Outline

- Lines and planes in space

The vector equation for a line:

$$\vec{r}(t) = \vec{r}_0 + t\vec{v} = \vec{r}_0 + t|\vec{v}|\frac{\vec{v}}{|\vec{v}|} \tag{1}$$

Ex: A helicopter is to fly from origion in the direction of the point $(1, 1, 1)$ at a speed of $60ft/s$. What is the position after $10sec$?

$$\frac{\vec{u}}{|\vec{u}|} = \frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k} \tag{2}$$

$$\vec{r}(t) = \vec{0} + t60(\frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}) = 20\sqrt{3}t(\vec{i} + \vec{j} + \vec{k}) \tag{3}$$

$$t = 10, \quad \vec{r} =< 200\sqrt{3}, 200\sqrt{3}, 200\sqrt{3} > \tag{4}$$

The distance from a point S to a line in space through P parallel to $\vec{v}$.

$$d = \frac{|\vec{PS} \times \vec{v}|}{|\vec{v}|} \tag{5}$$

Ex: Find the distance from the point $S(1,1,5)$ to line

$$L : x = 1 + t, \quad y = 3 - t, \quad z = 2t \tag{6}$$

$$\vec{v} = \vec{i} - \vec{j} + 2\vec{k} \tag{7}$$

Find a point on the line, $P(1,3,0)$.

$$\vec{PS} = -2\vec{j} + 5\vec{k} \tag{8}$$

$$\vec{PS} \times \vec{v} = \vec{i} + 5\vec{j} + 2\vec{k} \tag{9}$$

$$d = \frac{|\vec{PS} \times \vec{v}|}{|\vec{v}|} = \sqrt{5} \tag{10}$$

An equation for a plane in space:

The plane through $P_0(x_0, y_0, z_0)$ normal to $\vec{n} = A\vec{i} + B\vec{j} + C\vec{k}$ has

- vector equation $\vec{n} \cdot \vec{P_0P} = 0$
- component equation $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$
- simplified component equation
  $Ax + By + Cz = D = Ax_0 + By_0 + Cz_0$

Ex: Find an equation for a plane through $P_0(-3, 0, 7)$ perpendicular to $\vec{n} = 5\vec{i} + 2\vec{j} - \vec{k}$

$$5x + 2y - z = -22 \tag{11}$$

Ex: Find an equation for a plane through $A(0, 0, 1), B(2, 0, 0), C(0, 3, 0)$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -1 \\ 0 & 3 & -1 \end{vmatrix} = 3\vec{i} + 2\vec{j} + 6\vec{k} \tag{12}$$

$$3x + 2y + 6z = 6 \tag{13}$$

Lines of intersections of two planes

Ex: Find a vector parallel to the line of the intersection of the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$. Find the parametric equation for this line.

$$\vec{n_1} = 3\vec{i} - 6\vec{j} - 2\vec{k} \tag{14}$$

$$\vec{n_2} = 2\vec{i} + \vec{j} - 2\vec{k} \tag{15}$$

$$\vec{n_1} \times \vec{n_2} = 14\vec{i} + 2\vec{j} + 15\vec{k} \tag{16}$$

Find a point on the intersection line for $z = 0$.

$$3x - 6y = 15 \quad 2x + y = 5 \tag{17}$$

$$x = 3, \quad y = -1, \quad z = 0 \tag{18}$$

$$x = 3 + 14t, \quad y = -1 + 2t, \quad z = 15t \tag{19}$$

Find the point where the line $x = \frac{8}{3} + 2t, \quad y = -2t, \quad z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$.

$$t = -1 \tag{20}$$

The point is $(\frac{2}{3}, 2, 0)$.

The distance from a point to a plane.

If P is a point on a plane with $\vec{n}$, then the distance from a point S to a plane is

$$d = |\vec{PS} \cdot \frac{\vec{n}}{|\vec{n}|}| \tag{21}$$

Ex: Find the distance from $S(1, 1, 3)$ to the plane $3x + 2y + 6z = 6$

$$\vec{n} = 3\vec{i} + 2\vec{j} + 6\vec{k} \tag{22}$$

Find a point in plane $P(0, 3, 0)$

$$\vec{PS} = \vec{i} - 2\vec{j} + 3\vec{k} \tag{23}$$

$$d = |\vec{PS} \cdot \frac{\vec{n}}{|\vec{n}|}| = \frac{17}{7} \tag{24}$$

Angle between planes: the angle between two intersecting planes is the angle between their normal vector.

Ex: Find the angle between $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

$$\vec{n_1} = 3\vec{i} - 6\vec{j} - 2\vec{k} \tag{25}$$

$$\vec{n_2} = 2\vec{i} + \vec{j} - 2\vec{k} \tag{26}$$

$$\theta = \cos^{-1}(\frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}) = \cos^{-1}(\frac{4}{21}) \tag{27}$$