Chapter 14
TRANSMISSION LINES AND WAVES

14-1 Introduction and Objectives

Most communication systems contain components that utilize the theory of transmission lines and wave concepts. Transmission line phenomena are important whenever the dimensions of circuits are reasonable fractions of a wavelength. In particular, the line between the output of a transmitter and an antenna and the line between the receiving antenna and the receiver possess attributes that require transmission line concepts for proper design and analysis. Telephone lines and network connections are also dependent on these properties.

A large number of communication systems employ electromagnetic radiation between transmitting and receiving antennas. The phenomena involve electric and magnetic field concepts along with radiated power. The concept of a plane wave propagating in space is an essential part of the process.

Objectives

After completing this chapter, the reader should be able to

1. Discuss the concept of a *distributed parameter circuit* and explain how it differs from a *lumped parameter circuit.*.

2. Determine the *delay time* on a transmission line.

3. Estimate the frequency and/or wavelength conditions at which propagation effects must be considered.

4. Sketch the form of the incremental model of a transmission line and explain the effects of inductance and capacitance in establishing its behavior.

5. Define *characteristic impedance* for a transmission line.

6. Determine the characteristic impedance for a lossless transmission line in terms of inductance and capacitance per unit length.

7. Discuss *permittivity* and *permeability* as they relate to transmission lines.

8. Determine the velocity of propagation for a lossless transmission line.

9. Determine the various physical and electrical properties of a parallel-line transmission line.

10. Determine the various physical and electrical properties of a coaxial cable.

11. Discuss the requirements for optimum load impedance matching.

12. For a mismatched line, determine the load reflection coefficient and the standing-wave ratio.

13. Define and discuss the *electric field* and the *magnetic field* associated with a *plane wave.*

14. Define and compute *intrinsic impedance* for a lossless medium.

15. Define and compute the *power density* associated with a plane wave.

14-2 Propagation Time Effects

Consider the simple circuit of Figure 14-1 in which a dc source of value 200 V is to be connected through a 50-Ù resistance to a cable of length *d* and a 50-Ù resistance is connected across the output. The resistance of the cable is assumed to be negligible in comparison to the resistances, and any connections at the ends are assumed to be negligible in comparison with *d*.

When the switch is closed, the circuit appears to be a very simple series circuit containing two 50-Ù resistances in series, and by basic circuit theory, there should be a loop current whose value should be $I = 200 \text{ V}/(50\ \Omega + 50\Omega) = 2 \text{ A}$. However, that idealized case is based on the assumption that propagation time effects are negligible.

The fact is that the 2-A current is not immediately established throughout the circuit. When the switch is closed, a *wave* consisting of both voltage and current will propagate from the left-hand end of the line toward the load. The value of the current will depend on a property of the line called the *characteristic impedance*, whose properties will be introduced shortly. Upon reaching the load, if the value of the resistance is not equal to the characteristic impedance of the line, a second wave of voltage and current will be reflected from the load and will travel back toward the left. When it reaches the source, another wave would be generated and it would travel back to the right. This process is called the *transient response* of the line and the process will continue until steady-state conditions are reached.

In case the reader's faith in basic circuit theory has been disturbed, it should be stated that the current in the loop will eventually reach the value of 2 A predicted by Ohm's law. The time required for the circuit to settle to the steady-state value of 2 A is based on the propagation time of the line and certain conditions concerning the impedance match between the line and the source and load resistances. In the large body of lumped circuit theory familiar to the reader from dc and ac circuit courses, it is assumed that all propagation effects are negligible and can be ignored. However, when the concepts of a transmission line are introduced, it is necessary to consider *distributed circuit* effects.

The quantity t_1 will be defined as the *one-way propagation time*, which is the time required for a wave to propagate from one end of a line to the other end. In terms of the length *d* and the velocity of propagation *v*, this time is

$$t_1 = \frac{d}{v} \tag{14-1}$$

The velocity of propagation may or may not be equal to the speed of light c, depending on the physical properties of the line.

When are propagation time effects important, and when must they be considered in analyzing a circuit? It should be clear from the reader's experience that many situations do not require this degree of microscopic detail. In general, propagation and/or wave effects must be considered in either of the following situations:

1. Pulse waveforms are employed and the propagation times are of the same order of magnitudes as the widths of the pulses.

2. Modulated waveforms are employed and the lengths of connections are of the same order of magnitude as the wavelengths of the spectral components of the signal.

The first effect can be referred to as a *time-domain effect* and the second as a *frequency-domain effect*. Both effects are results of the same phenomenon, but they are different ways of viewing the problem.

Because communication systems generally focus heavily on the frequency domain representation of the signals, much of our emphasis here will be directed on the second effect.

Wavelength Review

The relationship between wavelength and frequency was provided in Chapter 1 and will be reviewed here for convenience. For an arbitrary velocity of propagation v, the wavelength l is related to the frequency f by the relationship

$$l = \frac{v}{f} \tag{14-2}$$

The velocity of free space is denoted as c and its value is $c = 3 \times 10^8 \text{ m/s}$, and whenever that value can be assumed, the relationship becomes

$$l = \frac{c}{f} = \frac{3 \times 10^8}{f} \tag{14-3}$$

It should be stressed that for transmission lines having a dielectric other than air, the velocity of propagation may be much lower than c as will be noted later.

Frequency Domain Effects

In general, propagation effects are significant when the length between two connection points is a reasonable fraction of a wavelength, but what do we mean by a "reasonable fraction"? Actually, it is somewhat arbitrary, and in microwave systems, the value of $l/16$ is sometimes used as a basis. We will choose here to use a more easily remembered value of $0.1l$ as the transition point. In other words, propagation effects must be considered if a line connecting two points is equal to or greater than one-tenth of a wavelength at any frequencies involved. Remember, however, that this is simply a rule of thumb and not an exact formula.

Example 14-1

A frequency of 1 MHz is near the center of the commercial AM broadcast band. Using the rule of thumb of $0.1l$ and the free-space velocity, determine the length of line at which transmission line effects must be considered.

Solution

The free-space wavelength is

$$l = \frac{3\times10^8}{f} = \frac{3\times10^8}{1\times10^6} = 300 \text{ m} \qquad (14\text{-}4)$$

Let l represent the length based on the rule of thumb indicated.

$$l = 0.1l = 0.1\times300 \text{ m} = 30 \text{ m} \qquad (14\text{-}5)$$

This is long enough that there is reasonable leeway in dealing with internal circuits at this frequency. However, the distance between a transmitter and the antenna in a commercial AM station is sufficiently great that it must still be treated as a transmission line.

