Slide 1

Transmission Lines

Most communication systems contain components that utilize transmission line concepts. Whenever any circuit dimensions are reasonable fractions of a wavelength, transmission line effects must be considered. The concepts will be introduced in this module and the emphasis will be directed toward lines that are assumed to be lossless or nearly lossless. Various terms associated with transmission lines and their operating conditions will be explored.

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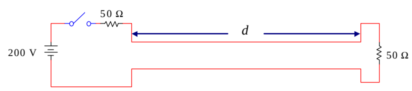
Limitation of Lumped Circuit Theory

Basic circuit theory indicates that the current in the circuit below should be 2 A after the switch is closed and it will be when steady-state conditions are reached. However, a transient interval exists in which one or more waves will propagate between source and load before the final conditions are established.



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Propagation Time and Review of Wavelength and Frequency

The one-way delay time t_1 is

$$t_1 = \frac{\text{length of line}}{\text{velocity of propagation}} = \frac{d}{n}$$

The wavelength l is

$$l = \frac{\text{velocity of propagation}}{\text{frequency}} = \frac{n}{f}$$

In free-space $n = c = 3\times10^8$ m/s

$$l = \frac{c}{f} = \frac{3\times10^8}{f}$$

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When are these effects important?

1. Pulse waveforms are used and propagation times are of the same order of magnitude as pulse widths.
2. Modulated waveforms are used and connection lengths are of the same order of magnitude as the wavelengths.

For 2, we will use the estimate of 0.1λ as the point where transmission line effects must be considered.

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__Example 1__. A frequency of 1 MHz is near the middle of the AM band. Estimate the length at which transmission line effects must be considered.

$$l = \frac{3\times10^{8}}{f} = \frac{3\times10^{8}}{1\times10^{6}} = 300 \text{ m}$$

The length l for $0.1\,l$ is

$$l = 0.1\,l = 0.1\times300 = 30 \text{ m}$$

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__Example 2__. A frequency of 100 MHz is near the middle of the FM band. Estimate the length at which transmission line effects must be considered.

$$l = \frac{3\times10^{8}}{f} = \frac{3\times10^{8}}{1\times10^{8}} = 3 \text{ m}$$

The length l for $0.1\,l$ is

$$l = 0.1\,l = 0.1\times3 = 0.3 \text{ m} = 30 \text{ cm}$$

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Example 3. **A frequency of 1 GHz is near where the *microwave region* begins. Estimate the length at which transmission line effects must be considered.**

$$l = \frac{3\times10^8}{f} = \frac{3\times10^8}{1\times10^9} = 0.3 \text{ m} = 30 \text{ cm}$$

The length l for $0.1\,l$ is

$$l = 0.1\,l = 0.1\times30 = 3 \text{ cm}$$

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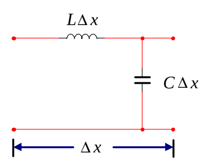
Incremental Model of Lossless Transmission Line

Let $L =$ inductance per unit length (H/m)

$C =$ capacitance per unit length (F/m)



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Characteristic Impedance Z_0

$$Z_0 = \frac{\text{voltage of a single wave}}{\text{current of a single wave}}$$

When line is lossless, the value is real.

For that case, we will set $Z_0 = R_0$

For a lossless line,

$$R_0 = \sqrt{\frac{L}{C}}$$

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Example 4. A lossless line has L = 320 nH/m and C = 90 pF/m. Determine R_0.

$$R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{320\times10^{-9}}{90\times10^{-12}}}$$

$$= 59.63\ \Omega$$

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Permittivity

Permittivity is a property of a dielectric material.

e = permittivity of material in farads/meter (F/m)

e_0 = permittivity of free space = $(1/36p)\times10^{-9}$ F/m

e_r = dielectric constant or relative permittivity

$$e = e_r e_0$$

$$\text{or } e_r = \frac{e}{e_0}$$

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Permeability

Permittivity is a magnetic property of a material.

m = permeability of material in henries/m (H/m)

m_0 = permeability of free space = $4p\times10^{-7}$ H/m

m_r = relative permeability

$$m = m_r m_0$$

Except for ferromagnetic materials, $m_r = 1$.

