Chapter 16
COMMUNICATION LINK ANALYSIS AND DESIGN

16-1 Introduction and Objectives

Any communication system must employ a transmission “channel” in which the signal from the information source is transferred to the destination. The simplest channel is the wire link in which the signal is transmitted over a pair of wires or a cable. The most obvious example of this nature is the standard telephone circuit within a local area. The use of fiber optics for transmitting signals is certainly a growing area and will likely see increased usage in the years ahead.

While various types of channels have their advantages, the channel concept that has traditionally provided the most capability for information transmission is the use of electromagnetic waves. The concepts of electromagnetic radiation and the use of antennas were introduced in the past two chapters. We are now ready to study the behavior of the link between the transmitting antenna and the receiving antenna. In particular, the study of direct-ray propagation will be heavily emphasized due to its accelerated use in satellite systems. However, some of the classical methods such as surface wave and sky wave propagation will also be discussed.

Objectives

After completing this chapter, the reader should be able to

1. State the Friis transmission formula and discuss the various parameters and their significance.
2. Define effective isotropic radiated power (EIRP).
3. State and apply the decibel forms of the one-way link equation.
4. Define and calculate path loss.
5. Determine the line-of-sight distance between transmitting and receiving antennas
6. Determine the minimum obstacle clearance required to avoid diffraction effects.
7. State the radar link equation and discuss the various parameters and their significance.
8. Define radar backscatter cross section.
9. State and apply the decibel form of the two-way link equation.
10. Explain the principle of pulse radar and how distance is measured.
11. For a pulse radar, determine the maximum unambiguous range and the resolution.
12. Explain the principle of Doppler radar and how velocity is measured.
13. Determine the Doppler shift for a given velocity and vice-versa.
15. Define refraction and the index of refraction
16. Discuss ground wave propagation in terms of its properties and applicable frequency range.
17 Discuss sky wave propagation in terms of its properties and applicable frequency range.
In this section, a development of the Friis transmission formula will be presented. This formula is most important in establishing a relationship between transmitted power, received power, antenna gain, and distance between transmitter and receiver. Its use permits a design trade-off between various system link parameters in establishing proper operating levels for a complete RF communication system.

The Friis transmission formula applies to a system in which there is a transmitting antenna, a receiving antenna, and a direct electromagnetic path without obstacles between the two antennas as shown in Figure 16-1. It does not apply to the various types of indirect propagation such as sky-wave bounce and tropospheric scatter. Such indirect propagation techniques have depended on empirical data to determine proper operating conditions, and such results are not always reliable. Consideration of some of the various types of indirect wave propagation will be made later in the chapter.

The trend of the future is definitely toward the use of direct wave propagation because of its predictability and reliability. Prior to the advent of communication satellites, direct ray propagation was limited to a very short range due to the curvature of the earth and the corresponding difficulty of locating antennas at sufficient heights to utilize direct rays. Following the development of satellite repeater systems, new dimensions for long-range communications became feasible. Direct ray propagation is now practical between the earth and a satellite, and the signal can then be retransmitted to points around the globe. As we will see with realistic examples later, the amount of power required is relatively modest with direct ray propagation utilizing high-gain antennas and microwave frequencies.

Development of Formula

The development of the several steps leading to the Friis formula will contribute to its understanding. Assume an isotropic point radiator emitting a power of $P_T$ watts. Consider an imaginary spherical surface of radius $d$ surrounding the radiator, and assume that the transmitter is at the center of the sphere. Let $p_d$ represent the power density in watts/meter$^2$ at any point on the surface. Since the power radiates equally well in all directions, it divides uniformly across the imaginary surface and is

$$p_d = \frac{P_T}{4\pi d^2} \quad (16-1)$$

where the denominator of (16-1) represents the surface area of the sphere.

An actual antenna having a gain $G_T$ with respect to an isotropic antenna would produce a power density $p_{d,\text{max}}$ in the direction of maximum radiation given by

$$p_{d,\text{max}} = G_T P_T = \frac{G_T T}{4\pi d^2} \quad (16-2)$$

This will be the power density at the location of the receiving antenna in Figure 16-1. The receiving antenna will "capture" a power $P_R$ given by

$$P_R = A_e p_{d,\text{max}} = \frac{A_e G_T P_T}{4\pi d^2} \quad (14-3)$$

where $A_e$ is the capture area of the receiving antenna. It is related to the receiving antenna gain by
where $G_R$ is the gain of the receiving antenna. Substitution of (16-4) in (16-3) yields

$$P_R = \frac{\lambda^2 G_T G_R P_T}{(4\pi)^2 d^2}$$

The result of (16-5) is the Friis transmission formula for one-way direct ray propagation from transmitter to receiver. It is also referred to as the one-way link equation. Later, we will see that there is a different version for signals that are transmitted to a destination and are "bounced back", such as in radar.