Example 14-2

A frequency of 100 MHz is near the center of the commercial FM broadcast band. Repeat the analysis of Example 14-1 at this frequency.

Solution

The free-space wavelength is

$$l = \frac{3\times10^8}{f} = \frac{3\times10^8}{1\times10^8} = 3 \text{ m} \qquad (14\text{-}6)$$

The value of l is

$$l = 0.1l = 0.1\times3 \text{ m} = 0.3 \text{ m} \qquad (14\text{-}7)$$

This value is about 1 foot and it clearly illustrates the fact that circuit connection lengths are much more critical in the frequency range of FM.

Example 14-3

The frequency of 1 GHz is the vicinity of where the *microwave region* begins. Repeat the analysis of Examples 14-1 and 14-2 at this frequency.

Solution

The free-space wavelength is

$$l = \frac{3\times10^8}{f} = \frac{3\times10^8}{1\times10^9} = 0.3 \text{ m} = 30 \text{ cm} \qquad (14\text{-}8)$$

The value of l is

$$l = 0.1 l = 0.1\times30 \text{ cm} = 3 \text{ cm} \qquad (14\text{-}9)$$

This very short length illustrates the difficulty in constructing lumped circuits to operate in the microwave frequency region. Much of the technology above about 1 GHz or so utilizes wave concepts as opposed to the low-frequency concepts of voltage and current. Even amplifier circuits utilize short sections of microstrip transmission lines to provide optimum coupling between different stages.

14-3 Transmission Line Electrical Properties

While all transmission lines have losses, the most common initial assumption made in dealing with RF transmission lines of moderate length is to assume that the line is *lossless*. This means that the effects of series resistance in the wires and shunt conductance in the dielectric will be assumed to be negligible. Thus, until stated otherwise later, the *lossless* transmission line model will be assumed.

Distributed Parameter Effects

A transmission line is an example of a *distributed-parameter circuit*, in which the effects of various parameters such as capacitance and inductance are spread out as a function of length. This type of circuit is in sharp contrast to a *lumped-parameter circuit*, in which the parameters appear in concentrated form at specific points in the circuit, and that is the assumption made in basic circuit theory..

Because distributed-parameter effects are more difficult to analyze, early researchers focused on incremental models, in which lumped components are assumed for an infinitesimally short distance.

Throughout the body of transmission line theory, the symbols L and C are used to used to denote, respectively, the *inductance and capacitance per unit length*. Thus, common units for L are henries/meter (H/m), and the corresponding units for C are farads/meter (F/m). This is in sharp contrast to lumped circuit theory where L and C represent total inductance in henries and total capacitance in farads. Within this chapter, the distributed parameter units will be assumed primarily.

The basic form of the incremental model of a lossless line of length Δx is shown in Figure 14-2. The model shown is an unbalanced model, which is easier to describe, but it could be readily adapted to a balanced form by placing half of the inductance in each of the horizontal conductors. An incremental section of the line is assumed to possess a series inductance and a shunt capacitance. Since L and C are each expressed in parameter units/length, the net inductance of the section is $L\Delta x$ henries, and the net capacitance is $C\Delta x$ farads.

Inductance

Inductance is a property of any circuit that encloses magnetic flux. In general, magnetic flux is measured in webers (Wb) and is a direct result of current flow. The basic definition of inductance is

$$\text{inductance(H)} = \frac{\text{flux(Wb)}}{\text{current(A)}} \qquad (14\text{-}10)$$

Capacitance

Capacitance is a property of any circuit in which an electric field exists between two conductors as a result of the movement of charge. In general, charge is measured in coulombs (C) and is a direct result of the potential difference established between the conductors. The basic definition of capacitance is

$$\text{capacitance (F)} = \frac{\text{charge (C)}}{\text{voltage (V)}} \qquad (14\text{-}11)$$

Referring again to Figure 14-2, note that current flow along the two conductors results in the creation of magnetic flux all around the region, and this effect is represented as series inductance. Voltage across the two conductors results in the displacement of charge and the creation of an electric field, and this is represented as shunt capacitance.

Characteristic Impedance

One of the most important parameters used in describing a transmission line is the *characteristic impedance*. The symbol Z_0 is the most widely used symbol for characteristic impedance. The basic definition of characteristic impedance is

$$Z_0 = \frac{\text{voltage value of a single wave}}{\text{current value of a single wave}} \tag{14-12}$$

This means that for a uniform transmission line, the ratio of voltage to current of a single wave traveling in one direction is a constant.

In general, the characteristic impedance is complex and contains both a resistance and a reactance. For most of the work presented here, the characteristic impedance will be assumed as a real number. To emphasis this point, we will use the symbol R_0 to represent the characteristic impedance when it is real. Thus, we define

$$Z_0 = R_0 \quad \text{(when } Z_0 \text{ is real)} \tag{14-13}$$

This is in contrast to many books that use Z_0 for both real and complex values, but we believe that the choice of using R_0 for all cases where it is a real value will make the concept clearer to the reader.

Rather interestingly, *the characteristic impedance of a lossless transmission line is always a real number*. This may seem puzzling since series resistance and shunt conductance are both assumed to be zero in the lossless case. Remember, however, that characteristic impedance is not a property of energy dissipation. Rather, it is a voltage-to-current ratio, and it is a real number when no dissipation occurs.

Characteristic Impedance in Terms of Inductance and Capacitance

The value of the characteristic impedance of a line is a function of all the incremental parameters. For a *lossless line*, it can be shown that the value is simply

$$R_0 = \sqrt{\frac{L}{C}} \tag{14-14}$$

where L and C are inductance and capacitance per unit length as established earlier.

In general, the inductance increases as the spacing between the lines increases, and the capacitance decreases for the same condition. Thus, the characteristic impedance increases as the spacing between the wires increases and decreases as the spacing decreases.

Example 14-4

An assumed lossless line has $L = 320$ nH/m and $C = 90$ pF/m . Determine the characteristic impedance.

Solution

The result of (14-14) is employed, but care must be used to ensure that both parameters be expressed in their basic units. We have

$$R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{320\times10^{-9}}{90\times10^{-12}}} = 59.63\ \Omega \qquad (14\text{-}15)$$

14-4 Permittivity and Permeability

In order to deal with some of the physical properties of transmission lines, and later to deal with electric and magnetic fields, it is necessary to introduce some terms that relate to their properties. These terms will be introduced in this section.

