Chapter 12
NOISE IN COMMUNICATION SYSTEMS

12-1 Introduction and Objectives

The analysis of communication systems thus far in the text has not considered the effects of interfering noise. With large transmission power levels and/or close range transmission, the effects of noise may be negligible, and the system may be considered as noise-free for most practical purposes. However, noise is always present, and when the power level is low and/or the transmission distance is great, noise effects must be considered. As we will see later, different modulation processes vary widely in their behavior with respect to noise.

Some of the different types of electrical noise will be surveyed early in this chapter. The techniques for analyzing noise effects will be developed through the use of thermal noise models. The concepts of effective noise temperature and noise figure will be introduced as a means of characterizing the overall noise present at the output of a receiver.

Objectives

After completing this chapter, the reader should be able to

1. List the different forms of external and internal noise and their properties.
2. Discuss thermal noise and its behavior.
3. State and apply the relationships for thermal noise voltage or current produced by a resistance, and draw the corresponding Thevenin and Norton equivalent circuit models.
4. Determine the noise voltage produced by an arbitrary combination of resistances.
5. Determine the available noise power produced by a resistance.
6. Determine the noise power spectral density produced by a resistance.
7. Utilize system gain to predict output noise power, voltage, and current.
8. Define effective noise temperature and apply it to determine the output noise produced by a system.
9. Define noise figure, both in absolute and decibel forms, and apply it to determine the output noise produced by a system.
10. Convert between noise temperature and noise figure.
11. Determine the noise temperature and/or noise figure for an attenuator or lossy component.
12. Determine the effective noise temperature or noise figure of a cascade of amplifier and/or attenuator stages.
12-2 Noise Classifications

On the broadest scale, noise can be classified as either external or internal. Each category consists of several different types.

External Noise

*External noise* represents all the different types that arise outside of the communication system components. It includes atmospheric noise, galactic noise, man-made noise, and interference from other communication sources.

Internal Noise

*Internal noise* represents all the different types that arise inside of the communication system components. It includes thermal noise, shot noise, and flicker noise. Although the components of both the transmitter and receiver are included in the definition, the region of primary concern is from the receiving antenna through the first several stages of the receiver. It is in this region of small signal amplitudes that internal noise is most troublesome.

Some of the most common types of external and internal noise will be described in the remainder of this section.

Atmospheric Noise

*Atmospheric noise* is produced mostly by lightning discharges in thunderstorms. It is usually the dominating external noise source in quite locations at frequencies below about 20 MHz or so. However, the power spectrum of atmospheric noise decreases rapidly as the frequency increases and the effect becomes relatively insignificant at frequencies well above this value. The level of atmospheric noise also decreases with increasing latitude on the surface of the globe, and it is particularly severe during the rainy season in regions near the equator.

Galactic Noise

*Galactic noise* is caused by disturbances originating outside the earth's atmosphere. The primary sources of galactic noise are the sun, background radiation along the galactic plane, and the many cosmic sources distributed along the galactic plane. The primary frequency range in which galactic noise is significant is from about 15 MHz to perhaps 500 MHz, and its power spectrum decreases with increasing frequency.

Man-Made Noise

*Man-made noise* is somewhat obvious from its title and consists of any source of electrical noise resulting from a man-made device or system. Among the chief offenders in this category are electric motors, automobile ignition systems, neon signs, and power lines. As one would likely suspect, the average level of man-made noise is significantly higher in urban areas than in rural areas. This fact has led to the selection of certain remote rural areas for the locations of many of the satellite tracking stations and radio astronomy observatories. The power spectrum of man-made noise decreases as the frequency increases, but the exact frequency range at which it becomes negligible is a function of its relative level. For example, in quite remote locations, the noise level from man-made sources will usually be below galactic noise in the frequency range from about 10 MHz or so.
Interference

One can debate as to whether or not interference from other communication sources should be classified as "noise." However, it produces many of the same interfering effects and can thus be classified as noise as far as the desired signal is concerned.

Thermal Noise

*Thermal noise* is the result of the random motion of charged particles (usually electrons) in a conducting medium such as a resistor. Since all circuits necessarily contain resistive devices, thermal noise sources appear throughout all electrical circuits. The power spectrum of thermal noise is quite wide and is essentially uniform over the RF spectrum of interest for most communications applications. Mathematical models for analyzing thermal noise are used either directly or indirectly for dealing with a variety of different types of noise. Consequently, much of this chapter is devoted to dealing with the development and application of thermal noise models, so we will postpone further consideration until the next section.

Shot Noise

*Shot noise* arises from the discrete nature of current flow in electronic devices such as transistors and tubes. For example, the electrons or holes crossing a semiconductor junction display a random variation of the time corresponding to the crossing, which in turn produces a random fluctuation of the current. The associated random variation in the current appears as a disturbance to the signal being processed by the device, and so the result is a form of noise. The power spectrum of shot noise is similar to that of thermal noise, and the two effects are usually lumped together for system analysis.

Flicker Noise

*Flicker noise* (also called $1/f$ noise) is a somewhat vaguely understood form of noise occurring in active devices such as transistors at very low frequencies. It is most significant near dc and a few hertz and is usually negligible above about 1 kHz or so. Flicker noise is often a limiting factor for the minimum signal level that can be processed by a direct-coupled (dc) amplifier.

There are other forms of noise that are peculiar to certain types of modulation system that will be discussed as the need arises. For example, digital PCM systems exhibit an inherent noiselike uncertainty called quantization noise.

Noise Pollution

There are many sources of pollution of which most people are aware, e. g., air pollution and water pollution. However, another form of pollution of major interest to the communications engineer or technologist, of which many people are not aware, is that of *spectral pollution*. Because of the large number of electromagnetic transmission sources, a large amount of spectral background radiation exists. The available frequency spectrum is another natural resource that is rapidly being depleted by the increasing utilization of so many different types of communications systems.

Two widely employed acronyms of interest in discussing noise effects are RFI (radio frequency interference) and EMC (electromagnetic compatibility). The latter term relates to the process of ensuring that different electronic equipment can coexist in the same environment without unwanted interference between units. It is sufficiently important that some engineers are classified as "EMC engineers."
12-3 Thermal Noise

In basic circuit theory, a resistance is considered as a passive device containing no energy. In reality, however, all resistive devices generate a small level of thermal noise as a result of the random motions of the electrons within the device. This effect is usually insignificant in applications where the signal levels are moderate to large. However, in communication systems, signal levels at the antenna and within the first few stages of a receiver are often of the order of microvolts. Thermal noise may completely overshadow a small signal and render it completely unintelligible.

Noise Properties

Consider the simple resistance $R$ shown in Figure 12-1(a). The waveform of (b) illustrates the random behavior of the small thermal voltage existing across the terminals of the resistance. The properties of the random thermal noise voltage have been studied extensively, both theoretically and practically, and the following properties have been deduced:

1. The \textit{dc value (mean or average value)} of the voltage is zero; i.e.,
   \[ v_{dc} = 0 \]  \hspace{1cm} (12-1)

2. For a given set of operating conditions, the noise voltage may be described by an \textit{rms or effective value} that can be used in describing or characterizing the voltage. This topic will be developed shortly.

3. The power spectrum of the noise before filtering is essentially constant over a wide frequency range encompassing virtually all the range used in conventional RF communications.

4. The voltage is random in nature and its instantaneous behavior can only be described on a statistical basis. We will sidestep the use of statistics within the text, but the interested reader may refer to Appendix A for further details.

Because of the presence of a broad spectrum and the corresponding analogy with white light, thermal noise is frequently referred to as \textit{white noise}.

Mean-Square or RMS Noise

It turns out that the noise voltage generated by a resistance is a function of the resistance, the absolute temperature, and the bandwidth over which the noise is transferred to an external circuit. The bandwidth concept needs some further clarification since the simple model shown in Figure 12-1 shows no bandwidth limiting parameters. However, in the actual transfer of noise power, there will be a finite bandwidth imposed by the receiver, amplifier, or instrument responding to the noise. For the moment, accept this notion, and further clarification of the exact meaning will be provided later.

