Analog Systems with Noise

A major consideration for any analog modulation method is the behavior in the presence of noise. In this module, the various common analog systems will be compared. As a common reference, a sinusoidal modulating signal will be assumed, but sufficient bandwidth will be assumed to accommodate a baseband bandwidth $W$. The two comparisons used are (1) the baseband comparison gain and (2) the receiver processing gain. Both concepts will be developed and explored.

Receiver Noise Terms

- $T_a$: Antenna noise temperature (based on integrated brightness temperature)
- $T_r$: Receiver noise temperature referred to input
- $T_{sys}$: System noise temperature $= T_a + T_r$
- $N_a$: Antenna noise power $= k T_a B$
- $N_r$: Receiver noise input power $= k T_r B$
- $N_{sys}$: System noise input power $= k T_{sys} B$

Receiver Input Parameters

- $P_s$: System input power
- $N_{sys} = N_r + N_{sys} = k T_r B + k T_{sys} B = k T_{sys} B$
- $T_{sys} = T_r + T_{sys}$
- $N_{in} = N_r = k T_r B$
- $T_{in} = T_r$
Slide 4

**Signal-to-Noise Ratios**

Overall System S/N Ratio at Detector Input:

\[
\frac{(S/N)_{\text{sys}}}{N_B} = \frac{P_s}{N_B} = \frac{P_s}{kT_B B} = \frac{P_s}{k(T_s + T_B) B}
\]

Detector Output S/N Ratio:

\[
(S/N)_{\text{output}} = \frac{P_s}{N_C}
\]

Reference Baseband S/N Ratio:

\[
(S/N)_{\text{baseband}} = \frac{P_s}{\eta_{\text{baseband}} W} = \frac{P_s}{kT_r W}
\]

Slide 5

**Two Comparison Gain Factors**

Baseband Comparison Gain

\[
G_B = \frac{(S/N)_{\text{output}}}{(S/N)_{\text{baseband}}}
\]

Receiver Processing Gain

\[
G_R = \frac{(S/N)_{\text{output}}}{(S/N)_{\text{sys}}}
\]

Slide 6

**TABULATION**

<table>
<thead>
<tr>
<th>Modulation Method</th>
<th>(A)</th>
<th>(G_B)</th>
<th>(G_R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM</td>
<td>20</td>
<td>(2^{10})</td>
<td>(2^{10})</td>
</tr>
<tr>
<td>AM</td>
<td>20</td>
<td>(2^{10})</td>
<td>(2^{10})</td>
</tr>
<tr>
<td>(100% modulation)</td>
<td>20</td>
<td>(2^{10})</td>
<td>(2^{10})</td>
</tr>
<tr>
<td>FM</td>
<td>20 + 0 dB</td>
<td>(2^{10})</td>
<td>(2^{10})</td>
</tr>
<tr>
<td>FM</td>
<td>20 + 0 dB</td>
<td>(2^{10})</td>
<td>(2^{10})</td>
</tr>
<tr>
<td>FS (preemphasis)</td>
<td>20 + 0 dB</td>
<td>(2^{10})</td>
<td>(2^{10})</td>
</tr>
</tbody>
</table>
Example 1. Determine baseband comparison gains for different methods.

Example 2. Determine receiver processing gains for different methods.

<table>
<thead>
<tr>
<th>Modulation Method</th>
<th>Baseband Comparison Gain</th>
<th>Receiver Processing Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSB</td>
<td>0 dB</td>
<td>0 dB</td>
</tr>
<tr>
<td>DSB</td>
<td>0 dB</td>
<td>3.01 dB</td>
</tr>
<tr>
<td>AM (50% Modulation)</td>
<td>-6.99 dB</td>
<td>-4.77 dB</td>
</tr>
<tr>
<td>AM (100% Modulation)</td>
<td>-4.77 dB</td>
<td>-1.76 dB</td>
</tr>
<tr>
<td>PM (5° Δ)</td>
<td>10.97 dB</td>
<td>21.76 dB</td>
</tr>
<tr>
<td>FM (5° Δ)</td>
<td>15.74 dB</td>
<td>26.53 dB</td>
</tr>
<tr>
<td>FM (5° Δ), Preemphasis, 15 kHz, 2 kHz Wf</td>
<td>21.42 dB</td>
<td>32.22 dB</td>
</tr>
</tbody>
</table>
**Threshold Effect**

- **Receiver operating input signal-to-noise ratio (dB scale)**
- **sys**
- **SN**

\[ SN_{\text{in}} = \frac{P_{\text{in}}}{N_{\text{sys}}} \]

**Example 3.** The signal power at a receiver input is 50 pW. Antenna noise temperature = 150 K and receiver noise temperature = 325 K. For \( W = 15 \text{ kHz} \) and \( D = 5 \), determine detected S/N ratio.

\[
T_{\text{in}} = T_{\text{a}} + T_{\text{n}} = 150 + 325 = 475 \text{ K}
\]

\[
B = 2(1 + D)W = 2(1 + 5) \times 15 = 180 \text{ kHz}
\]

\[
N_{\text{in}} = kT_{\text{in}}B = 1.38 \times 10^{-23} \times 475 \times 180 \times 10^3
\]

\[
= 1.180 \text{ fW}
\]

\[
(S/N)_{\text{in}} = \frac{P_{\text{in}}}{N_{\text{in}}} = \frac{50 \times 10^{-12}}{1.180 \times 10^{-15}} = 42.37 \times 10^3
\]

**Example 3. (Continuation)**

\[
G_s = 3(1 + D)D^2 = 3(1 + 5)5^2 = 450
\]

\[
(S/N)_{\text{sys}} = G_s \times (S/N)_{\text{in}} = 450 \times 42.37 \times 10^3
\]

\[
= 19.07 \times 10^6
\]

\[
(S/N)_{\text{sys,in}} = 10\log 19.07 \times 10^6 = 72.80 \text{ dB}
\]

*Alternate Approach*

\[
G_{\text{sys,in}} = 10\log 42.37 \times 10^3 = 46.27 \text{ dB}
\]

\[
(S/N)_{\text{sys,in}} = (S/N)_{\text{sys,in}} + G_{\text{sys,in}}
\]

\[
= 46.27 + 26.53 = 72.80 \text{ dB}
\]
Summary
- The baseband comparison gain compares the detected S/N ratio with direct baseband transmission for the same signal bandwidth.
- The receiver processing gain compares the detected S/N ratio with the receiver input S/N ratio.
- The two gain factors vary widely with the type of modulation.
- Angle modulation methods provide greater gains but require greater bandwidths.