Dielectric

A *dielectric* material is basically an insulating material, but the term is usually applied to those low-loss materials used to separate the conductors in a transmission line or a capacitor. Examples of dielectric materials are polyethylene, mylar, and, of course, free space.

Permittivity

The term *permittivity* refers to a basic property of a dielectric medium concerning its effect on capacitance and charge. The exact definition involves some terms that fall outside of the intended coverage here so we will deal with in an indirect way. For our purposes, we will say that capacitance between two surfaces is directly proportional to the permittivity of the dielectric medium and leave it at that.

The basic unit of permittivity is farads/meter (F/m). In general, three different quantities are used in describing permittivity:

$$\varepsilon = \text{permittivity of arbitrary material in farads/meter (F/m)}$$

$$\varepsilon_0 = \text{permittivity of free space}$$

$$= \frac{1}{36\pi}\times 10^{-9} \text{ F/m} = 8.842\times 10^{-12} \text{ F/m}$$

$$\varepsilon_r = \text{dielectric constant or relative permittivity}$$

The three quantities are related by

$$\varepsilon = \varepsilon_r \varepsilon_0 \qquad (14\text{-}16)$$

or

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \qquad (14\text{-}17)$$

Note that the dielectric constant ε_r has no units since it is the ratio of two actual permittivity values. Since $\varepsilon = \varepsilon_0$ in free space, the dielectric constant of free space is simply $\varepsilon_r = 1$. This value is also assumed as the dielectric constant for air in most applications.

Values of dielectric constant for most materials used in transmission lines range from 1 (for air lines) to about 10 or so.

Permeability

The term *permeability* refers to a magnetic property of any arbitrary material and is a measure of the flux density produced by a magnetizing current. As in the case of permittivity, we will sidestep the exact definition since some of the terminology is outside the scope of the text. Instead, it will be noted that an increased permeability for magnetic materials within an inductor will result in increased inductance.

The basic unit of permeability is henries/meter (H/m). In general, three different quantities are used in describing permeability:

$\mu =$ permeability of arbitrary material in henries/meter (H/m)

$\mu_0 =$ permeability of free space $= 4\pi \times 10^{-7}$ H/m

$\mu_r =$ relative permeability

The three quantities are related by

$$\mu = \mu_r \mu_0 \tag{14-18}$$

or

$$\mu_r = \frac{\mu}{\mu_0} \tag{14-19}$$

Note that the relative permeability has no dimensions since it is the ratio of two permeability values. Since $\mu = \mu_0$ in free space, the relative permeability of free space is $\mu_r = 1$.

Unlike permittivity, where many common materials have values that differ from that of free space, the vast majority of common materials have permeability values equal to that of free space. The primary exceptions are the ferromagnetic materials such as iron, nickel, and cobalt. Such materials are never used in the dielectric material of a transmission line. Therefore, *to simplify the notation, the assumption will be made that* $\mu = \mu_0$ *unless otherwise indicated.*

14-4 Velocity of Propagation

The velocity of propagation of a wave traveling on a transmission line is a function of the inductance and capacitance per unit length. It can be shown that the velocity of propagation for a *lossless* line is given by

$$v = \frac{1}{\sqrt{LC}} \tag{14-20}$$

where L is the inductance per unit length and C is the capacitance per unit length. The units for v depend on the length units for the inductance and capacitance, which must be the same. For example if the inductance and capacitance are given in henries/meter and farads/meter, respectively, the units for the velocity will be in meters/second (m/s).

Velocity in Terms of Permittivity and Permeability

It can also be shown that the velocity of propagation is related to the permittivity and permeability of the dielectric medium inside the line. For a lossless line, the velocity is given by

$$v = \frac{1}{\sqrt{\mu\epsilon}} \tag{14-21}$$

where the previous comments about the distance units in (14-20) apply here as well.

Free-Space Velocity

Let us establish some plausibility for (14-21) by assuming that $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m and $\epsilon = \epsilon_0 = (1/36\pi) \times 10^{-9}$ F/m and substituting these values in (14-21). The result is

$$v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times (1/36\pi) \times 10^{-9}}} = 3 \times 10^8 \text{ m/s} = c \tag{14-22}$$

Thus, the free-space velocity of propagation is obtained, as should be expected.

Velocity with Non-Unity Dielectric Constant

In general, $\mu = \mu_r \mu_0$ and $\epsilon = \epsilon_r \epsilon_0$. When these values are substituted in (14-21) and the result of (14-22) is utilized, the result may be expressed as

$$v = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{3 \times 10^8}{\sqrt{\mu_r \epsilon_r}} \quad \text{m/s} \tag{14-23}$$

As previously discussed, rarely will any value of μ other than μ_0 be encountered in transmission lines or other communications system work so for the remainder of the text, we will assume that $\mu = \mu_0$. Hence, the velocity of propagation may be expressed as

$$v = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{\epsilon_r}} \quad \text{m/s} \tag{14-24}$$

Example 14-5

Consider the lossless transmission line of Example 1-4, in which $L = 320$ nH/m and $C = 90$ pF/m. Determine the velocity of propagation.

Solution

From (14-20), we have

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{320\times10^{-9}\times90\times10^{-12}}} = 1.863\times10^{8} \text{ m/s} \tag{14-25}$$

Example 14-6

For the transmission line of Example 14-5, determine the dielectric constant. (Assume that $m = m_0$, which will always be the case, unless otherwise indicated.)

Solution

The velocity is related to the dielectric constant e_r by (14-24). Using the result of the previous example, we have

$$1.863\times10^{8} = \frac{3\times10^{8}}{\sqrt{e_r}} \tag{14-26}$$

This leads to

$$e_r = 2.59 \tag{14-27}$$

14-5 Properties of CommonTransmission Lines

The two most common types of transmission lines are (1) parallel lines and (2) coaxial cable. Their physical and electrical properties will be described in this section. Formulas concerning the inductance per unit length and capacitance per unit length will be presented. These formulas are derived in texts primarily devoted to electric and magnetic field theory. Once these parameters are known, the characteristic impedance may be readily determined.