**General Discussion of One-Way Link Equation**

A number of important deductions can be made from the Friis formula, some of which are obvious, and some of which are more subtle. Immediately it is noted that the power at the receiver is directly proportional to the product of the transmitted power, the gain of the transmitter antenna, and the gain of the receiver antenna. Within limits, these three parameters may be adjusted without affecting performance provided that the product remains the same. For example, a transmitter with a power output of 2 W with an antenna gain of 100 would produce the same received power in the direction of maximum radiation as a transmitter with a power output of 50 W and a gain of 4. As a second example, if a receiving antenna is replaced by one having half the gain, the original received power could be restored by either doubling the transmitted power or by replacing the transmitting antenna with one having twice the gain.

**Variation with respect to Distance**

The denominator of the link equation indicates that the received power varies inversely with the square of the distance $d$ between transmitter and receiver. Thus, if the distance between transmitter and receiver is doubled, the received power is reduced by a factor of one-fourth.

The decibel variation as a result of the distance variation is interesting. It can be readily shown that if the distance between the antennas is doubled, the received power level is reduced by about 6 dB. If the distance between the antennas is increased by a factor of 10, the received power level is reduced by 20 dB.

**Variation with respect to Wavelength**

The variation of received power with respect to wavelength $\lambda$ is more subtle than the other quantities and needs some interpretation to avoid misleading conclusions. From the link equation, it appears that the received power is directly proportional to $\lambda^2$ and this would be true if all other parameters remained the same. The fallacy is that if the wavelength were increased, corresponding to lowering the frequency, the antennas would need to be replaced with new ones having the same gains as the previous ones. As deduced in the previous chapter, lower frequencies require larger antennas for the same gain, so any change in wavelength would mandate a complete redesign of the system. Therefore, accept the $\lambda^2$ as a factor in the equation, but one that cannot easily be changed without a complete system overhaul.

**Losses**

In its basic form, the Friis transmission formula is valid under a wide range of operating conditions. In general, however, other factors have to be considered in a full system design for worst-case conditions.

Losses resulting from atmospheric absorption are important in certain frequency ranges. Such losses are shown as a function of frequency in Figure 16-2. At frequencies well below 20 GHz, water and oxygen
losses are very low. Losses due to water vapor absorption are most significant in the vicinities of 23 and 180 GHz. Similarly, losses due to oxygen absorption are most significant in the vicinities of 60 and 120 GHz. It is interesting, however, that there are certain regions in this general frequency range in which these losses are greatly reduced. Such frequencies are called windows, and two window locations are 33 and 110 GHz. Operation above about 20 GHz is still in a development mode, but such future operation might be forced to utilize these window regions to minimize losses.

Other factors that might have to be considered are antenna pointing errors, bending of waves due to atmospheric refraction, attenuation due to rainfall, and multipath fading. Multipath fading results from the successive reinforcement and cancellation of the signal when two or more rays having different path lengths (along with different phase shifts) combine at the receiving antenna.
Example 16-1

A communication system has the following parameters:

\[ P_T = 5 \text{ W} \]
\[ G_T (\text{dB}) = 13 \text{ dB} \]
\[ G_R (\text{dB}) = 17 \text{ dB} \]
\[ d = 80 \text{ km} \]
\[ f = 3 \text{ GHz} \]

Determine the value of the received power.

Solution

The basic form of the Friis transmission formula given by (16-5) will be used in this example. We must determine the wavelength and it is

\[ \lambda = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m} \] (16-6)

The transmitting and receiving antenna gains are given in decibels and they must be converted to absolute form. The values are

\[ G_T = 10^{(13/10)} = 10^{1.3} = 19.95 \] (16-7)

\[ G_R = 10^{(17/10)} = 10^{1.7} = 50.12 \] (16-8)

The units for \( d \) must be the same as the units for \( \lambda \), i.e., meters. Hence, \( d = 80 \times 10^3 \text{ m} \). Inserting all the preceding values in (16-5), we have

\[ P_R = \frac{(0.1)^2 (19.95)(50.12)(5)}{(4\pi)^2 (80 \times 10^3)^2} = 49.5 \text{ pW} \] (16-9)

Even with the "nice" rounded numbers chosen, the computations are quite messy. At the end of the next section, this example will be worked again using decibel forms throughout the process.
16-3 Decibel Forms for One-Way Link Equation

In the example at the end of the previous section, it was evident that the link equation involves working with a combination of very small numbers and very large numbers. While modern computational aids certainly make this chore easier than in the early days of communications, there has evolved a widespread use of decibel forms to simplify the analysis and interpretation of link analysis. Anyone who has occasion to search the literature or to review system specifications should be familiar with this approach. In this section, we will show how some of these forms provide a different approach to the analysis.

