Nonhomogeneous ODE: undetermined coefficients
Example 1:

$$y'' - 5y' + 4y = 8e^x \tag{1}$$
$$y_c = c_1e^x + c_2e^{4x} \tag{2}$$
$$y_p = Ae^x \tag{3}$$
$$(A - 5A + 4A)e^x = 8e^x \quad 0 = 8e^x \tag{4}$$

But it is also a solution of the associated homogeneous DE.

$$y_p = Axe^x \tag{5}$$
$$y_p' = (A + Ax)e^x \quad y_p'' = (2A + Ax)e^x \tag{6}$$
$$-3Ae^x = 8e^x \quad A = \frac{8}{3} \tag{7}$$
$$y = c_1e^x + c_2e^{4x} - \frac{8}{3}xe^x \tag{8}$$

Example 2:

$$y'' - 2y' + y = e^x \tag{9}$$
$$y_c = c_1e^x + c_2xe^x \tag{10}$$
$$y_p = Ae^x, \quad y_p = Axe^x \tag{11}$$

But they are also solutions of the associated homogeneous DE.

$$y_p = Ax^2e^x \tag{12}$$
$$y_p' = (2Ax + Ax^2)e^x \quad y_p'' = (4Ax + Ax^2 + 2A)e^x \tag{13}$$
$$2Ae^x = e^x \quad A = \frac{1}{2} \tag{14}$$
$$y = c_1e^x + c_2xe^x + \frac{1}{2}x^2e^x \tag{15}$$

Note:

- No function in the assumed particular solution is a solution of the associated homogeneous DE.
- y_p contains terms that duplicate terms in y_c, then y_p must be multiplied by x^n, where n is the lowest positive integer that eliminates that duplication.

Example 3:

$$y'' - 8y' + 25y = (5x^3 - 7)e^{-x} \tag{16}$$
$$y_c = e^{4x}(c_1\cos(3x) + c_2\sin(3x)) \tag{17}$$
$$y_p = e^{-x}(Ax^3 + Bx^2 + Cx + D) \tag{18}$$

Example 4:

$$y'' + 4y = x\cos x \tag{19}$$
$$y_c = c_1\cos(2x) + c_2\sin(2x) \tag{20}$$
$$y_p = (Ax + B)\sin x + (Cx + D)\cos x \tag{21}$$

Example 5:

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x} \tag{22}$$
$$y'' - 2y' - 3y = 4x - 5 \tag{23}$$
$$y'' - 2y' - 3y = 6xe^{2x} \tag{24}$$
$$y_c = c_1e^{-x} + c_2e^{3x} \tag{25}$$
$$y_{p1} = Ax + B \tag{26}$$
$$y'_{p1} = A \tag{27}$$
$$y''_{p1} = 0 \tag{28}$$
$$A = -\frac{4}{3}, \quad B = \frac{23}{9} \tag{29}$$
$$y_{p1} = -\frac{4}{3}x + \frac{23}{9} \tag{30}$$
$$y_{p2} = (Cx + D)e^{2x} \tag{31}$$
$$y'_{p2} = (2Cx + C + 2D)e^{2x} \tag{32}$$
$$y''_{p2} = (4Cx + 4C + 4D)e^{2x} \tag{33}$$
$$C = -2, \quad D = -\frac{4}{3} \tag{34}$$
$$y_{p2} = (-2x - \frac{4}{3})e^{2x} \tag{35}$$
$$y_p = y_{p1} + y_{p2} = -\frac{4}{3}x + \frac{23}{9} + (-2x - \frac{4}{3})e^{2x} \tag{36}$$

Example 6:

$$y'' + y = 4x + 10\sin x \tag{37}$$
$$y(\pi) = 0, \quad y'(\pi) = 2 \tag{38}$$
$$y_c = c_1\cos x + c_2\sin x \tag{39}$$
$$y_p = (Ax + B) + (Cx\sin x + Dx\cos x) \tag{40}$$

$$A = 4, \quad B = 0, \quad C = 0, \quad D = -5 \tag{41}$$

$$y = c_1 \text{cos}x + c_2 \text{sin}x + 4x - 5x\text{cos}x \tag{42}$$

$$y' = -c_1 \text{sin}x + c_2 \text{cos}x + 4 - 5\text{cos}x + 5x\text{sin}x \tag{43}$$

$$c_1 = 9\pi c_2 = 7 \tag{44}$$

$$y = -9\pi \text{sin}x + 7\text{cos}x + 4x - 5x\text{cos}x \tag{45}$$

About homework on Page 108, No. 2.
Using logistic model

$$\frac{dN}{dt} = N(a - bN) \tag{46}$$

$$N_0 = 500 \tag{47}$$

$$c = \frac{N_0}{a - bN_0} = \frac{500}{a - b500} \tag{48}$$

The physical meaning of logistic model is that modeled population cannot exceed $\frac{a}{b}$, so this is the limiting number. This corresponds to the value in the problem 50000, which is the limiting number of pepople in the community who will see the advertisement.

$$\frac{b}{a} = 50000 \tag{49}$$

$$b = 50000a \quad c = -\frac{500}{49999a} \tag{50}$$

$$N(t) = \frac{a(-\frac{500}{49999a})}{50000a(-\frac{500}{49999a}) + e^{at}} \tag{51}$$

Using t=1, N=1000 to determine the value of a.

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