Parallel Lines

Parallel lines represent the simplest type of geometry in that the two conductors are of equal size and are spaced apart by a constant separation as shown in Figure 14-3. The medium between the conductors may be air ("open-wire" lines), or it may be a material such as polyethylene ('twin-lead"). Parallel lines have been used extensively in the telephone industry and for 60-Hz power transmission.

A parallel line is referred to as a *balanced line*. This description means that when the parallel line is operating properly, all electric and magnetic fields are symmetrical with respect to ground and the impedance of each wire to ground is the same. This balance minimizes the amount of radiation from the line and other undesirable coupling effects. However, a balanced line should **not** be driven by an unbalanced source (i. e., a source with one side grounded), unless there is a circuit between the source and the line to convert between the forms. A transformer may be used in some cases. A device called a *balun* may also be used. The term *balun* is a contraction for ”balanced-to-unbalanced".

The fields of a parallel line extend some distance from the wires. As the frequency increases, the losses due to unwanted radiation on the line increases. Although parallel lines have been used extensively in the past at TV frequencies for coupling between antenna and receiver, where some losses are acceptable, their use is generally restricted to operation below about 100 MHz or so.

Capacitance, Inductance, and Characteristic Impedance of Parallel Lines

Referring to Figure 14-3, let

D = distance between centers of the two lines
d = diameter of each of the conductors

At RF frequencies, the inductance per meter of the line may be expressed as

$$L = \frac{m_0}{p} \ln \frac{2D}{d} = 4 \times 10^{-7} \ln \frac{2D}{d} \quad \text{H/m} \qquad (14\text{-}28)$$

where "ln" refers to the natural logarithm.

Under similar conditions, the capacitance per unit length is given by

$$C = \frac{pe}{\ln \frac{2D}{d}} = \frac{(1/36) \times 10^{-9} e_r}{\ln \frac{2D}{d}} = \frac{27.78 \times 10^{-12} e_r}{\ln \frac{2D}{d}} \quad \text{F/m} \qquad (14\text{-}29)$$

Assuming a lossless line, the characteristic impedance may then be determined by (14-14) as

$$R_0 = \frac{120}{\sqrt{e_r}} \ln \frac{2D}{d} \quad \Omega \qquad (14\text{-}30)$$

Coaxial Cable

The most common type of coaxial cable has the geometry shown in Figure 14-4. The two required conductors are realized by a center conductor and an enclosing conducting shield. The shield and conductor are separated by a *dielectric* material. The complete cable is enclosed with an insulating *jacket* such as vinyl.

The common form of coaxial cable that has been described is an example of an *unbalanced line*; i. e., the two conductors do not have symmetrical electric and magnetic properties with respect to ground. Instead the conducting shield is connected to the system ground. Ideally, this should result in an effective shield around the inner conductor and prevent any radiation losses. However, some losses in the dielectric material may still appear.

In general, coaxial cables are used in a wide number of applications at frequencies extending into the lower microwave region. Because they are unbalanced, coaxial cables may be readily connected to unbalanced sources. If the load is balanced, however, there may still be a need for a conversion circuit. The most significant disadvantages of coaxial cable are the cost and, in some applications, the physical size.

Capacitance, Inductance, and Characteristic Impedance of Coaxial Cable

Referring to Figure 14-4, let

D = inner diameter of outer conducting shield
d = diameter of inner conductor

At RF frequencies, the inductance per meter of the line may be expressed as

$$L = \frac{\mu_0}{2\pi} \ln \frac{D}{d} = 2 \times 10^{-7} \ln \frac{D}{d} \quad \text{H/m} \tag{14-31}$$

where "ln" refers to the natural logarithm.

Under similar conditions, the capacitance per unit length is given by

$$C = \frac{2\pi\varepsilon}{\ln \frac{D}{d}} = \frac{(1/18) \times 10^{-9} \varepsilon_r}{\ln \frac{D}{d}} = \frac{55.56 \times 10^{-12} \varepsilon_r}{\ln \frac{D}{d}} \quad \text{F/m} \tag{14-32}$$

Assuming a lossless line, the characteristic impedance may then be determined by (14-14) as

$$R_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln \frac{D}{d} \quad \Omega \tag{14-33}$$

Example 14-6

A coaxial cable has the following dimensions:

Diameter of inner conductor (d) = 0.29 cm
Inner diameter of outer conductor (D) = 1.05 cm
Polyethylene dielectric (e_r) = 2.25

Determine (a) the inductance per unit length, (b) the capacitance per unit length, (c) the characteristic impedance, and (d) the velocity of propagation.

Solution

Since $\ln(D/d)$ is required for the first three parameters, it will be determined first:

$$\ln\frac{D}{d} = \ln\frac{1.05}{0.29} = 1.287 \tag{14-34}$$

(a) The inductance per unit length is determined from (14-31).

$$L = 2\times10^{-7}\ln\frac{D}{d} = (2\times10^{-7})(1.287) = 257.4 \text{ nH/m} \tag{14-35}$$

(b) The capacitance per unit length is determined from (14-32).

$$C = \frac{55.56\times10^{-12}e_r}{\ln\frac{D}{d}} = \frac{55.56\times10^{-12}\times2.25}{1.287} = 97.13 \text{ pF/m} \tag{14-36}$$

(c) The characteristic impedance is determined from (14-33).

$$R_0 = \frac{60}{\sqrt{e_r}}\ln\frac{D}{d} = \frac{60}{\sqrt{2.25}}(1.287) = 51.48\ \Omega \tag{14-37}$$

(d) The velocity of propagation is determined from (14-24).

$$v = \frac{c}{\sqrt{e_r}} = \frac{3\times10^8}{\sqrt{2.25}} = \frac{3\times10^8}{1.5} = 2\times10^8 \text{ m/s} \tag{14-38}$$

These parameters are typical of realistic coaxial cables. A "standard" value of characteristic impedance assumed for many coaxial cables is $50\ \Omega$, and the velocity of propagation is 2/3 of the free space velocity.

14-6 Matched Load Impedance

Consider the situation depicted in Figure 14-5. An ac phasor source with internal impedance $\overline{Z_s}$ is connected through a lossless transmission line with characteristic impedance R_0 to a load impedance $\overline{Z_L}$. In this case, the load impedance is assumed to be purely resistance and equal to the characteristic impedance of the line; i. e., $\overline{Z_L} = R_L = R_0$. This is an ideal situation as far as the load is concerned. While there will be a delay between the source and the load, *no reflected wave will be generated*. Thus, there will only be a forward wave traveling from the source to the load.