The process may be initiated from the link equation, which is repeated here for convenience:

\[ P_R = \frac{\lambda^2 G_T G_R P_T}{(4\pi)^2 d^2} \]  

(16-10)

The wavelength may be readily expressed in terms of frequency as \( \lambda = \frac{3 \times 10^8}{f} \), in which case (16-10) may be expressed as

\[ P_R = \frac{569.93 \times 10^{12} G_T G_R P_T}{f^2 d^2} \]  

(16-11)

where the various constants have been lumped together. Next, a common power reference will be assumed for both transmitted and received power and the logarithm to the base 10 of both sides will be taken. Both sides can then be multiplied by 10 to give the results in decibel form. This will lead to

\[ P_R (\text{dB}) = P_T (\text{dB}) + G_T (\text{dB}) + G_R (\text{dB}) + 147.56 - 20 \log f - 20 \log d \]  

(16-12)

where dB represents decibels with respect to any standard reference, e.g., dBW, dBf, etc. However, it is essential that both \( P_R \) and \( P_T \) have the same reference.

Note that "20" appears in front of the logarithm of frequency and the logarithm of distance, a result of the squares for those variables in (16-11). In the form of (16-12), frequency is measured in hertz and distance is measured in meters. It is more convenient to work with frequency in either MHz or GHz and to work with distance in kilometers. The equations may be converted to those forms and this process results in certain additional constants. One form after some manipulation is

\[ P_R (\text{dB}) = P_T (\text{dB}) + G_T (\text{dB}) + G_R (\text{dB}) - 32.44 - 20 \log f (\text{MHz}) - 20 \log d (\text{km}) \]  

(16-13)

Note that in this form, frequency is expressed in MHz and distance is expressed in km.

Effective Isotropic Radiated Power

Returning momentarily to the basic form of (16-5), a term called the effective isotropic radiated power (EIRP) is defined as

\[ \text{EIRP} = G_T P_T \]  

(16-14)

The EIRP is the effective power in the direction of radiation resulting from the antenna gain in that direction. In many cases, this will be the maximum gain. In decibel form, the first two terms of (16-13) are the terms that produce this effect and we can express the decibel form of the EIRP as

\[ \text{EIRP}(\text{dB}) = P_T (\text{dB}) + G_T (\text{dB}) \]  

(16-15)
This is the effective decibel power level in the direction of radiation resulting from the action of the antenna. For our purposes, we will leave it expressed by the sum on the right hand side of (16-15), but the reader should be aware of the EIRP term since it is used in some forms of the link equation.

Path Loss

The last three terms on the right-hand side of (16-13) have negative signs in front of them. This means that they can be interpreted as loss terms. Leaving off the negative signs, we will denote the one-way path loss as $\alpha_1 (\text{dB})$ and it can be defined as

$$\alpha_1 (\text{dB}) = 20 \log f (\text{MHz}) + 20 \log d (\text{km}) + 32.44$$

(16-16)

where frequency is expressed in MHz and distance is expressed in km.

An alternate form of the path loss is

$$\alpha_1 (\text{dB}) = 20 \log f (\text{GHz}) + 20 \log d (\text{km}) + 92.44$$

(16-17)

where frequency is expressed in GHz and distance is expressed in km.

One-Way Link Equation Using Path Loss Definition

The form of the one-way link equation using the various preceding definitions is then

$$P_R (\text{dBx}) = P_T (\text{dBx}) + G_T (\text{dB}) + G_R (\text{dB}) - \alpha_1 (\text{dB})$$

(16-18)

or

$$P_R (\text{dBx}) = \text{EIRP} (\text{dBx}) + G_R (\text{dB}) - \alpha_1 (\text{dB})$$

(16-19)

where $\text{EIRP}(\text{dBx})$ was defined in (16-15)

While some of the preceding steps may seem like going around in circles, these final forms put everything in nice little decibel-form "packages" that simplify the process of communication system analysis and design, especially when investigating the various tradeoffs.
Example 16-2

Rework Example 16-1 using the decibel approach developed in this section.