Let $v_{rms}$ represent the \textit{rms} value of the noise voltage and let $v_{rms}^2$ represent the square of the \textit{rms} value. This latter quantity is known in statistics as the \textit{mean-squared value}. The \textit{mean-squared value} of the noise voltage is given by

\[ v_{rms}^2 = 4RkTB \]  \hspace{1cm} (12-2)

The corresponding \textit{rms} value is then

\[ v_{rms} = \sqrt{v_{rms}^2} = \sqrt{4RkTB} \]  \hspace{1cm} (12-3)
The parameters in these equations are as follows:

\[ \begin{align*}
R & = \text{resistance in ohms} \\
\kappa & = \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ joules per kelvin (J/K)} \\
T & = \text{absolute temperature in kelvins} \\
B & = \text{bandwidth in hertz}
\end{align*} \]

The absolute temperature \( T \) in kelvin is determined from the more familiar celsius temperature \( T_C \) by the addition of 273 K; i.e.

\[ T = T_C + 273 \] (12-4)

It should be stressed that this voltage is the value that would be measured under open-circuit conditions in a bandwidth \( B \); i.e., there is no loading by the external circuit used to measure the voltage. Loading effects will be considered later.

"Standard" Reference Temperature

While the preceding equations hold for any arbitrary temperature \( T \), many noise calculations and measurements utilize a "standard" reference temperature of \( T_0 = 290 \text{ K} \). This value corresponds to 17° C or 62.6° F and is widely used in system noise analysis and measurements. For the remainder of the text, the term "standard temperature" will refer to this value, and the symbol \( T_0 \) will be used.

For \( T = T_0 = 290 \text{ K} \), the product \( kT_0 = 1.38 \times 10^{-23} \text{ J/K} \times 290 \text{ K} = 4 \times 10^{-21} \text{ J} \). With the additional factor of 4 in (12-1), the expression for the mean-square noise voltage at \( T = T_0 \) reduces to

\[ \nu_{\text{rms}}^2 = 16 \times 10^{-21} RB \] (12-5)
Example 12.1

The purpose of this example is to illustrate typical values of the rms noise voltage for different values of resistance at a fixed bandwidth. Assume the bandwidth over which the noise is measured is 1 MHz and that the temperature is \( T_0 \). Determine the mean-square and rms values of voltage for the following values of \( R \): (a) 1 k\( \Omega \), (b) 100 k\( \Omega \), and (c) 10 M\( \Omega \).

Solution

Since \( T = T_0 = 290 \) K, the simplified form of (12-5) will be used.

(a) \( R = 1 \) k\( \Omega \)

\[
v_{rms}^2 = 16 \times 10^{-21} \times 10^3 \times 10^6 = 16 \times 10^{-12} \text{ V}^2
\]

\[
v_{rms} = \sqrt{16 \times 10^{-12}} = 4 \times 10^{-6} \text{ V} = 4 \mu \text{V}
\]

(b) \( R = 100 \) k\( \Omega \)

\[
v_{rms}^2 = 16 \times 10^{-21} \times 10^5 \times 10^6 = 16 \times 10^{-10} \text{ V}^2
\]

\[
v_{rms} = \sqrt{16 \times 10^{-10}} = 4 \times 10^{-5} \text{ V} = 40 \mu \text{V}
\]

(c) \( R = 10 \) M\( \Omega \)

\[
v_{rms}^2 = 16 \times 10^{-21} \times 10^7 \times 10^6 = 16 \times 10^{-8} \text{ V}^2
\]

\[
v_{rms} = \sqrt{16 \times 10^{-8}} = 4 \times 10^{-4} \text{ V} = 400 \mu \text{V}
\]

Before the reader decides to visit an electronics laboratory to attempt to verify some of the preceding results, it should be noted that the measurement of a noise voltage is not a routine process. First, the typical values are quite small and well below the range of most voltmeters except when resistance values and the bandwidth are very large. Second, if one attempts to amplify the noise to enhance the measurement, additional noise and stray pickup are introduced by the amplifier, which makes it difficult to separate the noise produced by the resistor. Third, the appropriate bandwidth parameter \( B \) for the system must be known. Finally, a true rms instrument with a wide bandwidth is required. Special noise-measuring instrumentation systems are available, and some consideration of such systems will be made later.
Example 12.2

Consider the 10-MΩ resistor of Example 12-1(c), and assume that it is connected across the input of an ideal noise-free amplifier with a voltage gain of 5000 and a bandwidth of 1 MHz. Assume that the input impedance of the amplifier is infinite so that there is no loading effect. Determine the rms value of the output noise voltage.

Solution

Let $v_{\text{rms}}$ represent the rms value of the amplifier output voltage, which is determined by multiplying the input rms voltage by the voltage gain of 5000.

$$v_{\text{rms}} = 400\times10^{-6} \times 5000 = 2 \text{ V}$$ \hspace{1cm} (12-12)

where the input voltage is 400 µV as given by (12-11). This voltage is relatively large in comparison with many signal levels and illustrates the potential difficulty with large resistances and wide bandwidths.

This example assumed a infinite input impedance for the amplifier in order to make a point about the noise voltage level. As we will see later, most communication circuits utilize matched impedances at input and output, and it is better to work with power transfer in that case.
In the last section, the open-circuit noise voltage across a resistance was the quantity of interest. Suppose, however, that there is a loading effect produced by another resistance. This leads us to some general models for representing the random noise based on standard circuit theorems.

Thevenin Noise Model

Both Thevenin's and Norton's theorems may be used to develop noise models for a resistance, although it is easier to apply Thevenin's theorem first due to the fact that the noise has been stated so far in terms of voltage. In this case the mean-square open circuit voltage is computed over a hypothetical rectangular bandwidth $B$ as given by (12-2). With this source deenergized, the equivalent impedance is readily determined to be simply the resistance $R$. Thus, the Thevenin equivalent circuit of the resistance plus thermal noise is shown in Figure 12-2(a). For noise analysis, it is best to work with mean-square voltage (rms value squared), so the usual plus and minus signs are irrelevant.

Norton Noise Model

Norton's theorem may now be applied directly to the Thevenin model by measuring the short-circuit current. However, it is best with noise analysis to work with mean-square values, so let $i_{\text{rms}}$ represent the rms noise current, and let $i_{\text{rms}}^2$ represent the mean-square value of the current. Since we are working with squared-quantities, we have

$$i_{\text{rms}}^2 = \frac{v_{\text{rms}}^2}{R^2} = \frac{4RkTB}{R^2} = \frac{4kTB}{R} = 4GTB$$  \hspace{1cm} (12-13)

where $G = 1/R$ is the conductance value in siemens associated with the resistance. The corresponding Norton equivalent circuit is shown in Figure 12-2(b). (Don't confuse the $G$ symbol for conductance with the fact that $G$ is also used to represent power gain. It should be clear in a given case which variable is under consideration.)

Addition of Noise Voltages

The two equivalent circuits developed in this section can be used to simplify noise computations in circuits containing several resistors. One important rule should be remembered when combining the effects of sources contained in more than one resistor. The rule is that the net mean-square (or power) effect produced by more than one independent noise source is obtained by adding individual mean-square (or power) effects. This concept is based on the fact that the random voltages produced by the individual resistances have no dependency between them. (In statistical terms, they are said to be statistically independent.)

For example, suppose that the rms noise voltages produced by two resistances are 3 V and 4 V, respectively. If the resistances are connected in series, one might be led to believe that the net voltage would be 7 V, but that is not the case. Instead, the squared values are added; i.e., $(3)^2 = 9$ $V^2$ is added to $(4)^2 = 16$ $V^2$ to yield 25 $V^2$. Taking the square root, the net rms voltage is 5 V.