Whether or not maximum power is transferred to the load will depend on the match between the source internal impedance and the line and that situation is usually easier to deal with than that of the load. Reactive impedance matching networks are often used to provide an impedance match at the output of a transmitter and they can be tuned to provide different conditions at different frequencies. It is also possible to provide reactive networks at the input to an antenna to match the impedance, but it is usually more difficult if the frequency is to be varied.

With a matched load, the input impedance $\overline{Z_{in}}$ at any point along the lossless line is equal to the load impedance and is

$$\overline{Z_{in}} = R_L = R_0 \tag{14-39}$$

This means that basic circuit theory may be used to predict the **magnitudes** of the steady-state voltage and current anywhere on the line. (The phase shift will be different at different points due to the time delay, but we will not be concerned with that phenomenon at the moment.)

The rms magnitude I_{rms} of the current at the input of the line under matched conditions is related to the rms magnitude of the source voltage V_{rms} by

(14-40)

where magnitude bars are used since the source impedance could be complex.

The input power P_{in} to the line must be the same as the power P_L dissipated in the load since the line is assumed to be lossless. Hence,

$$P_{in} = P_L = I_{rms}^2 R_0 = I_{rms}^2 R_L \tag{14-41}$$

Assuming that the source has a non-zero value of internal impedance, if there is a match at the source, this resulting power will be the maximum available power that can be obtained from the source.

It should be noted that even under matched conditions, there will be some losses in the line, especially if the line is reasonably long. Nevertheless, under matched conditions at the load, the approach used here is one of simplicity and can be modified slightly to take care of losses in the line without getting into the more complex conditions of mismatch to be considered in the next section.

Example 14-7

The circuit of Figure 14-6 represents the output of a transmitter connected through a transmission line to an antenna. The input impedance of the antenna is resistive and equal to 50 Ù and the characteristic impedance of the line is 50 Ù resistive. The non-zero output impedance of the transmitter has been matched to that of the line and is also 50 Ù resistive. The rms value of the source voltage referred to the output of the matching circuit is 200 V. Determine (a) the input impedance of the line, (b) the rms current flowing into the line, (c) the power accepted by the line, (d) the load power if the line is considered lossless.

Solution

(a) The line is matched at the load and the input impedance at any point along the line is simply

$$\overline{Z_{in}} = R_{in} = 50\ \Omega \tag{14-42}$$

(b) The rms current at the source is determined from basic circuit theory as

$$I_{rms} = \frac{400\ \text{V}}{50\ \Omega + 50\ \Omega} = 4\ \text{A} \tag{14-43}$$

(c) The input power accepted by the line is

$$P_{in} = I_{rms}^2 R = (4)^2 \times 50 = 800\ \text{W} \tag{14-44}$$

(d) If the line is lossless, the load power will be equal to the power input to the line and is

$$P_L = P_{in} = 800\ \text{W} \tag{14-45}$$

If the line is lossless and matched, as assumed here, the magnitudes of both the voltage and current will not vary along the line, although the phase shift will change.

Example 14-8

Assume all the conditions given in Example 14-7 except that the transmission line has a loss of 0.01 dB/m. Determine the load power if the length of the transmission line is 50 m.

Solution

Strictly speaking, once the line is assumed to be lossy, the characteristic impedance will become complex and will have both a real and an imaginary part. Hence, the assumption of a perfect match at the load might have to be questioned.

In practice, as long as the length is moderate and the losses not too great, it is reasonable to assume that the matching is essentially perfect and the effect of losses can be added "after the fact". With the length and loss value given here, that should be a reasonable assumption.

The net loss L_{dB} is given by

$$L_{\text{dB}} = 50 \text{ m} \times 0.01 \text{ dB/m} = 0.5 \text{ dB} \qquad (14\text{-}46)$$

This corresponds to an absolute loss factor L as given by

$$L = 10^{0.5/10} = 1.122 \qquad (14\text{-}47)$$

The actual power reaching the load will then be

$$P_L = \frac{P_{in}}{L} = \frac{800}{1.122} = 713.0 \text{ W} \qquad (14\text{-}48)$$

This means that 800 - 713 = 87 W will be dissipated in the line.

The approach taken here should be used with caution if the line is very long or if the loss is very great.

14-7 Mismatched Load Impedance

While the matched situation discussed in Section 14-6 is almost always a desired goal in designing a transmission system between an transmitter and an antenna or between an antenna and a receiver, there are situations where it is not practical. A good example is an antenna that must be used over a wide frequency range where matching is not possible at all frequencies. Another example is that of a portable unit in which the necessary length of a proper antenna is not feasible. Some of the problems associated with a mismatched load will be discussed in this section.

Consider the situation depicted in Figure 14-7. A source with internal impedance $\overline{Z_s}$ is connected through a lossless transmission line with characteristic impedance R_0 to an arbitrary load impedance $\overline{Z_L}$. Since we are considering a mismatched situation, we may as well assume that $\overline{Z_L}$ is complex and that it contains both a real and an imaginary part; i. e.,

$$\overline{Z_L} = R_L + jX_L \tag{14-49}$$

where R_L is the resistive part of the load impedance and X_L is the reactive part. Most antennas will exhibit such a complex impedance when they are operated at other than their design frequency range.

Reflection Coefficient at Load

We will now introduce a complex parameter $\overline{\Gamma_L}$ that will be denoted as the *voltage reflection coefficient at the load*. It is defined as

$$\overline{\Gamma_L} = \frac{Z_L - R_0}{Z_L + R_0} \tag{14-50}$$

The reflection coefficient is the ratio of the wave reflected from the load to the wave incident to it. We could also define a reflection coefficient at the source, but that is not necessary for our purposes.

Reflection Coefficient for Matched Load

When $\overline{Z_L} = R_0$, the value of the reflection coefficient is quickly determined as

$$\overline{\Gamma_L} = 0 \quad \text{for a matched load} \tag{14-51}$$

General Range

In general, the magnitude of the reflection coefficient is bounded by

$$0 \le \Gamma_L \le 1 \tag{14-52}$$

At one extreme, the reflection coefficient is 0, indicating a matched. load. Two other conditions are worth noting:

(1) When the load is a short-circuit; i. e, when $\overline{Z_L} = 0$,

$$\overline{\Gamma_L} = -1 \tag{14-53}$$

(2) When the load is an open-circuit; i. e, when $\overline{Z_L} \to \infty$

$$\overline{\Gamma_L} = 1 \tag{14-54}$$

For either a short-circuit or an open-circuit, all the power reaching the load is reflected back to the source and the line acts as a reactance. Circuits composed of short-circuit and or open-circuit lines may be used in impedance matching and filter circuits.