Solution

First, the transmitted power of 5 W must be converted to a decibel reference form. We will choose to use units of dBW as the basis since the power is expressed directly in watts. Hence,

\[ P_T (\text{dBW}) = 10 \log \left( \frac{P_T (\text{W})}{1 \text{ W}} \right) = 10 \log 5 = 6.99 \text{ dBW} \quad (16-20) \]

Next, the path loss must be determined. Since the frequency is given in GHz, the form of \((16-17)\) will be used. We have

\[ \alpha_i (\text{dB}) = 20 \log f (\text{GHz}) + 20 \log d (\text{km}) + 92.44 \]
\[ = 20 \log 3 + 20 \log 80 + 92.44 \]
\[ = 9.54 + 38.06 + 92.44 \]
\[ = 140.04 \text{ dB} \quad (16-21) \]

Since the transmitted power in expressed in dBW, the link equation will be expressed in that form, and it is

\[ P_R (\text{dBW}) = P_T (\text{dBW}) + G_T (\text{dB}) + G_R (\text{dB}) - \alpha_i (\text{dB}) \]
\[ = 6.99 + 13 + 17 - 140.04 \]
\[ = -103.05 \text{ dBW} \quad (16-22) \]

Conversion of this value back to watts is accomplished as follows:

\[ P_R (\text{W}) = 10^{\frac{P_R (\text{dBW})}{10}} = 10^{-103.05/10} = 49.5 \text{ pW} \quad (16-23) \]

which agrees with the result of Example 16-1.

Was this problem any easier to work in decibel form as compared to the basic analysis of Example 16-1? It probably was not in this case. In fact, it may have even been more unwieldy to work it out this way.

The true beauty of this approach is best seen when an overall system containing many parts must be designed or analyzed. In such cases, the various tradeoffs can be more easily visualized and their effects weighted and compared. Since many system component specifications are given directly in decibel form, they can be easily adapted to this form in working with a complete system. Additional losses not considered here such as antenna pointing error, rain attenuation, etc., may be more easily entered into the analysis when the decibel form is used. In fact, many system designers utilize a so-called link budget, in which all of these terms in decibel form may be considered almost in the same way as a financial budget.

Irrespective of any discussions concerning advantages or disadvantages of working with decibel forms, the fact is that the entire communications industry utilizes this approach, so one must learn to work with such forms in order to deal with system specifications, design, and analysis.
Example 16-3

The distance from the earth to the moon is approximately 240,000 miles. Determine the path loss at (a) 100 MHz, (b) 1 GHz, and (c) 10 GHz. (Note: 1 mile = 1.609 km)

Solution

First, the distance in miles must be converted to kilometers to use the formulas developed.

\[ d(\text{km}) = 240,000 \text{ miles} \times 1.609 \text{ km/mile} = 386.2 \times 10^3 \text{ km} \]  

(a) At 100 MHz, the form of (16-16) will be used.

\[ \alpha_1(\text{dB}) = 20 \log f (\text{MHz}) + 20 \log d(\text{km}) + 32.44 \]
\[ = 20 \log 100 + 20 \log 386.2 \times 10^3 + 32.44 \]
\[ = 40 + 111.7 + 32.44 \]
\[ = 184.1 \text{ dB} \]  

(b) At 1 GHz, the form of (16-17) is slightly easier to use.

\[ \alpha_1(\text{dB}) = 20 \log f (\text{GHz}) + 20 \log d(\text{km}) + 92.44 \]
\[ = 20 \log 1 + 20 \log 386.2 \times 10^3 + 92.44 \]
\[ = 0 + 111.7 + 92.44 \]
\[ = 204.1 \text{ dB} \]  

(c) At 10 GHz, the form of (16-17) will be used again.

\[ \alpha_1(\text{dB}) = 20 \log f (\text{GHz}) + 20 \log d(\text{km}) + 92.44 \]
\[ = 20 \log 10 + 20 \log 386.2 \times 10^3 + 92.44 \]
\[ = 20 + 111.7 + 92.44 \]
\[ = 224.1 \text{ dB} \]

From these results, it can be readily deduced that the path loss increases by 20 dB for each increase in the frequency by a factor of 10. However, within limits, it is easier to build antennas having a higher gain as the frequency increases, which can offset the increasing path losses.
Example 16-4

At a frequency of 1 GHz, determine the path loss at the following distances: (a) 1 km, (b) 10 km, and (c) 100 km.