The process just described is the same as employed in circuit theory for determining the net rms value of two sine waves at different frequencies. This is the main reason why in noise analysis it is easier to work with mean-square or power values since they add directly.
Combining Resistances

To illustrate some of the preceding concepts, consider the simple series connection of two resistors, both at the same temperature $T$, as shown in Figure 12-3(a). The two resistors can be represented by their Thevenin models as shown in Figure 9-3(b). In view of the simple series connection, both the respective resistances and the mean-square voltages can be added. Thus, the net resistance $R$ is given by

$$R = R_1 + R_2$$

(12-14)

The net mean-square voltage $v_{rms}^2$ is expressed in terms of the separate mean-square voltages as

$$v_{rms}^2 = v_{rms1}^2 + v_{rms2}^2 = 4R_1kTB + 4R_2kTB = 4(R_1 + R_2)kTB = 4RkTB$$

(12-15)

A resulting equivalent circuit is shown in Figure 12-3(c).

Several comments about this development will now be made. First, note in (12-15) that the combined effect of the sources was obtained by adding mean-square values as previously discussed. Polarity is not important for this purpose since the mean-square values are all positive. Next, note that the net effective resistance is simply the sum of the individual resistances as noted in (12-15) and that is the equivalent resistance of two resistors in series. While this circuit represents a single simple example, it turns out that the concept is more general and can be stated as follows: The net thermal noise effect of any arbitrary combination of simple resistances all at the same temperature is the same as that of a single resistance whose value is the equivalent resistance of the combination at the reference terminals of interest.

This concept reduces considerably the amount of effort involved with analyzing thermal noise in circuits containing several resistors. If the temperatures are different, it is necessary to utilize the Thevenin and/or Norton models and perform a more complex analysis.
Example 12.3

Consider the resistance combination shown in Figure 12-4(a) and assume that all resistors have a temperature $T = T_0$. Determine the net rms voltage across the terminals $A-A'$ over a bandwidth of 2 MHz.

Solution

The first step in the problem is an exercise in simple dc circuit analysis consisting of determining the equivalent resistance at the terminals. First, the parallel combination of 4 MΩ and 12 MΩ is determined as 3 MΩ. Next this value is series combined with 12 MΩ to yield 15 MΩ. This value is parallel combined with 10 MΩ to yield 6 MΩ. The net mean-square noise is then

$$
u_{rms}^2 = 16 \times 10^{-21} R_{eq} B = 16 \times 10^{-21} \times 6 \times 10^6 \times 2 \times 10^6 = 192 \times 10^{-9} \text{ V}^2 \quad (12-16)$$

The rms voltage is then

$$v_{rms} = \sqrt{192 \times 10^{-9}} = 438.2 \mu\text{V} \quad (12-17)$$
12-5 Noise Power

The emphasis on noise analysis up to this point has been on noise voltage and current since these are the variables of common familiarity. However, most communication system components and subsystems are designed around the concept of maximum power transfer. Not only does this usually ensure maximum signal strength, but it also serves to minimize reflections produced by interconnecting sections of transmission lines. Beginning in this section and continuing throughout the chapter, the focus will be primarily aimed at the process of signal and noise power transfer as opposed to voltage and current considerations.

Available Power

All practical signal sources have an internal impedance that will limit the amount of power that can be extracted from the source. Consider the source of Figure 12-5 in which the Thevenin equivalent circuit is a voltage source with rms value \( V_{rms} \) in series with a resistive impedance \( R_s \). It is important to realize in this model that \( R_s \) is internal and cannot be changed or eliminated. Under this constraint, an external load resistance \( R_L \) is connected to the terminals and adjusted for maximum power transfer to the load.

From the maximum power transfer theorem, maximum power is delivered to \( R_L \) when its value is equal to the internal source resistance; i.e., when \( R_L = R_s \). For this condition, the current around the loop is

\[
I = \frac{V_{rms}}{R_s + R_L} = \frac{V_{rms}}{R_s + R_s} = \frac{V_{rms}}{2R_s}
\]

(12-18)

The power \( P_L \) in the load resistance \( R_L \) is

\[
P_L = I^2 R_L = \left( \frac{V_{rms}}{2R_s} \right)^2 R_L = \left( \frac{V_{rms}}{2R_s} \right)^2 R_s = \frac{V_{rms}^2}{4R_s}
\]

(12-19)

This value of power is the maximum amount that can be extracted from the source into an external load and is referred to as the maximum available power \( P_{av} \). Hence,

\[
P_{av} = \frac{V_{rms}^2}{4R_s}
\]

(12-20)

This value is the maximum average power that can be extracted from the source into an external load, and that power will be delivered to the load only when the load resistance is matched to the internal resistance of the source. It should be noted that there is additional power dissipated in the internal resistance which, at the point of maximum power transfer, is equal to the power delivered to the external load.

Available Noise Power

Consider now a resistance \( R \) at temperature \( T \) and its equivalent noise circuit as shown in Figure 12-6. Assume that an external load \( R_L \) is connected to \( R \) as shown. To simplify the development, assume that the external resistance is noise-free, which is equivalent to it having an absolute temperature of 0 K.

When \( R_L \) is connected to \( R \), some of the available noise power from \( R \) is transferred to \( R_L \). Maximum power will be transferred when \( R_L = R \). Let \( N_{av} \) represent the available noise power, and it is
\[ N_{av} = \frac{v_{rms}^2}{4R} = \frac{4RkTB}{4R} = kTB \] (12-21)

This result is very interesting in that the available noise power is completely independent of the value of the resistance! Thus, while the noise voltage and noise current are both dependent on the resistance, the available noise power is independent of the resistance. Moreover, this available power will be delivered to the load resistance if the load resistance is equal to \( R \).

For \( T = T_0 = 290 \) K, (12-21) may be expressed as

\[ N_{av} = kT_0B = 4 \times 10^{-21} B \] (12-22)

If the resistance \( R_L \) is not noise-free, it will transfer a certain amount of noise power back to \( R \). In fact, when the resistances are equal and their temperatures are the same, each delivers an equal amount of power to the other.

A result of the preceding development is that the analysis of noise transfer through a system is greatly simplified when all junctions in the system are matched. Instead of having to work with the somewhat clumsy forms of mean-square voltage and current, one can simply compute the available power \( kTB \) and assume that value as the basis for noise power transfer.

In many of the subsequent developments, matched conditions will be assumed and stated. Such conditions are widely assumed by communication engineers and technologists since this is part of the design strategy.

**Power Gain**

The power gain \( G \) of a system could be defined as

\[ G = \frac{\text{output power}}{\text{input power}} \] (12-23)

There are several variations on power gain definitions, depending primarily on whether the input and output ports are terminated in matched impedances. To simplify the development in this text, we will assume, unless otherwise stated, that both input and output ports are terminated correctly for maximum power transfer. This means that the available power from a source will be delivered to the input of the system and the available output power from the system is delivered to the load. As we will see, this assumption greatly simplifies the analysis of the system, and it is usually a condition around which much of the design is aimed. Unless otherwise stated, the symbol \( G \) with appropriate subscripts will denote a power gain based on both input and output ports matched for maximum power transfer.

Let \( P_i \) represent the input power and let \( P_o \) represent the output power. The output power is

\[ P_o = GP_i \] (12-24)

where both power values represent maximum values based on the assumed conditions. When the input is a noise source with temperature \( T \), the output noise power \( N_o \) is then

\[ N_o = GN_{av} = GkTB \] (12-25)

where \( B \) is the bandwidth of the system.
**Example 12-4**

Determine the available noise power contained in a simple 50-Ω resistance at the standard reference temperature in a bandwidth of 2 MHz.

**Solution**

Since the temperature is at the standard reference level, the form of (12-22) will be used.

\[
N_{av} = kT_0B = 4 \times 10^{-21} \times 2 \times 10^6 = 8 \times 10^{-15} \text{ W} = 8 \text{ fW}
\]  

(12-26)

Note that it was not necessary to specify the resistance value for the determination of available power, but the resistance value will be important in a later example.

**Example 12-5**

The resistance of Example 12-4 is connected across the input of an ideal noise-free amplifier whose input and output impedances are resistive value of 50 Ω. The amplifier has a matched gain of 60 dB and a bandwidth of 2 MHz. Determine the output noise power in a 50-Ù load resistance

**Solution**

The amplifier is obviously matched at both input and output. The decibel gain of 60 dB corresponds to an absolute power gain of \(10^6\). The output noise power \(N_o\) is

\[
N_o = GN_{av} = GkT_0B = 10^6 \times 8 \times 10^{-15} = 8 \times 10^{-9} \text{ W} = 8 \text{ nW}
\]  

(12-27)

where the result of (12-26) was used as the available input power

**Example 12-6**

Determine the rms noise voltage across the load resistance in the amplifier of Example 12-5.