Voltage-Standing Wave Ratio

It has been stated that for a matched load, the magnitude of the voltage or current at any point on a lossless line is the same. However, when there is a mismatch, the voltage and current vary along the line. The greater the mismatch, the greater the variation of either voltage or current. Voltage is easier to measure so it will be used as the basis for the discussion that follows.

A quantity called the voltage-standing wave ratio is widely used in describing the behavior of a transmission line under operating conditions. Some references use VSWR, but we will use the simpler term S to represent this quantity. The basic definition is

$$S = \frac{V_{\max}}{V_{\min}} \tag{14-55}$$

Where $V_{\max}$ is the maximum voltage on the line and $V_{\min}$ is the minimum voltage on the line. The standing-wave ratio is characterized by the inequality

$$1 \leq S \leq \infty \tag{14-56}$$

When the line is perfectly matched, $S = 1$, meaning that there is no variation of the voltage along the line. Conversely, when $S = \infty$, the voltage varies between a minimum value of 0 and a finite maximum value, providing an infinite ratio for (14-55).

Relationship Between Reflection Coefficient and Standing-Wave Ratio

It turns out that there is a well-defined relationship between reflection coefficient and standing-wave ratio. The standing-wave ratio may be determined from the magnitude of the reflection coefficient by the following relationship:

$$S = \frac{1+\Gamma_L}{1-\Gamma_L} \tag{14-57}$$

The inverse relationship can be determined by solving for the magnitude of reflection coefficient in terms of the standing-wave ratio

$$\Gamma_L = \frac{S-1}{S+1} \tag{14-58}$$

Note that only the magnitude of the reflection coefficient may be determined from (14-58). The angle must be determined by some other means.

Resistive Load and Lossless Line

When the load is purely resistive and the line is lossless, there is a particularly simple relationship for the standing-wave ratio. Thus, let $\overline{Z_L} = R_L$ and continue the assumption that the line is lossless. It can be shown that the standing-wave ratio is given by one of the following relationships:

$$S = \frac{R_L}{R_0} \tag{14-59}$$

or

$$S = \frac{R_0}{R_L} \tag{14-60}$$

depending on which value is greater than one, since $S \geq 1$.

Reflected Power and Transmitted Power

Let P_{inc} represent the incident power propagating on a lossless transmission toward a load and assume that it encounters a mismatch at the load. Depending on the extent of the mismatch, some or all of the power will be reflected, and some will be absorbed by the load, with the exception that it is all reflected when there is a short-circuit or an open-circuit at the load. Let P_{ref} represent the power reflected and let P_L represent the power absorbed by the load. By the conversation of power, the following relationship must be satisfied:

$$P_{inc} = P_{ref} + P_L \tag{14-61}$$

It can be shown that the reflected power is related to the incident power by

$$P_{ref} = \Gamma_L^2 P_{inc} \tag{14-62}$$

where Γ_L is the magnitude of the reflection coefficient at the load. Substituting (14-62) in (14-61), the load power can be expressed as

$$P_L = \left(1 - \Gamma_L^2\right) P_{inc} \tag{14-63}$$

Aside from ohmic losses, any mismatch at the load will contribute to an additional reduction in load power as evident in (14-63). The *decibel mismatch loss* can be defined as

$$\text{decibel mismatch loss} = -10 \log\left(1 - \Gamma_L^2\right) \tag{14-64}$$

The negative sign in (14-64) is required to make the loss be a positive number.

Another quantity sometimes used in characterizing a transmission line is the *decibel return loss.* It is defined as

$$\text{decibel return loss} = -10 \log \Gamma_L^2 \tag{14-65}$$

Again, the negative sign results in a positive value for the decibel return loss.

<u>Input Impedance</u>

When a line is mismatched at the load, the input impedance varies alone the line. If the line is lossless, the pattern is periodic and repeats at intervals of one-half wavelength. It can be shown that the input impedance $\overline{Z_{in}}$ at any distance d from the load for a lossless line is given by

$$\overline{Z_{in}} = R_0 \frac{\left(1+\overline{\Gamma_L} e^{-j4p d/l}\right)}{\left(1-\overline{\Gamma_L} e^{-j4p d/l}\right)} \qquad (14\text{-}66)$$

For a matched load, this equation reduces to simply $\overline{Z_{in}} = R_0$ as previously stated. For a mismatch, however, the input impedance can vary considerably with the length of the line, particularly when there is a high standing-wave ratio. For any given length, the real part of the input impedance can be considered as accepting the input power for a given rms current, and if the line is lossless, this will be the power delivered to the load. Thus, for a high standing wave ratio, the power delivered to the load may be a very sensitive function of the line length.

<u>Example 14-9</u>

A load impedance given by $\overline{Z_L} = 50 + j100\ \Omega$ is connected through a lossless transmission line with a characteristic impedance of $50\ \Omega$. Determine (a) the load reflection coefficient and (b) the standing-wave ratio.

<u>Solution</u>

(a) The reflection coefficient at the load in this case is complex and is given by

$$\overline{\Gamma_L} = \frac{Z_L - R_0}{Z_L + R_0} = \frac{50 + j100 - 50}{50 + j100 + 50} = \frac{j100}{100 + j100} = \frac{100\angle 90^o}{141.4\angle 45^o} \qquad (14\text{-}67)$$
$$= 0.7071\angle 45^o$$

(b) The standing-wave ratio is given by

$$S = \frac{1+\Gamma}{1-\Gamma} = \frac{1+0.7071}{1-0.7071} = 5.828 \qquad (14\text{-}68)$$

Example 14-10

A load impedance give by $\overline{Z_L} = R_L = 100\ \Omega$ is connected through a lossless transmission line with a characteristic impedance of $300\ \Omega$. Determine (a) the load reflection coefficient and (b) the standing wave ratio.

