You have 75 minutes (the full class period) to answer the following 6 (six) questions on 2 (two) pages. Show all your work. There is a total of 200 points on the test.

1. (20 + 20 = 40 points)
   (a) Find det(A), where

   \[
   A = \begin{bmatrix}
       1 & 4 & 0 & 3 \\
       0 & 0 & 2 & 0 \\
       3 & 1 & 0 & 4 \\
       0 & 3 & 5 & 0 
   \end{bmatrix}
   \]

   (b) Find det(B), where

   \[
   B = \begin{bmatrix}
       0 & 0 & 2 & 0 \\
       1 & 4 & 0 & 3 \\
       3 & 1 & 0 & 4 \\
       0 & 3 & 5 & 0 
   \end{bmatrix}
   \]

   (note: B is obtained from A by swapping the first two rows)

2. (20 points)
   Find det(A), where

   \[
   A = \begin{bmatrix}
       1 & 4 & 3 & 2 \\
       1 & 2 & 5 & 4 \\
       1 & 4 & 1 & 6 \\
       1 & 4 & 3 & 5 
   \end{bmatrix}
   \]
3. (20 + 20 + 20 = 60 points)
Given the matrix
\[
A = \begin{bmatrix}
3 & -5 & -2 \\
1 & -3 & -\frac{2}{5} \\
0 & 0 & -2 \\
\end{bmatrix}
\]
(a) find all the eigenvalues of \(A\)
(b) find the eigenvectors associated with each eigenvalue of \(A\)
(c) if possible, find an invertible \(S\) matrix and diagonal \(D\) matrix such that \(D = S^{-1}AS\)

4. (10 + 20 + 20 = 50 points)
Given the matrix
\[
A = \begin{bmatrix}
1 & a & 1 \\
0 & 1 & 0 \\
0 & 0 & 2 \\
\end{bmatrix}
\]
(a) find all the eigenvalues of \(A\)
(b) if \(a=0\), what are the eigenvectors of \(A\), and is there an eigenbasis?
(c) if \(a\neq0\), what are the eigenvectors of \(A\), and is there an eigenbasis?

5. (15 + 5 = 20 points)
Given the matrix
\[
A = \begin{bmatrix}
1 & 4 \\
0 & 2 \\
\end{bmatrix}
\]
(a) find formulas for the entries of \(A^t\) (where \(t\) is a positive integer)
(b) find the vector
\[
A^t \begin{bmatrix} 4 \\ 1 \end{bmatrix}
\]

6. (10 points)
Given that \(A\) is a 4×4 matrix, and \(\det(A) = 3\).
If \(B = 2 \ A^T\) (\(A^T\) is the transpose matrix of \(A\)), what is \(\det(B)\)?