**Solution**

One might be initially tempted to return to the basic formula of (12-3) but note that what is desired is the voltage across the load resistance resulting from the input source. Moreover, (12-3) is based on an open-circuit voltage, while the present situation involves a terminated resistance.

The desired result can be achieved by reverting back to basic circuit analysis and expressing the power in the load in terms of the rms voltage across the load and the load resistance. We have

\[
N_o = 8 \times 10^{-9} = \frac{v_{rms}^2}{50}
\]  

(12-28)

Solving for \(v_{rms}\), we obtain

\[
v_{rms} = 632 \mu \text{V}
\]  

(12-29)
12-6. **Power Spectrum Concepts**

In all computations made thus far, a bandwidth $B$ has been assumed. In practice, the noise spectrum will be altered by the frequency response of the system, and the value of the bandwidth used for noise purposes may not be immediately evident. In this section, this phenomenon will be investigated, and the process by which an equivalent noise bandwidth is established will be described.

**Power Spectral Density**

The power spectral density is based on a spectral representation of the power and is defined at any frequency as the power per unit bandwidth measured in watts per hertz (W/Hz). It is possible to use either a one-sided or a two-sided spectral form. However, the treatment here will utilize the one-sided form since it relates more to the practical interpretation of the concept for our purposes.

In general, the power spectral density is a frequency dependent function and will be denoted as $S(f)$ with appropriate subscripts added as needed. For the case of a simple resistance with available noise power $kTB$, the power spectral density is easily obtained by dividing by $B$, and the resulting value will be denoted as $\eta$. Hence, for a simple resistance

$$S(f) = \eta = kT \quad \text{W/Hz} \quad (12-30)$$

as shown in Figure 12-7. At the reference temperature $T = T_0 = 290$ K, this value is

$$S(f) = \eta_0 = kT_0 = 4 \times 10^{-21} \quad \text{W/Hz} \quad (12-31)$$

where additional subscripts have been added for clarity.

In some cases, it is desirable to treat power spectral density functions as the input and output variables for an amplifier or other linear system. Let $S_i(f)$ represent the input power spectral density and let $S_o(f)$ represent the corresponding output density. For a constant power gain $G$, these functions are related in exactly the same fashion as the power values; i.e.

$$S_o(f) = GS_i(f) \quad (12-32)$$

One situation in which spectral density functions are appropriate is when the bandwidths of different stages have different values. In such situations, not all of the available power at the input is amplified and delivered to the load. However, the spectral density functions may still serve as a meaningful way to relate output to input.

**Equivalent Noise Bandwidth**

For non-ideal filters, a term called the *equivalent noise bandwidth* is used to predict the output noise power produced by a flat input spectrum. Assume a non-ideal amplitude response $A(f) = |H(f)|$ and let $A_o^2(f) = |H_o(f)|^2$ as shown in Figure 12-8. Let $A_o^2$ represent the maximum level of the amplitude-squared response, which for the case shown, is at dc.

A fictitious ideal block characteristic having the same maximum level as the actual filter amplitude response is sketched on the same scale as the actual filter response. The *equivalent noise bandwidth* $B_N$ is
the bandwidth of the ideal block response that would produce the same output noise power as the real filter. It can be shown that this value is

\[ B_N = \frac{1}{A_N^2} \int_0^\infty A^2(f)df \] (12-33)

This is the quantity that has been denoted simply as \( B \) in all computations involving noise power up to this point. To keep the notation as simple as possible, we will continue to use \( B \), but it should be understood that it is the noise bandwidth that is being used for noise computations.

An obvious question is how does \( B_N \) compare with the usual bandwidth parameter for a given amplifier or other linear system? For circuits with a low to moderate rate of cutoff in the amplitude response, the equivalent noise bandwidth may be much greater than, say, the 3-dB bandwidth. As the order of the filter increases, the equivalent noise bandwidth becomes much closer to the usual bandwidth value. In the theoretical limit of an ideal block amplitude response, the two values are equal.

An abbreviated set of noise bandwidth values for low-pass Butterworth filters up to 10 poles is provided in Table 12-1. In each case \( f_c \) is the 3-dB bandwidth.

**Noise Temperature**

We have seen that the available noise power density for a resistive source can be expressed simply as \( kT \). In the context of a resistance \( R \), the temperature \( T \) is the physical temperature. The fact is that there are many noise sources that are not simple resistances. An antenna, for example, may absorb different forms of radiation within its field pattern and produce an output noise much greater than the value predicted by the \( kT \) factor based on physical temperature alone. Likewise, an antenna pointed out into deep space may absorb an equivalent noise near zero.

It has been customary in the communications field to represent noise sources by an *equivalent noise temperature* that may or may not represent a true physical temperature. For any noise source that produces a flat noise power spectrum \( \eta_s \), an equivalent noise source temperature \( T_s \) may be defined as

\[ T_s = \frac{\eta_s}{k} \] (12-34)

Once \( T_s \) is known, the available noise power density may always be computed simply as

\[ \eta_s = kT_s \] (12-35)
Example 12-7

A simple 50-Ω resistance at a temperature of 290 K is connected across the 50-Ω input of a noise-free amplifier having a matched gain of 80 dB. Determine the power spectral density at the output.

Solution

The gain of 80 dB corresponds to an absolute power gain of $10^8$. From (12-31), the input power spectral density is simply $4 \times 10^{-21}$ W/Hz. The output spectral density is then

$$S_o(f) = GkT_0 = 10^8 \times 4 \times 10^{-21} = 4 \times 10^{-13} \text{ W/Hz} = 400 \text{ fW/Hz}$$

Example 12-8

A noise generator produces white noise having a power spectral density of $6 \times 10^{-3}$ fW / Hz. (a) Determine the equivalent source noise temperature. (b) If the generator is connected as a matched input to a noise-free amplifier having a flat gain of 43 dB over a wide band, determine the output power spectral density over the flat region. (c) If the noise bandwidth of the amplifier is 12 MHz, determine the output noise power due to the source.

Solution

(a) The equivalent source temperature is determined from (12-34) as

$$T_s = \frac{\eta_e}{k} = \frac{6 \times 10^{-18}}{1.38 \times 10^{-23}} = 434.8 \times 10^3 \text{ K} = 434,800 \text{ K}$$

(b) The absolute power gain of the amplifier is $20 \times 10^3$, and the output power spectral density over the flat region is

$$S_o(f) = \eta_o = GS_o(f) = 20 \times 10^3 \times 6 \times 10^{-3} = 120 \text{ fW/Hz}$$

(c) The total output noise power $N_o$ in a bandwidth of 12 MHz is

$$N_o = \eta_o B = 120 \times 10^{-15} \times 12 \times 10^6 = 1.44 \times 10^{-6} \text{ W} = 1.44 \mu\text{W}$$
Example 12.9

Consider the system shown in Figure 12-9 and assume that impedances at all junctions are matched. The input is a thermal noise source having a one-sided power spectral density of 1 pW/Hz. Assume that any internal noise is negligible in comparison to the noise produced by the source. The bandwidths are all low-pass in nature and have different values for the three stages. Determine the output noise power.

Solution

The net decibel gain $G_{\text{net}}$ is readily determined as

$$G_{\text{net}} = 10 + 15 + 25 = 50 \text{ dB}$$

(12-40)

This corresponds to an absolute power gain $G = 10^5$. The bandwidths of the three stages are different but are all low-pass in nature. Therefore, the smallest bandwidth, i.e. 10 kHz, determines the effective bandwidth for the noise delivered to the output. Thus,

$$N_o = \eta GB = 10^5 \times 10^{-12} \times 10^4 = 1 \text{ mW}$$

(12-41)
12-7 Models for Internally Generated Noise

All of the noise sources within an amplifier or other subsystem can be collectively specified as a single noise parameter, which can be used to predict the noise performance of the system. The two most widely used approaches for specifying the total noise are the (1) the noise temperature method and (2) the noise figure method. Often, specifications are given that utilize both parameters so it is necessary to investigate both methods. The author has a preference for the noise temperature approach, and it will be considered first.