Unless otherwise stated,

we will assume $m = m_0$

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Velocity of Propagation

For a transmission line,

$$n = \frac{1}{\sqrt{LC}}$$

For a plane wave,

$$n = \frac{1}{\sqrt{me}} = \frac{c}{\sqrt{e_r}} = \frac{3\times 10^8}{\sqrt{e_r}}$$

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<u>Example 5</u>. For line of Example 4 with *L* = 320 nH/m and *C* = 90 pF/m, determine velocity of propagation.

$$n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{320\times 10^{-9} \times 90\times 10^{-12}}}$$

$$= 1.863\times 10^{8} \text{ m/s}$$

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<u>Example 6</u>. For line of Examples 4 and 5 with velocity of 1.863x10^8 m/s, determine the dielectric constant.

$$n = \frac{3\times 10^8}{\sqrt{e_r}}$$

$$1.863\times 10^8 = \frac{3\times 10^8}{\sqrt{e_r}}$$

$$e_r = 2.59$$

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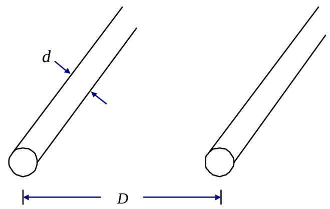
Parallel Lines



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Formulas for Parallel Lines

$$L = \frac{m_0}{p} \ln \frac{2D}{d} = 4 \times 10^{-7} \ln \frac{2D}{d} \quad \text{H/m}$$

$$C = \frac{pe}{\ln \frac{2D}{d}} = \frac{(1/36) \times 10^{-9} e_r}{\ln \frac{2D}{d}} = \frac{27.78 \times 10^{-12} e_r}{\ln \frac{2D}{d}} \quad \text{F/m}$$

$$R_0 = \frac{120}{\sqrt{e_r}} \ln \frac{2D}{d} \quad \Omega$$

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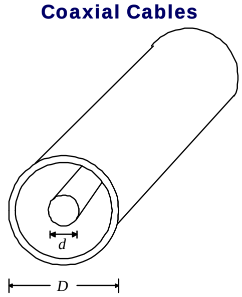
Coaxial Cables



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Formulas for Coaxial Cables

$$L = \frac{m_0}{2p}\ln\frac{D}{d} = 2\times10^{-7}\ln\frac{D}{d} \quad \text{H/m}$$

$$C = \frac{2pe}{\ln\frac{D}{d}} = \frac{(1/18)\times10^{-9}e_r}{\ln\frac{D}{d}} = \frac{55.56\times10^{-12}e_r}{\ln\frac{D}{d}} \quad \text{F/m}$$

$$R_0 = \frac{60}{\sqrt{e_r}}\ln\frac{D}{d} \quad \Omega$$

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Example 6. A coaxial cable has d = 0.29 cm, D = 1.05 cm and a dielectric constant of 2.25. Find (a) L, (b) C, (c) R_0 and (d) v.

The quantity $\ln\frac{D}{d}$ is required in 3 formulas.

$$\ln\frac{D}{d} = \ln\frac{1.05}{0.29} = 1.287$$

$$\text{(a) } L = 2\times10^{-7}\ln\frac{D}{d} = (2\times10^{-7})(1.287)$$

$$= 257.4 \text{ nH/m}$$

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Example 6. (Continuation)

$$\text{(b) } C = \frac{55.56\times10^{-12}e_r}{\ln\frac{D}{d}} = \frac{55.56\times10^{-12}\times2.25}{1.287}$$

$$= 97.13 \text{ pF/m}$$

$$\text{(c) } R_0 = \frac{60}{\sqrt{e_r}}\ln\frac{D}{d} = \frac{60}{\sqrt{2.25}}(1.287) = 51.48\ \Omega$$

$$\text{(d) } n = \frac{c}{\sqrt{e_r}} = \frac{3\times10^8}{\sqrt{2.25}} = \frac{3\times10^8}{1.5} = 2\times10^8 \text{ m/s}$$

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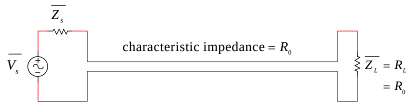
Matched Load Impedance



At any point on line, $\overline{Z}_{\text{in}} = R_L = R_0$

$$I_{rms} = \frac{V_{rms}}{\left|\overline{Z}_s + R_o\right|} = \frac{V_{rms}}{\left|\overline{Z}_s + R_L\right|}$$

$$P_{in} = P_L = I_{rms}^2 R_0 = I_{rms}^2 R_L$$

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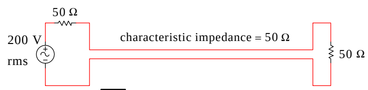
Example 7. Determine Z_{in}, I_{rms}, P_{in} and P_L



$$\overline{Z}_{in} = R_{in} = 50\ \Omega$$

$$I_{rms} = \frac{400\ \text{V}}{50\ \Omega + 50\ \Omega} = 4\ \text{A}$$

$$P_{in} = I_{rms}^2 R = (4)^2 \times 50 = 800\ \text{W}$$

$$P_L = P_{in} = 800\ \text{W}$$

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<u>Example 8</u>. Assume system of Example 7 except that line has loss of 0.01 dB/m. Find load power for line length of 50 m.

