1. Solve first order ODEs.

- $$xy' + y = 3x^2 \tag{1}$$
$$y' + \frac{y}{x} = 3x \tag{2}$$
$$\mu = e^{\int p(x)dx} = e^{\int \frac{1}{x}dx} = x \tag{3}$$
$$\frac{d}{dx}[xy] = 3x^2 \tag{4}$$
$$xy = x^3 + c \tag{5}$$
$$y = x^2 + \frac{c}{x} \tag{6}$$

- $$e^x y\frac{dy}{dx} = e^{-y} + e^{-2x-y} \tag{7}$$
$$ye^y \text{dy} = (e^{-x} + e^{-3x})\text{dx} \tag{8}$$
$$e^y(y-1) = -e^{-x} - \frac{1}{3}e^{-3x} + c \tag{9}$$

2. Show that the equation is exact, and solve it.

$$(2x+y)dx + (2y+x)dy = 0 \tag{10}$$
$$M = 2x + y \quad N = 2y + x \tag{11}$$
$$\frac{\partial M}{\partial y} = 1 \quad \frac{\partial N}{\partial x} = 1 \tag{12}$$
$$\frac{\partial f}{\partial x} = 2x + y \tag{13}$$
$$f = x^2 + xy + h(y) \tag{14}$$
$$\frac{\partial f}{\partial y} = x + h'(y) = 2y + x \tag{15}$$
$$h(y) = y^2 \tag{16}$$
$$f = x^2 + xy + y^2 \tag{17}$$
$$x^2 + xy + y^2 = c \tag{18}$$

3. Solve second order ODE with initial values.

$$y'' + 2y' - 3y = e^x, \quad y(0) = 0, \quad y'(0) = 0 \tag{19}$$
$$m^2 + 2m - 3 = 0 \tag{20}$$
$$m_1 = -3, \quad m_2 = 1 \tag{21}$$
$$y_c = c_1e^{-3x} + c_2e^x \tag{22}$$
$$y_p = Axe^x \tag{23}$$

$$A = \frac{1}{4} \tag{24}$$

$$y(0) = 0, \quad y'(0) = 0 \tag{25}$$

$$c_1 = \frac{1}{16}, \quad c_2 = -\frac{1}{16} \tag{26}$$