times 0.866 + F_{Ay} = 0. \quad \therefore F_{Ay} = 283$

$\therefore F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{193^2 + 283^2} = 343$

8.16 d) $\sum M_A = 0. \quad \therefore 2F_B + 1.2\times 200 - 141.4\times 2 - 141.4\times 1.2 + 50 = 0. \quad \therefore F_B = 81.2$

$\sum F_x = 0. \quad \therefore F_{Ax} - 200 + 141.4 = 0. \quad \therefore F_{Ax} = 58.6$

$\sum F_y = 0. \quad \therefore F_{Ay} + 81.2 - 141.4 = 0. \quad \therefore F_{Ay} = 60.2$

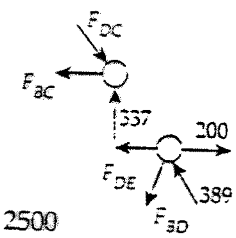
$\therefore F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{58.6^2 + 60.2^2} = 84.0$

8.17 a) $\sum M_A = 0. \quad \therefore 500\ell + 200\times 0.866\ell - F_C\times 2\ell = 0. \quad \therefore F_C = 337$

$0.866F_{DC} = 337. \quad \therefore F_{DC} = 389$

$0.866\times 389 = 0.866F_{BD}. \quad \therefore F_{BD} = 389$

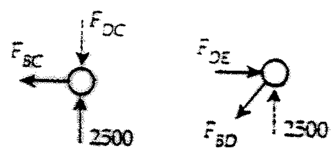
$-F_{DE} + 200 - 389\times 0.5 - 389\times 0.5 = 0. \quad \therefore F_{DE} = -189$




8.18 d) $\sum M_A = 0. \quad \therefore 5\times 5000 = 10\times F_C. \quad \therefore F_C = 2500 \quad \therefore F_{DC} = 2500$

$0.707F_{BD} = 2500. \quad \therefore F_{BD} = 3536$

$0.707\times 3536 = F_{DE}. \quad \therefore F_{DE} = 2500$

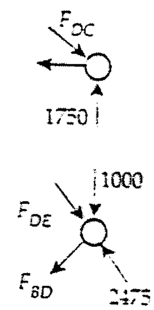



8.19 b) $\sum M_A = 0. \quad \therefore 4\times 2000 + 6\times 1000 = 8F_C. \quad \therefore F_C = 1750$

$0.707F_{DC} = 1750. \quad \therefore F_{DC} = 2475$

Sum forces in dir. of $F_{DE}$: $F_{DE} - 2475 + 1000\times 0.707 = 0.$

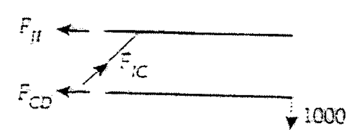
$\therefore F_{DE} = 1768$




8.20 a) Sum forces in dir. of $F_{FB}$ at $F$. $F_{FB} = 0.$

8.21 c) $\sum M_B = 0. \quad \therefore 12F_F = 3\times 4000. \quad \therefore F_F = 1000 \downarrow$

$\sum F_y = 0. \quad \therefore 0.8F_{IC} = 1000. \quad \therefore F_{IC} = 1250$

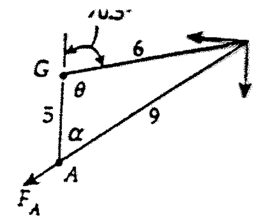



8.22 d) Cut vertically through link $KA$. Then $F_{KA} = 5000.$
Obviously, $F_{AL} = 0. \quad \therefore F_{AB} = 3000. \quad \therefore F_{BC} = 3000$

8.23 d) At $E$ we see that $F_{EC} = 0. \quad \therefore$ At C, $F_{FC} = 0$

8.24 c) $9^2 = 6^2 + 5^2 - 2\times5\times6\cos\theta.\quad \therefore \theta = 109.5°$

$6^2 = 9^2 + 5^2 - 2\times9\times5\cos\alpha.\quad \therefore \alpha = 38.9°$




From pts $E, C, F, B$ we see that $F_{EC} = F_{FC} = F_{FB} = F_{GB} = 0.$

Also, $F_A = F_{BC}.\quad \sum M_G = 0.$

$\therefore 5\times F_A \sin 38.9° + 5000\times 6\sin 70.5° = 5000\times 6\cos 70.5°.$

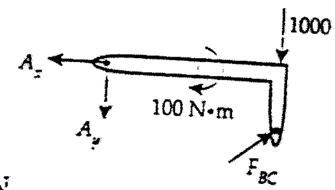
$\therefore F_A = -5817$ N

8.25 b) Recognize that link BC is a two-force member. $\sum M_A = 0.$

$\therefore 0.2\times1000 + 100 = 0.08\times F_{BC}\times0.8 + 0.2\times F_{BC}\times0.6$