Effective Noise Temperature Method

Consider the block diagram of an amplifier as shown in Figure 12-10 having an equivalent noise bandwidth $B$. Assume that both input and output ports are matched for maximum power transfer and that the power gain is $G$. Assume that the resistance across the input port has a temperature $T_i$.

The total output noise power $N_o$ can be considered as the sum of two terms.

$$N_o = N_{o1} + N_{o2}$$

(12-42)

The first term $N_{o1}$ represents the noise resulting from the input source and the second term $N_{o2}$ represents the noise arising within the amplifier. A technique that is widely used to specify this noise is to define a fictitious effective temperature $T_e$ at the input to the amplifier that would produce the noise power arising within the amplifier. Such a temperature then would then satisfy the equation

$$N_{o2} = GkT_e B$$

(12-43)

The total output noise power is then

$$N_o = GkT_i B + GkT_e B$$

(12-44a)

$$= Gk(T_i + T_e) B$$

(12-44b)

$$= GkT_{sys} B$$

(12-44c)

where

$$T_{sys} = T_i + T_e$$

(12-45)

is called the system temperature. It is also called the operating temperature in some references, in which case the symbol $T_{op}$ may be used.

The process of determining the output noise power for an amplifier can be summarized as follows:

(1) The system temperature is determined by adding the input source noise temperature to the amplifier effective noise temperature referred to the input.

(2) The system temperature is treated in the same manner as the physical temperature of a resistance at the input (although it will often be drastically different than any real physical temperature). Thus, the equivalent available noise power at the input $kT_{sys} B$ is multiplied by the power gain $G$ of the amplifier to determine the output noise power.
It should be stressed again that the effective noise temperature of the amplifier is not the physical temperature of the unit. It is simply a number that is treated like a physical temperature for the purpose of computing noise power. In fact, even the source temperature \( T_i \) need not correspond to a physical temperature. Source and amplifier noise temperatures vary from a few kelvin to thousands of kelvin.

**Noise Figure Method**

Next, we consider the concept of the *noise figure* \( F \) (also called the *noise factor*). There are a number of variations in the literature on the definition of noise figure depending on whether it is defined on a frequency dependent (or "spot") basis or on an overall average basis. In accordance with most manufacturers' specifications, we will consider only the average basis representation.

Refer to Figure 12-11, which is very similar to Figure 12-10 except that a few additional quantities are labeled on this figure. Along with the noise source input, a signal input source is also assumed. The available power corresponding to the signal input source is denoted as \( P_i \), and the available signal output power is denoted as \( P_o \). The output noise power \( N_o \) is the same as with the noise temperature development, and \( N_i = kT_iB \) is the input available noise source power defined over the equivalent noise bandwidth.

The following signal-to-noise ratios are defined:

\[
(S/N)_{input} = \frac{P_i}{N_i} = \text{signal-to-noise power ratio of input source} \quad (12-46)
\]

\[
(S/N)_{output} = \frac{P_o}{N_o} = \text{signal-to-noise power ratio of output} \quad (12-47)
\]

The *noise figure* \( F \) is then defined as

\[
F = \left. \frac{(S/N)_{input}}{(S/N)_{output}} \right|_{T_i = T_0} \quad (12-48)
\]

Stated in words, the *noise figure is the input source signal-to-noise ratio divided by the output signal-to-noise ratio determined with the input at the standard reference temperature of \( T_0 = 290 \text{ K} \).

The noise figure is often stated in decibels and the value \( F_{\text{db}} \) is given by

\[
F_{\text{db}} = 10 \log F \quad (12-49)
\]

Since both input and output power levels are measured over the same reference bandwidth, the smallest possible value of the noise figure is \( F = 1 \) or \( F_{\text{db}} = 0 \text{ dB} \) and this would correspond to a noise-free device. In general, the larger the value of the noise figure (either absolute or in dB), the more the degradation imposed by the unit.

**Use of Noise Figure at Reference Temperature**
Assume for the moment that the input signal source has the reference temperature \( T_i = T_0 \). In this case, (12-48) may be rearranged as

\[
(S / N)_{\text{output}} = \frac{(S / N)_{\text{input}}}{F} \tag{12-50}
\]

The decibel form of (12-50) is obtained by taking the logarithms of both sides and multiplying by 10. This leads to

\[
(S / N)_{\text{output, dB}} = (S / N)_{\text{input, dB}} - F_{\text{dB}} \tag{12-51}
\]

**Misuse of Noise Figure in Literature**

Both (12-50) and (12-51) are easy to apply and they clearly illustrate the degradation imposed by the noise figure. Unfortunately, however, they are often misused. Since the noise figure is defined at the standard reference temperature, these equations are correct only when the input source temperature is equal to the standard reference temperature. Throughout the literature, there are many examples in which (12-50) or (12-51) are used at different source temperatures and this has led to a lot of confusion.

Provided that the temperature is not too different from the reference temperature, (12-50) and (12-51) may be used as reasonable approximations. However, the potential error that can arise when the source temperature is drastically different from the standard reference temperature is the primary reason that the author prefers the noise temperature approach.

**Relationship Between Noise Temperature and Noise Figure**

We will now develop a relationship between the noise temperature and the noise figure such that if one is known, the other may be determined. Start with (12-48), insert the definitions of (12-46) and (12-47), and rearrange as follows:

\[
F = \frac{P_i}{P_o} \frac{N_o}{N_i} \bigg|_{T_i = T_0} \frac{N_o}{GkT_0B} = \frac{N_o}{GkT_0B} \tag{12-52}
\]

where the ratio \( P_o / P_i = G \) and the value \( T_i = T_0 \) have been substituted. Next, substitute the expression of (12-44b) for \( N_o \) in (12-52). This results in

\[
F = \frac{GkT_0B + GkT_oB}{GkT_0B} \tag{12-53}
\]

After cancellation of the surplus common factors, we obtain

\[
F = \frac{T_o + T_c}{T_0} = 1 + \frac{T_c}{T_0} = 1 + \frac{T_c}{290} \tag{12-54}
\]

The inverse relationship is determined by solving for \( T_c \) in terms of \( F \).

\[
T_c = (F - 1)T_0 = (F - 1) \times 290 \tag{12-55}
\]
The preceding two relationships permit conversion from noise temperature to noise figure and vice-versa.
Example 12-10

A low-noise amplifier has an effective noise temperature of 50 K. Determine the (a) absolute noise figure and (b) decibel noise figure.

Solution

(a) To convert noise temperature to noise figure, we use (12-54).

\[
F = \frac{T_0 + T_e}{T_0} = 1 + \frac{T_e}{T_0} = 1 + \frac{50}{290} = 1.172
\]

(12-56)

(b) The decibel value is

\[
F_{\text{db}} = 10 \log F = 10 \log 1.172 = 0.689 \text{ dB}
\]

(12-57)

Example 12-11

An amplifier has a specified noise figure of 5 dB. Determine the effective noise temperature referred to the input.

Solution

We must first convert the decibel noise figure to the absolute value. We have

\[
F = 10^{F_{\text{db}}/10} = 10^{5/10} = 10^{0.5} = 3.162
\]

(12-58)

To convert from absolute noise figure to noise temperature, we use (12-55).

\[
T_e = (F - 1)T_0 = (3.162 - 1) \times 290 = (3.162 - 1) \times 290 = 627 \text{ K}
\]

(12-59)
Example 12-12

An RF amplifier has a matched gain of 50 dB, a noise figure of 9.03 dB, and an equivalent noise bandwidth of 2 MHz. The input signal level is 8 pW and the input source effective noise temperature is $T_i = T_0 = 290$ K. Using the noise temperature approach, determine the (a) input source noise power, (b) input signal-to-noise ratio, (c) output signal power, (d) output noise power, and (e) output signal-to-noise ratio.