Strictly speaking, line now has complex $\overline{Z}_0$, but approach is reasonable for low losses.

$$L_{\text{dB}} = 50\ \text{m} \times 0.01\ \text{dB/m} = 0.5\ \text{dB}$$

$$L = 10^{0.5/10} = 1.122$$

$$P_L = \frac{P_{in}}{L} = \frac{800}{1.122} = 713.0\ \text{W}$$

Line dissipates 800-713 = 87 W

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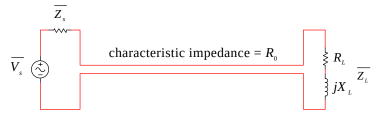
Mismatched Load Impedance



Reflection Coefficient at Load

$$\overline{\Gamma_L} = \frac{Z_L - R_0}{Z_L + R_0} \qquad 0 \le \Gamma_L \le 1$$

For matched load, $\overline{\Gamma_L} = 0$

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Voltage Standing Wave Ratio

$$\text{VSWR } = S = \frac{V_{max}}{V_{min}} \qquad 1 \le S \le 1$$

$$S = \frac{1+\Gamma_L}{1-\Gamma_L} \qquad \Gamma_L = \frac{S-1}{S+1}$$

The angle of $\overline{\Gamma_L}$ cannot be determined from S.

For resistive load, $S = \dfrac{R_L}{R_0}$ or $\dfrac{R_0}{R_L}$

depending on which value is $\ge$ 1.

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Power Relationships at Mismatch

Let P_{inc} = power incident to load

P_{ref} = power reflected by load

P_L = power absorbed by load

$$P_{inc} = P_{ref} + P_L \qquad P_{ref} = \Gamma_L^2 P_{inc}$$

$$P_L = \left(1 - \Gamma_L^2\right) P_L$$

decibel mismatch loss $= -10\log(1-\Gamma_L^2)$

decibel return loss $= -10\log \Gamma_L^2$

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Input Impedance

$$\overline{Z_{in}} = R_0 \frac{\left(1+\overline{\Gamma_L} e^{-j4pd/l}\right)}{\left(1-\overline{\Gamma_L} e^{-j4pd/l}\right)}$$

The input impedance of a lossless line with a mismatch can be determined by the equation above. This means that the power reaching the load is a function of the line length. The pattern repeats at intervals of λ/2.

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<u>Example 9</u>. A 50-Ω lossless line is terminated with an impedance of 50+j100 Ω. Determine the reflection coefficient and the standing-wave ratio.

$$\overline{\Gamma_L} = \frac{Z_L - R_0}{Z_L + R_0} = \frac{50 + j100 - 50}{50 + j100 + 50}$$

$$= \frac{j100}{100 + j100} = \frac{100\angle 90^o}{141.4\angle 45^o} = 0.7071\angle 45^o$$

$$S = \frac{1+\Gamma}{1-\Gamma} = \frac{1+0.7071}{1-0.7071} = 5.828$$

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<u>Example 10</u>. A 300-Ω lossless line is terminated with a resistance of 300 Ω. Determine the reflection coefficient and the standing-wave ratio.

$$\overline{\Gamma_L} = \frac{R_L - R_0}{R_L + R_0} = \frac{100-300}{100+300} = -0.5$$

$$S = \frac{R_0}{R_L} = \frac{300}{100} = 3$$

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<u>Example 11</u>. For the system of the preceding example, determine the decibel mismatch loss.

decibel mismatch loss

$$= -10\log\left(1-\Gamma_L^2\right)$$

$$= -10\log\left[1-(0.5)^2\right]$$

$$= -10\log 0.75$$

$$= -10(-0.125) = 1.25 \text{ dB}$$

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Summary

- Transmission line effects must be considered when a connection is an appreciable fraction of a wavelength.
- Important properties of a transmission line are the *characteristic impedance* and the *velocity of propagation*.
- Important properties of a line with load are the *reflection coefficient* at the load and the *voltage standing wave ratio*.
- The ideal condition at the load is for the load impedance to be real and equal to the characteristic impedance of the line.