1. (15pts) Let \( T: U^{2 \times 2} \to U^{2 \times 2} \), where \( U^{2 \times 2} \) is the space of all \( 2 \times 2 \) upper triangular matrices, be defined by
\[
T(M) = M \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} M.
\]
(i) Show that \( T \) is a linear transformation.
(ii) Is it an isomorphism?
(iii) Find the matrix of \( T \) with respect to the basis \( \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right\} \).

2. (10pts) Show that \( f_1(x) = 1 + 2x + 3x^2 \), \( f_2(x) = 4 + 5x + 6x^2 \), \( f_3(x) = 7 + 8x + 10x^2 \) are linearly independent in \( P_2 \), the linear space of all polynomials of degree \( \leq 2 \).

3. (10pts) Find the determinant of the following matrix. Explain the method which you used in detail.
\[
\begin{pmatrix}
1 & 0 & 0 & 9 & 8 \\
1 & 1 & 0 & 5 & 4 \\
1 & 1 & 1 & 4 & 2 \\
0 & 0 & 0 & 7 & 9 \\
0 & 0 & 0 & 3 & 1
\end{pmatrix}
\]

4. (10pts) Find the eigenvalue(s) and the corresponding eigenvector(s) for the following matrix \( A \). Specify their algebraic and geometric multiplicities.
\[
A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}
\]

5. (10pts) True or False?
(a) The equation \( \det(-A) = \det(A) \) holds for all \( 10 \times 10 \) matrices.
(b) \( \det \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = 1 \).
(c) There is a \( 3 \times 3 \) matrix \( A \) such that \( A^2 = -I_3 \).
(d) The eigenvalues of a \( 2 \times 2 \) matrix \( A \) are the solutions of the equation \( \lambda^2 - (tr \ A) \lambda + (det \ A) = 0 \).
(e) If \( 0 \) is an eigenvalue of a matrix \( A \), then \( \det(A) = 0 \).

Bonus: (5pts) Prove or disprove the statements in Exercise # 5.