Solution

As is often the case with practical problems, the values are given in decibels, and they must be converted to absolute quantities before proceeding. The given decibel and absolute values are determined as follows:

<table>
<thead>
<tr>
<th>Decibel Value</th>
<th>Absolute Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplifier Gain</td>
<td>50 dB</td>
</tr>
<tr>
<td>Amplifier Noise Figure</td>
<td>9.03 dB</td>
</tr>
</tbody>
</table>

To use the noise temperature approach, we first determine the effective noise temperature of the amplifier.

$$T_e = (F - 1) \times 290 = (8 - 1) \times 290 = 2030 \text{ K} \quad (12-60)$$

(a) The input signal source has a temperature equal to the standard reference temperature of 290 K, in which case the available noise power is

$$N_i = kT_B = kT_0B = 4 \times 10^{-21} \times 2 \times 10^6 = 8 \times 10^{-15} \text{ W} = 8 \text{ fW} \quad (12-61)$$

(b) The input source signal-to-noise ratio is

$$\frac{S}{N}_{\text{input}} = \frac{P_i}{N_i} = \frac{8 \times 10^{-12}}{8 \times 10^{-15}} = 1000 \quad (12-62)$$

The corresponding decibel value is

$$\left( \frac{S}{N} \right)_{\text{input, dB}} = 10 \log(1000) = 30 \text{ dB} \quad (12-63)$$

(c) The output signal power is given by

$$P_o = GP_i = 10^5 \times 8 \times 10^{-12} = 800 \text{ nW} \quad (12-64)$$

(d) The output noise power is given by

$$N_o = GkT_{sys}B = Gk(T_i + T_e) = 10^5 \times 1.38 \times 10^{-23} \times (290 + 2030) \times 2 \times 10^6 = 6.403 \text{ nW} \quad (12-65)$$

(e) The output signal-to-noise ratio is then

$$\left( \frac{S}{N} \right)_{\text{output}} = \frac{800 \text{ nW}}{6.403 \text{ nW}} = 124.9 \quad (12-66)$$

The corresponding dB value is

$$\left( \frac{S}{N} \right)_{\text{output, dB}} = 10 \log(124.9) = 20.97 \text{ dB} \quad (12-67)$$
The signal-to-noise ratio is thus degraded by about 9 dB.

Example 12-13

For the system of Example 12-12, use the noise figure approach to predict the output signal-to-noise ratio.

Solution

Many of the results obtained in Example 12-12 will be used in this analysis. The input source signal-to-noise ratio was determined as 1000 on an absolute scale or 30 dB on a decibel scale. The output signal-to-noise ratio can then be determined most easily by (12-51). This relationship yields

\[
(S/N)_{\text{output,dB}} = (S/N)_{\text{input,dB}} - F_{\text{dB}} = 30 - 8.03 = 21.97 \text{ dB}
\]

(12-68)

This result is exactly the same as obtained using the noise temperature approach, and this is a result of the fact that the source temperature is exactly the same as used in the noise figure definition. If the input source noise temperature is different than 290 K, the only truly correct approach is the noise temperature method. In practice, however, the method illustrated in (12-68) is used casually because of its simplicity and it yields approximate results whenever the source temperature is reasonably close to 290 K.
In the previous section, the concepts of effective noise temperature and noise figure were established as a basis for representing the noise within a single amplifier. In this section the concept will be extended to include any number of individual amplifiers (or possibly attenuators) connected in cascade, each of which has an individual noise temperature or figure. The ultimate goal is to determine a net noise temperature or figure that applies to the whole cascaded system so that it may be treated in the same manner as a single unit.

Cascade of Three Units

The concept will be developed for the case of a system with three cascaded units. This is sufficiently large to permit the concept to be generalized, and yet it is small enough to permit a straightforward solution. Consider then the system shown in Figure 12-12 containing three amplifiers and an input noise source with temperature $T_i$. It is assumed that the three effective noise temperatures of the individual stages are $T_{e1}$, $T_{e2}$, and $T_{e3}$, respectively. The corresponding matched power gains are $G_1$, $G_2$, and $G_3$. The net gain from input to output is

$$G = G_1 G_2 G_3$$  \hfill (12-69)

The actual noise output $N_o$ can be represented as the sum of several separate noise effects as follows: (1) the input noise source amplified by the total gain $G$; (2) the effective noise produced by the first amplifier referred back to its input amplified by the total gain $G$; (3) the effective noise produced by the second stage referred back to its input multiplied by the gain of the last two stages $G_2 G_3$; and (4) the effective noise of the last stage referred back to its input multiplied by the gain of the last stage $G_3$. In the same order as just listed, the output noise can be expressed as

$$N_o = G_1 G_2 G_3 k T_i B + G_1 G_2 k T_{e1} B + G_2 G_3 k T_{e2} B + G_3 k T_{e3} B$$  \hfill (12-70)

where the definition of (12-69) is used in the first two terms on the right.

Effective Input Noise Temperature

We now define an effective noise temperature $T_e$ for the cascade that will produce the noise effect for the system with gain $G$. This effective noise temperature must be the value which, when added to the source temperature at the input, must produce the effect at the output as given by (12-70). The equation will read

$$N_o = G k T_{sys} B = G_1 G_2 G_3 k (T_i + T_e) B$$  \hfill (12-71)

Equating (12-70) to (12-71) and canceling common factors, we obtain

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2}$$  \hfill (12-72)

This result is a single effective noise temperature referred to the input of the first stage that, in combination with the input source noise temperature, can be considered as the net system temperature for predicting the output noise.

The form of (12-72) can be readily generalized to the case of an arbitrary number of stages in cascade. The first term is the noise temperature of the input stage. The second term is the noise temperature of the second stage divided by the gain up to that input, which is the gain of the first stage. The third term is the
noise temperature of the third stage divided by the gain up to that input, which is the product of the gains of the first two stages. In general, the additive term for any stage is the noise temperature of that stage divided by the product of all preceding gains up to, but not including, that particular stage.

**Most Critical Stages**

For the moment, assume that all values of gain are greater than unity (which is not always the case). From (12-72), it is apparent that the effects of terms farther to the right are less significant since the temperature values are being divided by successively increasing gain factors. This means that in a cascade of amplifier stages, the input stage is usually the most important one in establishing the noise temperature (or noise figure) for the system. In fact, if all the gain factors (especially $G_1$) are very large, the total noise temperature may be only slightly higher than that of the first stage. Therefore, the first stage of a receiver should be selected to have the combination of a low noise temperature (or noise figure) and a high gain whenever feasible.

It should be noted that any unit that provides a loss, such as a transmission line, attenuator, or passive mixer, is treated as a gain less than one, and this tends to increase the noise temperature. This effect will be considered shortly.

**Combined Noise Figure**

Since a combined effective noise temperature can be obtained from the cascaded system, it is also possible to obtain a combined noise figure from the individual noise figures. The most direct way to accomplish this is to take (12-72) and substitute for each noise temperature the corresponding expression for the noise figure. Let $F_1$, $F_2$, and $F_3$ represent the three noise figures corresponding to $T_{e1}$, $T_{e2}$, and $T_{e3}$, respectively. Using (12-55) as the basis in each case, we have

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$  \hspace{1cm} (12-74)

A form for noise figure similar to (12-72) for the noise temperature is obtained. However, it is noted that all $F$ terms on the right except the $F_1$ term have unity subtracted from the value. There is an interesting reason for this, and it is based on the fact that the noise figure definition includes both the source noise and the amplifier noise in its definition. Since the source is applied only to the first stage, all noise figures except $F_1$ have to be reduced to compensate.

**Noise Temperature of Matched Attenuator**

An important topic of consideration is the effect of an attenuation element on the overall noise. For the purpose of this development, the term **matched attenuation network** will include any network having an input and an output that satisfies the following requirements: (1) Impedances are matched both at the input and the output. (2) The network is *passive* in the conventional sense that the only source of energy within the network is that produced by thermal noise effects. (3) Some of the power delivered to the input is dissipated within the network so that the power delivered to the load from the source is less than the power delivered to the input.
A block diagram of such a system is shown in Figure 12-13. The most common element that produces this type of effect in a receiver is a transmission line (or waveguide at microwave frequencies). A passive transmission line is required to couple between an antenna and a receiver, and some distance physically separates the units, which introduces losses into the system. Within a receiver, many mixers introduce losses and must be treated in the same manner as attenuators.