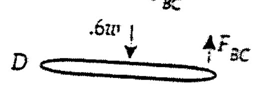
$\therefore F_{BC} = 1630.\quad A_x = 1630\times0.8 = 1304.\quad A_y = 1630\times0.6 - 1000 = -22$

$\therefore F_A = \sqrt{1304^2 + 22^2} = 1304$ N




8.26 d) $1800\times0.8 = 0.6F_{BC}.\quad \therefore F_{BC} = 2400$ N

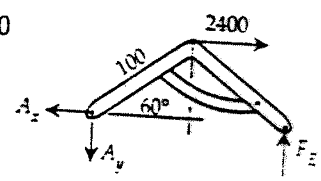
$0.3\times0.6w = 0.6\times2400.\quad \therefore w = 8000$ N/m




8.27 d) $\sum M_A = 0.\quad \therefore 1.2F_E = 0.8\times2400.\quad \therefore F_E = 1600$

$\therefore A_x = 2400.\quad A_y = 1600$

$\therefore F_A = \sqrt{2400^2 + 1600^2} = 2884$ N



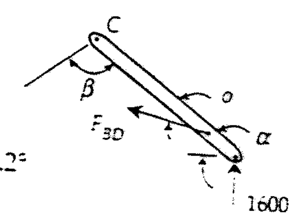

8.28 a) Link BD is a two-force member. $\therefore$ the force acts from D to B. Hence, the angles are found.

$120^2 = 100^2 + 100^2 - 2\times100\times100\cos\beta.\quad \therefore \beta = 73.7°$

$\overline{BD}^2 = 60^2 + 40^2 - 2\times60\times40\cos73.7°.\quad \therefore BD = 62.1$

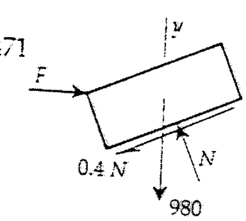
$\frac{62.1}{\sin 73.7°} = \frac{40}{\sin\phi}.\quad \therefore \phi = 38.2°.\quad \alpha = (180 - 73.7)/2 = 53.2°$

$\sum M_C = 0.\quad 1600\times100\cos53.2° = 60\times F_{BD}\sin38.2°.\quad \therefore F_{BD} = 2587$

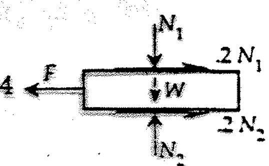



8.29 d) $\sum F_y = 0.\quad N\times0.866 - 980 - 0.4N\times0.5 = 0.\quad \therefore N = 1471$

$\sum F_x = 0.\quad F = 1471\times0.5 + 0.4\times1471\times0.866 = 1245$ N



8.30 b) $N_1 = 490. \quad N_2 = 980. \quad \therefore F = 0.2(490 + 980) = 294$

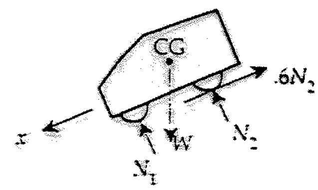



8.31 c) $\sum M_{\text{front wheel}} = 0. \quad \therefore 400N_2 - W\cos\theta \times 200 + W\sin\theta \times 50 = 0$

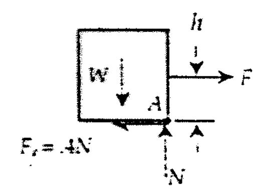
$\sum F_x = 0. \quad \therefore 0.6N_2 = W\sin\theta.$

$\therefore 400(W\sin\theta)/0.6 + 50W\sin\theta = 200W\cos\theta$

$\therefore \dfrac{\sin\theta}{\cos\theta} = \dfrac{200}{716.7} = \tan\theta. \quad \therefore \theta = 15.6°$




8.32 b) If $h < h_{\min}$ then sliding occurs, and $F_f = 0.4N$.
If $h > h_{\min}$ tipping occurs and $F_f < 0.4N$.
When $h = h_{\min}$, $F_f = 0.4N = 0.4W = F$.
$\sum M_A = 0. \quad \therefore 4W = hF = h \times 0.4W. \quad \therefore h = 10 \text{ cm}$