Solution

(a) The reflection coefficient at the load in this case is real and is given by

$$\overline{\Gamma_L} = \frac{R_L - R_0}{R_L + R_0} = \frac{100-300}{100+300} = -0.5 \tag{14-69}$$

(b) While (14-58) could be used to determine the standing-wave ratio, it is simpler to use either (14-59) or (14-60) since the load impedance is real. The applicable one in this case is (14-60) and we have

$$S = \frac{R_0}{R_L} = \frac{300}{100} = 3 \tag{14-70}$$

Example 14-11

For the system of Example 14-10, determine the mismatch loss in dB.

Solution

The value is determined from (14-64) as

$$\begin{aligned}\text{decibel mismatch loss} &= -10\log\left(1-\Gamma_L^2\right) = -10\log\left[1-(0.5)^2\right] \\ &= -10\log 0.75 = -10(-0.125) = 1.25\text{ dB}\end{aligned} \tag{14-71}$$

14-8 Electric and Magnetic Fields

In order to deal with electromagnetic radiation, we need to introduce two important field quantities: (1) the electric field and (2) the magnetic field. In a sense, the electric field in space corresponds to voltage in a circuit, and the magnetic field in space corresponds to current in a circuit.

Electric Field

Strictly speaking, the definition of electric field is the force per unit charge existing between two points or surfaces of electrical charge. For example, a charged capacitor can be thought of as having invisible lines of electric field between the plates, with the lines originating on the positively charged plate and terminating on the negatively charged plate.

The symbol for electrical field intensity is E and various subscripts and phasor notation may be added as required. The unit is volts/meter (V/m).

Magnetic Field

A magnetic field in the case of static fields arises from current flow. For example, invisible lines of magnetic field that encircle the conductor will surround a current-carrying conductor. Unlike an electric field, magnetic field lines do not have any point sources.

The symbol for magnetic field intensity is H and various subscripts and phasor notation may be added as required. The unit is amperes/meter (A/m).

Time-Varying Electric and Magnetic Fields

Electromagnetic radiation as utilized in communications is a result of time-varying electric and magnetic fields. It turns out that a time-varying electric field causes a time-varying magnetic field and vice-versa. These concepts serve as the basis for wave propagation.

14-9 Plane Wave Propagation in Lossless Medium

We will now turn our attention to an important situation called *plane wave propagation*. It is closely related to our preceding development of transmission line phenomena because of certain analogous properties. The radiation from an antenna at normal receiving distances assumes the form of a plane wave. Light waves and other forms of radiation tend to be of the form of a plane wave.

Direction of Wave Propagation

Refer to Figure 14-7(a) for the discussion that follows. A plane wave is characterized by an electric field in one direction, a magnetic field perpendicular to the electric field, and the direction of propagation perpendicular to both the electric and magnetic fields. In the frame of reference chosen, the electric field has only an *x*-component, and it is denoted by E_x. The magnetic field has only a *y*-component, and it is denoted by H_y. The direction of propagation in this case is in the *z*-direction as shown.
In general, the direction of propagation for a plane wave may be determined by taking the right hand (with the thumb on the left) and rotating from E to H. The direction of the thumb aligned perpendicular to the plane containing E and H will represent the direction of propagation. The simplified diagram of Figure 14-7(b) is useful for this purpose.

Intrinsic Impedance

Assume now that the medium is lossless, i. e., that it is a perfect dielectric. In this case there is a linear algebraic relation between E_x and H_y. The relationship, which is derived in texts devoted primarily to electromagnetic field theory, is

$$\frac{E_x}{H_y} = h \qquad (14\text{-}72)$$

The quantity h is called *the intrinsic impedance*. Since the units of E_x are volts/meter and the units of H_y are amperes/meter, there is a cancellation of meters in the ratio. Thus, the units of h are volts/ampere = ohms, which leads to its definition as an impedance. *The intrinsic impedance is to a plane wave what characteristic impedance is to a single wave of voltage and current on a transmission line.*

For a lossless dielectric medium, the intrinsic impedance for a plane wave is a real number given by

$$h = \sqrt{\frac{m}{e}} \qquad (14\text{-}73)$$

where m is the permeability and e is the permittivity.

When free-space plane wave propagation is considered, $m = m_0$, $e = e_0$, and the intrinsic impedance is denoted as h_0. Substitution of these free-space values in (14-73) yields

$$h_0 = 120p \approx 377\ \Omega \qquad (14\text{-}74)$$

Unless indicated otherwise, this value will be assumed as the intrinsic impedance of air.

Assuming that $m = m_0$, the intrinsic impedance for any dielectric can be expressed as

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} \approx \frac{377}{\sqrt{\varepsilon_r}} \tag{14-75}$$

Poyting Vector

The plane wave is actually propagating energy in the form of electromagnetic radiation. This energy serves as the basis for the signal captured by a receiver antenna. It also serves as the basis for the warmth felt by light rays from the sun and many other phenomena, both natural and artificial.

The propagated energy is best measured in terms of power, which is, of course, the rate of energy transmitted. For a plane wave, the concept of *power density* is used. The power density is the power per unit area measured over an area normal to the direction of propagation. It is measured in watts/meter^2 (W/m^2) and, in general, is a vector quantity called the *Poyting vector*.

For the orientation of our assumed plane wave, the electric field E_x is in the *x*-direction, the magnetic field H_y is in the *y*-direction, and the Poyting vector will be in the *z*-direction. This power density will be denoted simply as p_z and it is given by

$$p_z = E_x H_y \tag{14-76}$$

Note the dimensions in (14-76). We have (volts/meter) x (amperes/meter) = volt.amperes/meter^2 = watts/meter^2 as required.

Similarity to Circuit Power

We see that (14-76) has the same form as $P = VI$ in circuit theory. In the same sense that the power in a resistance can also be expressed as V^2 / R or $I^2 R$, (14-76) can also be expressed in terms of either E_x or H_y by using the definition of intrinsic impedance in (14-72), and two alternate expressions for the power density are obtained:

$$p_z = \frac{E_x^2}{\eta} \tag{14-77}$$

and

$$p_z = H_y^2 \eta \tag{14-78}$$

Power Over a Surface Area

Assume that the power density vector is normal to a surface area *A* as illustrated in Figure 14-9 and assume that the value of the power density is constant over that area. The total power *P* passing through the area is given by

$$P = p_z A \tag{14-79}$$

If the power density varies over the area, integration is required to determine the total power.

Example 14-12

The rms magnitude of the electric field intensity of a plane wave in free space is $E_x = 2 \text{ V/m}$. Determine (a) H_y, (b) p_z, and (c) the total power transmitted through a rectangular surface with dimensions of 20 m by 30 m in the *x*-*y* plane over which the fields are constant.