The most convenient way to represent the losses in a matched attenuator is through the use of an insertion loss factor $L$. Let $P_i$ represent the power delivered to the input and let $P_o$ represent the power delivered to the output. The relationship between these two values is

$$P_o = \frac{P_i}{L} \quad (12-75)$$

The factor $L$ for a true attenuator must satisfy $L \geq 1$, with the lower bound corresponding to no attenuation at all, and the upper bound (infinity) corresponding to complete absorption of the signal in the network. The factor $L$ is often given as a decibel value $L_{dB}$, where

$$L_{dB} = 10 \log L \quad (12-76)$$

The factor $1/L$ for a matched attenuator is treated in much the same way as available gain $G$ for an amplifier. This association will help in some of the results that occur later.

Now consider the situation depicted in Figure 12-14 with the following conditions imposed: (1) The attenuator has a constant physical temperature $T_p$, which means that all lossy elements within the network possess that temperature. (2) A noise source with an effective source temperature $T_i$ is connected to the input. Assume for the purpose of this development that $T_i \geq T_p$, although the results apply in general.

If the input noise source were at the same temperature as the network, the effective output noise would simply be $kT_p B$. However, since the input noise source has a greater temperature, there is an excess noise temperature $T_i - T_p$ supplied to the input and it is reduced by the factor $1/L$. Hence, the net output noise is given by

$$N_o = kT_p B + \frac{k(T_i - T_p)B}{L} \quad (12-77)$$

This equation may also be expressed as

$$N_o = \frac{kTB}{L} + \left(1 - \frac{1}{L}\right)kT_p B \quad (12-78)$$

This latter form displays the relative effects of the input temperature and the physical temperature as a function of the loss. While the assumption was made that the source temperature was greater than the physical temperature for convenience in the development, the results applies for opposite inequality as well.

An effective temperature $T_e$ referred to the input may be defined such that
\[ N_o = \frac{1}{L} k(T_i + T_e)B \]  

which is the standard form for evaluating the output noise power in terms of the effective temperature with the "gain" set as $1/L$. Equating (12-79) and (12-78), we obtain

\[ T_e = (L - 1)T_p \]  

These results indicate that a matched attenuator can be handled in the same way as an amplifier by defining an effective noise temperature at the input. Note that $T_e$ increases linearly with changes in $L$. This means that the greater the attenuation, the greater the effective noise temperature resulting from the attenuation.

**Noise Figure of Attenuator**

The noise figure $F$ corresponding to (12-80) is obtained from the application of (12-54). The result is

\[ F = 1 + (L - 1) \frac{T_p}{T_0} = 1 + (L - 1) \frac{T_p}{290} \]

An interesting special case occurs when $T_p = T_0$. In this case (12-77) reduces to

\[ F = L \]

*For the special case when the attenuator is at the standard temperature of 290 K, the noise figure is equal to the insertion power loss factor.* Because of the simplicity of this relationship and the fact that in many systems the actual physical temperatures of lossy elements may be reasonably close to the standard reference temperature, this result is often used as an estimate of the noise figure for a variety of cases.

The noise temperature associated with a loss increases as the loss factor increases. Thus, it is very desirable in any receiving system to have a high-gain, low-noise amplifier as close to the front-end as possible.

**Choice of Reference Points**

Assume that there is a loss associated with the input to a receiver, such as might be encountered in the transmission line connecting the antenna to the receiver. There are two approaches to dealing with the problem: (1) The effective input temperature may be referred all the way back to the transmission line input as the reference point using the effective noise temperature of the transmission line and treating the line with loss $L$ as if it were a gain of value $1/L$. (2) The receiver input may be used as the reference point, in which case the effective noise temperature is that of the receiver alone, but the output noise of the transmission line is considered as a linear combination of the antenna source temperature and the ohmic losses of the transmission line as indicated by (12-78). Both approaches are equally valid and both will be encountered later in subsequent sections of the text.
**Example 12-14**

For the system of Figure 12-15, determine the equivalent noise temperature referred to the input.

**Solution**

Note that all the gains are given in absolute form so we do not need to convert from decibel values in this example. Since there are three stages, the form of (12-72) may be directly applied.

\[
T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} = 100 + \frac{200}{20} + \frac{300}{20 \times 15}
\]

\[
= 100 + 10 + 1 = 111 \text{ K}
\]

(12-83)

This problem was obviously "rigged" to produce simple values, but the pattern should be of educational value. These amplifier gains are relatively low. Even so, it is clear that the most dominating noise temperature of the first stage. Moreover, the effects of the noise temperatures diminish as we move from the input stage to the output stage.

**Example 12-15**

Rework Example 12-14 by first determining the noise figures for the individual stages and then determining the net noise figure.

**Solution**

The noise figure for each stage may be determined from the relationship

\[
F = 1 + \frac{T_e}{290}
\]

(12-84)

The values obtained are readily determined as \( F_1 = 1.345 \), \( F_2 = 1.690 \), and \( F_3 = 2.034 \).

\[
F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 1.345 + \frac{1.690 - 1}{20} + \frac{2.034 - 1}{20 \times 15}
\]

\[
= 1.3448 + 0.0345 + 0.0034 = 1.3827
\]

(12-85)

As a check, the noise temperature may be determined from this value as

\[
T_e = (F - 1) \times 290 = (1.3827 - 1) \times 290 = 111 \text{ K}
\]

(12-86)

Understand, of course, that the total noise figure could have been determined from the total noise temperature in one step using (12-84) and the result of Example 12-14. This exercise was provided for its educational value.
Example 12-16

The circuit of Figure 12-16 represents a bad design situation which has been somewhat exaggerated to make a point. The transmission line between the antenna and the preamplifier has a loss of 6.02 dB. The complete receiving system contains both a preamplifier and a receiver. Determine (a) the effective noise temperature at the input to the transmission line (at antenna output) and (b) the noise figure (absolute and in dB).

Solution

The absolute value of the loss factor is

\[ L = 10^{rac{L_{dB}}{10}} = 10^{\frac{6.02}{10}} = 4 \quad (12-87) \]

The absolute gain of the preamplifier stage is readily determined as 100 and the absolute gain of the receiver is determined as \(10^6\), although the latter value is not required at this time.

(a) The effective noise temperature \(T_{e,L}\) of the lossy transmission line referred to the input is determined from (12-80).

\[ T_{e,L} = (L-1)T_p = (4-1) \times 290 = 870 \text{ K} \quad (12-88) \]

The effective noise temperature referred to the input of the transmission line is determined from (12-72). However, the "gain" of the first stage is the factor \(1/L\) for the transmission line. The factor \(1/L\) in the denominator brings \(L\) back to the numerator, and the form of (12-72) adapted to this case is

\[ T_e = T_{e,L} + LT_{e,pre} + \frac{LT_{e,rec}}{G_{pre}} \quad (12-89) \]

where \(T_{e,pre}\) is the noise temperature of the preamplifier, \(T_{e,rec}\) is the noise temperature of the receiver, and \(G_{pre}\) is the gain of the preamplifier. Substituting these values, we have

\[ T_e = 870 + 4 \times 50 + \frac{4 \times 200}{100} = 870 + 200 + 8 = 1078 \text{ K} \quad (12-90) \]

It is obvious that the lossy transmission line has caused a severe increase in the effective noise temperature.

(b) The corresponding noise figure is

\[ F = 1 + \frac{T_e}{290} = 1 + \frac{1078}{290} = 4.717 \quad (12-91) \]

The decibel value is

\[ F_{dB} = 10 \log 4.717 = 6.74 \text{ dB} \quad (12-92) \]
Example 12-17

A young electronics specialist who studied out of this book decided to switch the arrangement to the form shown in Figure 12-17. It is assumed that the preamplifier is weather protected and capable of being placed right at the output of the antenna. Determine (a) the effective noise temperature at the antenna output and (b) the noise figure (absolute and in dB).

