P73 No. 3 Solve the exact DE:

$$(5x+4y)\,dx + (4x-8y^3)\,dy = 0$$

$$M = 5x+4y \qquad N = 4x-8y^3$$

$$\frac{\partial M}{\partial y} = 4 \qquad \frac{\partial N}{\partial x} = 4$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

We find f: that $df = M\,dx + N\,dy$

$$= \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

$$\frac{\partial f}{\partial x} = M \qquad \frac{\partial f}{\partial y} = N$$

$$f = \int M\,dx = \int (5x+4y)\,dx = \frac{5}{2}x^2 + 4yx + h(y)$$

$$\frac{\partial f}{\partial y} = 4x + h'(y) = 4x - 8y^3$$

$$h'(y) = -8y^3$$

$$h(y) = -2y^4$$

So $f = \frac{5}{2}x^2 + 4xy - 2y^4$

The solution will be $\frac{5}{2}x^2 + 4xy - 2y^4 = C$

P78 No. 17 Solve $\frac{dy}{dx} = y(xy^3-1)$

$$\frac{dy}{dx} + y = xy^4 \qquad n=4$$

$$u = y^{1-n} = y^{1-4} = y^{-3}$$

$$y = u^{-\frac{1}{3}} \qquad \frac{dy}{dx} = -\frac{1}{3}u^{-\frac{4}{3}}\frac{du}{dx}$$

$$-\frac{1}{3}u^{-\frac{4}{3}}\frac{du}{dx} + u^{-\frac{1}{3}} = x\,u^{-\frac{4}{3}}$$

$$\frac{du}{dx} - 3u = -3x \qquad \text{Linear 1st ODE}$$

$$e^{\int p(x)\,dx} = e^{\int -3\,dx} = e^{-3x}$$

$$\frac{d}{dx}\left(e^{-3x}u\right) = -3x\,e^{-3x}$$

$$u e^{-3x} = x e^{-3x} + \frac{1}{3} e^{-3x} + c$$

$$y^{-3} = x + \frac{1}{3} + c e^{3x}$$

P78 No. 30 $\quad \dfrac{dy}{dx} = \dfrac{3x+2y}{3x+2y+2} \qquad y(-1) = -1$

Suggestion: Use substitution $u = 3x + 2y$