Solution

(a) From (14-72) with $\eta = \eta_0$, the value of H_y is

$$H_y = \frac{E_x}{\eta_0} = \frac{2}{377} = 5.305 \times 10^{-3} \text{ A/m} \tag{14-80}$$

(b) The power density could be determined from either (14-76), (14-77), or (14-78). We will choose (14-77) since it uses only the original data and does not depend on the result of part (a).

$$p_z = \frac{E_x^2}{\eta_0} = \frac{(2)^2}{377} = 10.61 \text{ W/m}^2 = 10.61 \text{ mW/m}^2 \tag{14-81}$$

(c) The area *A* of the surface involved is

$$A = 20 \text{ m} \times 30 \text{ m} = 600 \text{ m}^2 \tag{14-82}$$

The power passing through this surface is

$$P = p_z A = (10.61 \times 10^{-3}) \times 600 = 6.366 \text{ W} \tag{14-83}$$

PROBLEMS

14-1 The Citizen's band is near 27 MHz. Based on the rule of thumb provided in the text, determine the length of line at which transmission lines must be considered.

14-2 Determine the length of line at which transmission line effects must be considered for an operating frequency of 300 MHz.

14-3 An assumed lossless transmission line has $L = 600 \text{ nH/m and } C = 40 \text{ pF/m}$. Determine the characteristic impedance.

14-4 An assumed lossless transmission line has $L = 1000 \text{ nH/m and } C = 20 \text{ pF/m}$. Determine the characteristic impedance.

14-5 Determine the velocity of propagation of the lossless line of Problem 14-3.

14-6 Determine the velocity of propagation of the lossless line of Problem 14-4.

14-7 Determine the dielectric constant for the transmission line of Problem 14-3 and 14-5.

14-8 Determine the dielectric constant for the transmission line of Problem 14-4 and 14-6.

14-9 A two-wire parallel transmission line has the following physical dimensions:

Diameter of each conductor = 0.2 cm
Spacing between the two conductors = 4 cm
Dielectric constant of medium = 2.3

Determine the following : (a) inductance per unit length, (b) capacitance per unit length, characteristic impedance, and (d) velocity of propagation.

14-10 An open (air dielectric) two-wire parallel transmission line has the following physical dimensions:

Diameter of each conductor = 0.4 inches
Spacing between the two conductors = 8 inches

Determine the following : (a) inductance per unit length, (b) capacitance per unit length, characteristic impedance, and (d) velocity of propagation.

14-11 A coaxial cable has the following physical dimensions:

Diameter of inner conductor = 0.1 inches
Inner diameter of outer conductor = 0.4 inches
Polystyrene dielectric with dielectric constant = 2.7

Determine the following : (a) inductance per unit length, (b) capacitance per unit length, characteristic impedance, and (d) velocity of propagation.

14-12 A coaxial cable has the following physical dimensions:

Diameter of inner conductor = 5 mm
Inner diameter of outer conductor = 3 cm
Polystyrene dielectric with dielectric constant = 2.3

Determine the following : (a) inductance per unit length, (b) capacitance per unit length, characteristic impedance, and (d) velocity of propagation.

14-13 Determine the following for a coaxial cable that has a velocity factor of 66.7% of the velocity of free space and characteristic impedance of 50 Ù: (a) inductance per unit length, (b) capacitance per unit length, and (c) dielectric constant of the insulating material between the inner and outer conductors.

14-14 Determine the following for a coaxial cable that has a velocity factor of 66.7% of the velocity of free space and characteristic impedance of 72 Ù: (a) inductance per unit length, (b) capacitance per unit length, and (c) dielectric constant of the insulating material between the inner and outer conductors.

14-15 The circuit of Figure P14-15 represents the output of a transmitter connected through a transmission line to an antenna. Determine (a) the input impedance of the line, (b) the rms current flowing into the line, (c) the power accepted by the line, and (d) the load power if the line is considered lossless.

14-16 The circuit of Figure P14-16 represents the output of a transmitter connected through a transmission line to an antenna. Determine (a) the input impedance of the line, (b) the rms current flowing into the line, (c) the power accepted by the line, and (d) the load power if the line is considered lossless.

14-17 Assume all the conditions in Problem 14-15 except that the transmission line has a loss of 0.005 dB/m. Determine the load power if the length of the transmission line is 80 m.

14-18 Assume all the conditions in Problem 14-16 except that the transmission line has a loss of 0.01 dB/m. Determine the load power if the length of the transmission line is 30 m.

14-19 A load impedance is given by $\overline{Z_L} = 100 - j100\ \Omega$ is connected through a lossless transmission line with a characteristic impedance of $75\ \Omega$. Determine (a) the reflection coefficient at the load and (b) the standing wave ratio.

14-20 A load impedance is given by $\overline{Z_L} = 100 - j100\ \Omega$ is connected through a lossless transmission line with a characteristic impedance of $300\ \Omega$. Determine (a) the reflection coefficient at the load and (b) the standing wave ratio.

14-21 For the system of Problem 14-19, determine the mismatch loss in dB.

14-22 For the system of Problem 14-20, determine the mismatch loss in dB.

14-23 For the system of Example 14-9 within the chapter, determine the mismatch loss in dB.

14-24 For the system of Example 14-9 within the chapter, determine the return loss in dB.

14-25 The rms magnitude of the magnetic field of a plane wave in air is $H_y = 200\ m\text{A/m}$. Assuming that E is in the positive *x*-direction, determine the following for a circular surface of diameter 50 m in the *x*-*y* plane over which the fields are constant: (a) E_x, (b) p_z, (c) total power transmitted through the area.

14-26 The rms magnitude of the electric field of a plane wave in sea water is $E_x = 3\ \text{V/m}$ and the dielectric constant is 80. Assuming that H is in the positive *y*-direction, determine the following for a square surface with sides of 15 m each in the *x*-*y* plane over which the fields are constant: (a) H_y, (b) p_z, and (c) total power transmitted through the area.

14-27 In a lossless dielectric medium, the rms electric field is 18.97 mV/m and the rms magnetic field is 158.1 ìA/m. Determine the following (a) intrinsic impedance, (b) power density, and (c) dielectric constant.

14-28 The power density of a plane wave propagating in free space is 120 ìW/m^2. Determine (a) the electric field and (b) the magnetic field.