For this assignment, you will need to write a MATLAB program that takes as input a real $2 \times 2$ matrix $A$, and calculates the singular values and singular vectors of $A$.

Apply your program to each of the matrices given below,

$$
\begin{bmatrix}
2 & 0 \\
0 & 3
\end{bmatrix},
\begin{bmatrix}
3 & 0 \\
0 & 1/2
\end{bmatrix},
\begin{bmatrix}
1 & 2 \\
0 & 2
\end{bmatrix},
\begin{bmatrix}
1 & 2 \\
0 & 0
\end{bmatrix},
\begin{bmatrix}
0 & 1 \\
4 & 0
\end{bmatrix},
\begin{bmatrix}
1 & 2 \\
4 & 3
\end{bmatrix},
\begin{bmatrix}
0 & 5 \\
2 & 0
\end{bmatrix}
$$

and produce the following MATLAB output for each:

(a) plot the unit circle ($S$) on the left and the corresponding elliptical image of $S$ under $A$ ($AS$) on the right,

(b) plot the right singular vectors, $\tilde{v}_1$ and $\tilde{v}_2$, superimposed on the unit circle,

(c) plot the left singular vectors, $\tilde{u}_1$ and $\tilde{u}_2$, superimposed on the ellipse,

(note: for this part, find the singular values and singular vectors “manually,” rather than using the built-in SVD function in MATLAB).

For each of the matrices, also complete the following:

(a) what are the singular values, $\sigma_1$ and $\sigma_2$, of $A$,

(b) how many nonzero singular values are there,

(c) what is the rank of $A$,

(d) compare the singular values and singular vectors obtained above to those obtained using the built-in SVD function in MATLAB.