Solution

It should be clear that the net gain between antenna output and the receiver output will not be changed by this switch. However, let's see what happens to the noise temperature and noise figure.

(a) Using the form of (12-72) adapted to this case, the effective noise temperature is now given by

\[ T_e = T_{e,pre} + \frac{T_{e,L}}{G_{pre}} + \frac{LT_{e,rec}}{G_{pre}} \]  

(12-93)

Substituting values, we have

\[ T_e = 50 + \frac{870}{100} + \frac{4 \times 200}{100} = 50 + 8.7 + 8 = 66.7 \text{ K} \]  

(12-94)

This is a drastic reduction in noise temperature as compared with the preceding system form. Yet the output signal should have the same level since the net gain between the antenna output and receiver output is the same.

(b) The noise figure is

\[ F = 1 + \frac{T_e}{290} = 1 + \frac{66.7}{290} = 1.23 \]  

(12-95)

The decibel value is

\[ F_{db} = 10 \log 1.23 = 0.90 \text{ dB} \]  

(12-96)

The major point illustrated by this example is that it is very undesirable to have a lossy circuit at the front-end of any receiving system dealing with very low signal levels. When practical, a preamplifier can be placed right at the antenna output ahead of the lossy circuit and this is done in many practical systems.
PROBLEMS

12-1 (a) Determine the rms noise voltage produced by a 47-kΩ resistance in a 60-kHz bandwidth at the standard temperature $T_0 = 290$ K.

(b) Determine the rms noise voltage if the resistance is changed to 470 kΩ.

(c) Determine the rms noise voltage if the resistance is changed to 4.7 MΩ.

(d) With $R = 4.7$ MΩ, determine the rms noise voltage if the bandwidth is changed to 600 kHz.

(e) With $R = 4.7$ MΩ, determine the rms noise voltage if the bandwidth is changed to 6 MHz.

(f) With the values of $R$ and $B$ of part (e), determine the rms noise voltage if the temperature is increased to 310 K.

12-2 (a) Determine the rms noise voltage produced by a 120-kΩ resistance in an 80-kHz bandwidth at the standard temperature $T_0 = 290$ K.

(b) Determine the rms noise voltage if the resistance is changed to 1.2 MΩ.

(c) Determine the rms noise voltage if the resistance is changed to 12 MΩ.

(d) With $R = 12$ MΩ, determine the rms noise voltage if the bandwidth is changed to 800 kHz.

(e) With $R = 12$ MΩ, determine the rms noise voltage if the bandwidth is changed to 8 MHz.

(f) With the values of $R$ and $B$ of part (e), determine the rms noise voltage if the temperature is decreased to 270 K.

12-3 The 4.7-MΩ resistance of Problem 12-1(e) is connected to the input of an ideal noise-free amplifier with a voltage gain of $10^4$, a bandwidth of 6 MHz, and infinite input impedance. Determine the output rms noise voltage.

12-4 The 12-MΩ resistance of Problem 12-2(e) is connected to the input of an ideal noise-free amplifier with a voltage gain 2000, a bandwidth of 8 MHz, and infinite input impedance. Determine the output rms noise voltage.

12-5 Determine the net rms voltage in a 50-kHz bandwidth appearing across the series combination of two 10-kΩ resistances.

12-6 Determine the net rms voltage in a 50-kHz bandwidth appearing across the parallel combination of two 10-kΩ resistances.

12-7 Determine the available noise power contained in a 600-Ω resistance at the standard reference temperature in a bandwidth of 5 MHz.

12-8 Determine the available noise power contained in a 75-Ω resistance at the standard reference temperature in a bandwidth of 12 MHz.

12-9 The resistance of Problem 12-7 is connected across the input of an ideal noise-free amplifier whose input and output impedances are resistive values of 600 Ω. The amplifier has a matched gain of 50 dB and a bandwidth of 5 MHz. Determine the output noise power in a 600-Ω resistance.
12-10 The resistance of Problem 12-8 is connected across the input of an ideal noise-free amplifier whose input and output impedances are resistive values of 75 $\Omega$. The amplifier has a matched gain of 46 dB and a bandwidth of 12 MHz. Determine the output noise power in a 75-$\Omega$ resistance.

12-11 Determine the rms noise voltage across the load resistance in the amplifier of Problem 12-9.

12-12 Determine the rms noise current in the load resistance in the amplifier of Problem 12-10.

12-13 For the system of Problems 12-7 and 12-9, determine the power spectral densities at the (a) input and (b) output.

12-14 For the system of Problems 12-8 and 12-10, determine the power spectral densities at the (a) input and (b) output.

12-15 A source produces white noise having a power spectral density of 0.05 fW/Hz. Determine the equivalent source noise temperature.

12-16 A white noise source has an equivalent noise temperature of $10^5$ K. Determine the power spectral density.

12-17 Consider the system shown in Figure P12-17 and assume that impedances at all junctions are matched. The input is a thermal noise source having an equivalent noise temperature of 50,000 K. Assume that any internal noise is negligible in comparison to the noise produced by the source. The bandwidths are defined on the blocks. Determine the output noise power.

12-18 Consider the system shown in Figure P12-18 and assume that impedances at all junctions are matched. The input is a thermal noise source having a power spectral density of 5 fW/Hz. Assume that any internal noise is negligible in comparison to the noise produced by the source. The bandwidths are defined on the blocks. Determine the output noise power.

12-19 At the standard reference temperature, the signal-to-noise ratio at the input to an amplifier is 20 dB, and the output signal-to-noise ratio is 15 dB. Determine (a) decibel noise figure and (b) absolute noise figure.

12-20 At the standard reference temperature, the signal-to-noise ratio at the input to an amplifier is 600, and the output signal-to-noise ratio is 80. Determine (a) absolute noise figure and (b) decibel noise figure.

12-21 Determine the effective noise temperature of the amplifier of Problem 12-19 referred to the input.

12-22 Determine the effective noise temperature of the amplifier of Problem 12-20 referred to the input.

12-23 Calculate the effective noise temperature for each of the following noise figures: (a) $F = 1$, (b) $F = 2$, and (c) $F_{\text{dB}} = 8$ dB.

12-24 Calculate the effective noise temperature for each of the following noise figures: (a) $F = 3$, (b) $F = 4$, and (c) $F_{\text{dB}} = 7$ dB.

12-25 Calculate the absolute noise figure $F$ and the decibel value $F_{\text{dB}}$ for each of the following effective noise temperatures referred to the input: (a) 0 K, (b) 145 K, and (c) 290 K.
12-26 Calculate the absolute noise figure $F$ and the the decibel value $F_{dB}$ for each of the following effective noise temperatures referred to the input: (a) 580 K, (b) 1000 K, and (c) 2900 K.

12-27 (a) For the system of Figure P12-27, determine the effective noise temperature referred to the input. (b) From the result of (a), determine the noise figure.

12-28 (a) For the system of Figure P12-28, determine the effective noise temperature referred to the input. (b) From the result of (a), determine the noise figure.

12-29 For the system of Problem 12-27, determine the noise figure by first determining the noise figure for each stage and then combining their effects.

12-30 For the system of Problem 12-28, determine the noise figure by first determining the noise figure for each stage and then combining their effects.

12-31 A passive mixer, matched at both input and output, has a loss of 6.02 dB and physical temperature of 290 K. Determine (a) the effective noise temperature referred to the input and (b) the noise figure in dB.

12-32 A matched attenuator has a loss of 4 dB and physical temperature of 290 K. Determine (a) the effective noise temperature referred to the input and (b) the noise figure in dB.

Table 12-1. Equivalent Noise Bandwidths of Butterworth Filters.

<table>
<thead>
<tr>
<th>Poles</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_N / f_i$</td>
<td>1.571</td>
<td>1.111</td>
<td>1.047</td>
<td>1.026</td>
<td>1.017</td>
<td>1.012</td>
<td>1.008</td>
<td>1.006</td>
<td>1.005</td>
<td>1.004</td>
</tr>
</tbody>
</table>