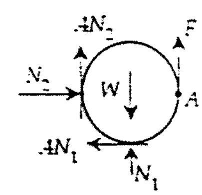



8.33 d) $\sum F_x = 0. \quad \therefore N_2 = 0.4N_1. \quad$ Also, $W = 980$

$\sum M_A = 0. \quad \therefore W \cdot r = (N_1 + 0.4N_1 + 2 \times 0.4N_2)r.$

$\therefore N_1 = 0.5814W = 570$

$\sum F_y = 0. \quad \therefore F = 980 - 570 - 0.16 \times 570 = 319$

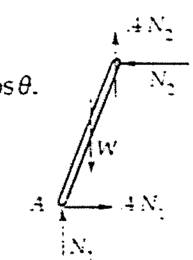



8.34 b) $\sum F_x = 0. \quad \therefore N_2 = 0.4N_1. \quad \sum F_y = 0. \quad \therefore N_1 + 0.4N_2 = W.$

$\therefore N_2 = 0.345W$

$\sum M_A = 0. \quad \therefore \frac{L}{2} \times W\cos\theta = N_2 \times L\sin\theta + 0.4N_2 \times L\cos\theta.$

This gives $\tan\theta = 1.049. \quad \therefore \theta = 46.4°$

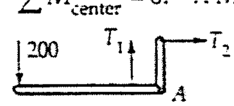



8.35 a) $F_B = F_D e^{-\mu\theta} = 800e^{-0.5\pi} = 166 \text{ N}$

8.36 a) $\sum M_A = 0. \quad \therefore 200 \times 0.6 = 0.1 \times T_1 + 0.1 \times T_2.$

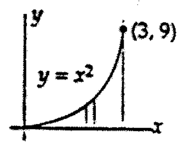
$T_1 = T_2 e^{0.4 \times 3\pi/2} = 6.59T_2. \quad$ Thus, $T_2 = 158$ and $T_1 = 1042.$

$\sum M_{\text{center}} = 0. \quad \therefore M = 0.1 \times (1042 - 158) = 88.4 \text{ N} \cdot \text{m}$



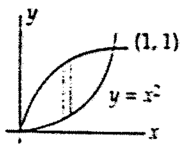

8.37 d) Let $h$ = long end. $m$ = mass/unit length. Then,
$(12 - 1.88 - h)mge^{0.5\pi} = hmg. \quad \therefore h = 8.38 \text{ m}$

8.38 d) $\bar{x} = \dfrac{\int_0^3 xy\,dx}{\int_0^3 y\,dx} = \dfrac{\int_0^3 x^3 dx}{\int_0^3 x^2 dx} = \dfrac{3^4/4}{3^3/3} = 2.25$

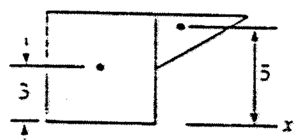



8.39 a) $\bar{y} = \dfrac{\int_0^3 \frac{y}{2} y\,dx}{\int_0^3 y\,dx} = \dfrac{\frac{1}{2}\int_0^3 x^4 dx}{\int_0^3 x^2 dx} = \dfrac{3^5/10}{3^3/3} = 2.7$

8.40 c) $\bar{x} = \dfrac{\int_0^1 \left(\sqrt{x} - x^2\right) x\,dx}{\int_0^1 \left(\sqrt{x} - x^2\right) dx} = \dfrac{\frac{1}{5/2} - \frac{1}{4}}{\frac{1}{3/2} - \frac{1}{3}} = 0.45$




8.41 b) $\bar{y} = \dfrac{24\times 3 + 6\times 5}{6\times 4 + 4\times 3/2} = 3.4$




8.42 d) $\bar{y} = \dfrac{48\times 3 + 12\times 7 - \pi\times 6}{8\times 6 + 3\times 4 - \pi} = 3.68$

8.43 b) $\bar{x} = \dfrac{10\times \frac{1}{2} + 5\times 3.5 + 3\times 7}{10 + 5 + 3} = 2.42$

8.44 b) $I_x = \int_0^3 y^3\,dx/3 = \int_0^3 x^6\,dx/3 = 3^7/21 = 104.1.$

With a horizontal strip: $I_x = \int_0^9 y^2(3-x)dy = \int_0^9 y^2\left(3-\sqrt{y}\right)dy = 9^3 - \dfrac{9^{7/2}}{7/2} = 104.1$

8.45 d) $I_x = 8\times 6^3/3 + \left(8\times 3^3/36 - 12\times 7^2\right) - \left(\pi\times 1^4/4 + \pi\times 6^2\right) = 1056$

8.46 a) $I_y = 12\times 12^3/3 - \left(8\times 8^3/12 - 64\times 6^2\right) = 4267$. Or, alternatively:

$I_y = 8\times 2^3/3 + 4\times 12^3/3 - 8\times 2^3/12 + 16\times 11^2 = 4267$

8.47 a) $I_{edge} = I_{c.g.} + Md^2 = \dfrac{1}{12}M\left(b^2 + b^2\right) + M\dfrac{b^2}{2} = \dfrac{2}{3}Mb^2$

