A-D and D-A Conversion

In order to convert an analog signal to a digital form, it is necessary to perform an analog-to-digital conversion. A/D converters are available as off-the-shelf items. They may be connected for unipolar encoding or for bipolar offset encoding. In the first case, the signal is assumed to be positive within a range from 0 to $V_{fs}$, the full-scale voltage. In the second case, the signal is assumed to be symmetrical about 0 with a range from $-V_{fs}$ to $V_{fs}$. The properties of various A/D and D/A converters will be studied in this module.

Quantization—a process unique to all digital sampling of analog signals in which the sample is replaced by the closest "standard" value.

Let $N =$ number of bits in each word.

$m = \text{number of possible levels} = 2^N$

$N = \log_2 m = 3.32 \log_{10} m$

Example 1. Determine the number of possible levels that can be encoded for (a) 4 bits, (b) 8 bits, and (c) 16 bits.

(a) $m = 2^4 = 16$ levels
(b) $m = 2^8 = 256$ levels
(c) $m = 2^{16} = 65,536$ levels
Example 2. It is desired to represent a signal in no less than 100 levels. Determine the minimum number of bits.

\[ N = \log_2 100 = 3.32\log_{10} 100 = 3.32 \times 2 = 6.64 \text{ bits} \]

Thus, 7 bits are required, which permits 128 levels.

Basic PCM Encoding

It is convenient to define a normalized input voltage as follows:

normalized input analog voltage = \( \frac{\text{actual input voltage}}{\text{full-scale voltage of A/D converter}} \)

Basic PCM Encoding, (continued).

\( \left( \frac{\text{actual output analog voltage}}{\text{normalized value of digital word}} \right) = \left( \frac{\text{full-scale voltage of D/A converter}}{V_n} \right) \)

\( V_n = \text{full-scale voltage of A/D and D/A converter.} \)
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**Unipolar Encoding**

Normalized analog signal varies from 0 to 1.

\[ \Delta X_u = 2^{-N} \]

Step Size = \( \Delta V_u = \Delta X_u V_p = 2^{-N} V_p \)

\[ X_u(\text{max}) = 1 - \Delta X_u = 1 - 2^{-N} \]

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**Bipolar Offset Encoding**

Normalized analog signal varies from -1 to 1.

\[ \Delta X_b = 2^{-N+1} \]

Step Size = \( \Delta V_b = \Delta X_b V_p = 2^{-N+1} V_p \)

\[ X_b(\text{max}) = 1 - \Delta X_b = 1 - 2^{-N+1} \]

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**Unipolar and Bipolar Offset Encoding**

<table>
<thead>
<tr>
<th>Natural Binary</th>
<th>Decoded Value</th>
<th>Unipolar Normalized Decimal Value</th>
<th>Bipolar Offset Normalized Decimal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1111</td>
<td>15</td>
<td>1111.000 x 0.5</td>
<td>0.75</td>
</tr>
<tr>
<td>1110</td>
<td>14</td>
<td>1110.000 x 0.5</td>
<td>0.625</td>
</tr>
<tr>
<td>1101</td>
<td>13</td>
<td>1101.000 x 0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>1100</td>
<td>12</td>
<td>1100.000 x 0.0</td>
<td>0</td>
</tr>
<tr>
<td>1011</td>
<td>11</td>
<td>1011.000 x 0.25</td>
<td>-0.25</td>
</tr>
<tr>
<td>1010</td>
<td>10</td>
<td>1010.000 x 0.0</td>
<td>-0.5</td>
</tr>
<tr>
<td>1001</td>
<td>9</td>
<td>1001.000 x 0.125</td>
<td>-0.625</td>
</tr>
<tr>
<td>1000</td>
<td>8</td>
<td>1000.000 x 0.0</td>
<td>-0.75</td>
</tr>
<tr>
<td>0111</td>
<td>7</td>
<td>0111.000 x 0.5</td>
<td>-0.875</td>
</tr>
<tr>
<td>0110</td>
<td>6</td>
<td>0110.000 x 0.125</td>
<td>-0.9375</td>
</tr>
<tr>
<td>0101</td>
<td>5</td>
<td>0101.000 x 0.0</td>
<td>-0.9375</td>
</tr>
<tr>
<td>0100</td>
<td>4</td>
<td>0100.000 x 0.375</td>
<td>-0.625</td>
</tr>
<tr>
<td>0011</td>
<td>3</td>
<td>0011.000 x 0.75</td>
<td>-0.875</td>
</tr>
<tr>
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<td>0010.000 x 1.25</td>
<td>-1</td>
</tr>
<tr>
<td>0001</td>
<td>1</td>
<td>0001.000 x 2.0</td>
<td>-1</td>
</tr>
<tr>
<td>0000</td>
<td>0</td>
<td>0000.000 x 3.0</td>
<td>-1</td>
</tr>
</tbody>
</table>
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Unipolar Encoding

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Bipolar Offset Encoding

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Quantization Error (Rounding)

\[ E_n = \frac{\Delta X_u}{2} = 2^{-(N+1)} \]
\[ E_p = \frac{\Delta X_b}{2} = 2^{-N} \]
\[ e_u = E_u V_u = 2^{-(N+1)} V_u = \frac{\Delta u}{2} \]
\[ e_p = E_p V_p = 2^{-N} V_p = \frac{\Delta p}{2} \]
Example 3. An 8-bit A/D converter with a full-scale voltage of 20 V is connected for unipolar operation. Find step size and maximum quantization error (rounding).

\[ V_u = 20 \text{ V} \]
\[ \Delta X_u = 2^{-N} = 2^{-8} = 0.003906 \]
\[ \Delta V_u = \Delta X_u V_u = 0.003906 \times 20 = 78.12 \text{ mV} \]
\[ e_u = \frac{\Delta V_u}{2} = \frac{78.12}{2} = 39.06 \text{ mV} \]

Example 4. The converter of Example 3 is connected for bipolar offset. Repeat calculations of Example 3.

\[ V_b = 10 \text{ V} \]
\[ \Delta X_b = 2^{-N+1} = 2^{-7} = 0.007812 \]
\[ \Delta V_b = \Delta X_v V_u = 0.007812 \times 10 = 78.12 \text{ mV} \]
\[ e_b = \frac{\Delta V_b}{2} = \frac{78.12}{2} = 39.06 \text{ mV} \]

Summary
- An A/D converter may be used to convert analog data to digital data.
- A D/A converter may be used to convert digital data to analog data.
- The two major forms of encoding for A/D converters are unipolar and bipolar offset.
- Unipolar encoding is based on the signal having one polarity only while bipolar encoding assumes a nearly symmetrical signal about a zero level.