8.48 d) $I_x = \dfrac{1}{3}(6m)\times 6^2\times 2 + 8m\times 6^2 = 432$ with $m = 1$

8.49 a) All lengths must also be equal. Sum moments about $C$:

$\Sigma M_C = 0.\quad 400\times 0.866\times 2L - 600L = 2L\times F_{Ay}.\quad \therefore F_{Ay} = 46.4\text{ N}\downarrow$

$\Sigma F_x = 0.\quad \therefore F_{Ax} = 400\text{ N}.\quad \therefore F_A = \sqrt{46.4^2 + 400^2} = 403\text{ N}$

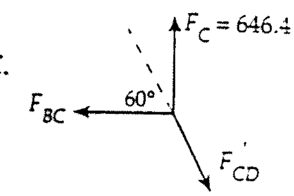
8.50 b) Find the force acting at $C$:

$\Sigma M_A = 0.\quad 2L \times F_C = 2L \times 0.866 \times 400 + 600L.$

$\therefore F_C = 646.4$ N. Sketch the force situation at $C$.

$0.866 \times F_{CD} = 646.4. \quad \therefore F_{CD} = 746.4.$

$F_{BC} = F_{CD} \sin 60° = 746.4 \times 0.5 = 373.2$ N

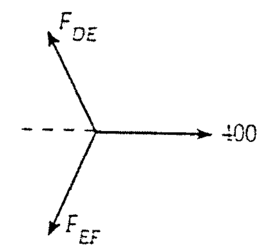



8.51 d) Consider the force situation at $E$:

$\Sigma F_x = 0.$

$\therefore F_{EF} \cos 60° = \frac{1}{2} \times 400.$

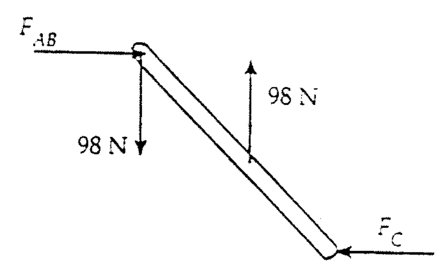
$\therefore F_{EF} = 400$ N




8.52 b) Link $AB$ is a two-force member so that the force in link $AB$ is in the $x$-direction only. A free-body diagram of link $BC$ *provides the solution:*

Since there are only two vertical forces acting on the link, they must be equal :

$\therefore F_{wire} = 10 \times 9.8 = 98$ N




8.53 c) The force in the two-force member is found from moments on the free-body of Problem 8.52:

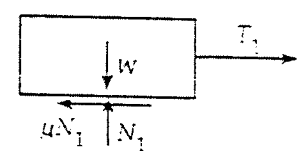
$$98 \times 40 \sin 45° = F_{AB} \times 80 \sin 45°. \quad \therefore F_{AB} = 49 \text{ N}$$

8.54 c) The force would reduce to zero since the two links would be free to simply fall and assume a vertical position.

8.55 b) Draw a free-body diagram of the upper block:

$N_A = W = 40 \times 9.8 = 392$ N

$\mu_A N_A = 0.3 \times 392 = 117.6 \text{ N} = T_1$

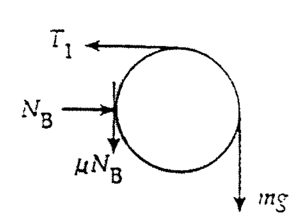



8.56 a) From a free-body of the lower block we see that $N_B = 117.6$ N. Draw a free-body diagram of the pulley:

$\therefore N_B = 117.6. \quad \mu_B N_B = 0.5 \times 117.6 = 58.8$

$\Sigma M_{pulley} = 0. \quad \therefore m \times 9.8 = T_1 + \mu_B N_B$

$\therefore m = (117.6 + 58.8) / 9.8 = 18$ kg




53

8.57 c) Use two rectangles, one 3 cm by 12 cm and the other 3 cm by 9 cm:

$$I_x = \left(\frac{bh^3}{3}\right)_1 + \left(\frac{bh^3}{3}\right)_2 = \frac{3\times 12^3}{3} + \frac{9\times 3^3}{3} = 1809 \text{ cm}^4$$

8.58 b) Use the two rectangles of Problem 8.57:

$$y_c = \frac{y_{c_1}A_1 + y_{c_2}A_2}{A_1 + A_2} = \frac{6\times 36 + 1.5\times 27}{36 + 27} = 4.07 \text{ cm}$$

8.59 d) Use the parallel-axis-transfer theorem:

$$I_{x_c} = I_x - a y_c^2 = 1809 - 63\times 4.07^2 = 765 \text{ cm}^4$$