Solution

Since the frequency is given in GHz, the form of (16-17) is easier to use.

(a) At a distance of 1 km, the path loss is

\[
\alpha_i (\text{dB}) = 20 \log f (\text{GHz}) + 20 \log d (\text{km}) + 92.44
\]

\[
= 20 \log 1 + 20 \log 1 + 92.44
\]

\[
= 0 + 0 + 92.44
\]

\[
= 92.44 \text{ dB}
\]  

(b) At a distance of 10 km, the path loss is

\[
\alpha_i (\text{dB}) = 20 \log f (\text{GHz}) + 20 \log d (\text{km}) + 92.44
\]

\[
= 20 \log 10 + 20 \log 10 + 92.44
\]

\[
= 0 + 20 + 92.44
\]

\[
= 112.4 \text{ dB}
\]

(c) Finally, at a distance of 100 km, the path loss is

\[
\alpha_i (\text{dB}) = 20 \log f (\text{GHz}) + 20 \log d (\text{km}) + 92.44
\]

\[
= 20 \log 100 + 20 \log 100 + 92.44
\]

\[
= 0 + 40 + 92.44
\]

\[
= 132.4 \text{ dB}
\]

These results indicate that for each increase in distance by a factor of 10, the path loss increases by 20 dB.
**Example 16-5**

A certain analog system requires an antenna signal power of 50 pW to meet the required detected signal-to-noise ratio. Other system parameters are given as follows:

- $G_T$ (dB) = 3 dB
- $G_R$ (dB) = 4 dB
- $f$ = 500 MHz
- $d$ = 80 km

Assuming direct ray propagation, determine the minimum value of the transmitted power required.

**Solution**

Once again, we will choose to work in the units of dBW. The value 50 pW corresponds to a level in dBW of

$$P_r (\text{dBW}) = 10 \log \left( \frac{P_r (\text{W})}{1 \text{ W}} \right) = 10 \log 50 \times 10^{-12} = -103.01 \text{ dBW} \quad (16-31)$$

The path loss is determined from (16-16) as

$$\alpha_1 (\text{dB}) = 20 \log f (\text{MHz}) + 20 \log d (\text{km}) + 32.44$$

$$= 20 \log 500 + 20 \log 30 + 32.44$$

$$= 53.98 + 38.06 + 32.44$$

$$= 124.48 \text{ dB} \quad (16-32)$$

The link equation from (16-18) is then rearranged to determine the transmitter power as

$$P_T (\text{dBW}) = P_r (\text{dBW}) + \alpha_1 (\text{dB}) - G_T (\text{dB}) - G_R (\text{dB})$$

$$= -103.01 + 124.48 - 3 - 4 \quad (16-33)$$

$$= 14.47 \text{ dBW}$$

The power level in watts is then

$$P_T = 10^{P_T (\text{dBW})/10} = 10^{14.47/10} = 10^{1.447}$$

$$= 28.0 \text{ W} \quad (16-34)$$
Example 16-6

In a certain binary digital communication system, the average signal carrier power at the receiver terminals for the specified probability of error is required to be 200 fW. For the link portion of the system, assume the following parameters:

- $G_T (\text{dB}) = 30 \, \text{dB}$
- $G_R (\text{dB}) = 20 \, \text{dB}$
- $f = 4 \, \text{GHz}$
- $d = 40,000 \, \text{km}$

Assuming direct ray propagation, determine the required transmitter power.

Solution

To illustrate a slightly different approach in this problem, the received power will be determined in units of dBf and we have

$$P_R (\text{dBf}) = 10 \log \left( \frac{P_R (\text{fW})}{1 \, \text{fW}} \right) = 10 \log 200 = 23.01 \, \text{dBf}$$  \hspace{1cm} (16-35)

Since the frequency is given in GHz, it is simpler to employ the path loss form of (16-17), and the value is

$$\alpha_t (\text{dB}) = 20 \log f (\text{GHz}) + 20 \log d (\text{km}) + 92.44$$
$$= 20 \log 4 + 20 \log 40,000 + 92.44$$
$$= 12.04 + 92.04 + 92.44$$
$$= 196.52 \, \text{dB}$$

The required transmitted power is

$$P_T (\text{dBf}) = P_R (\text{dBf}) + \alpha_t (\text{dB}) - G_T (\text{dB}) - G_R (\text{dB})$$
$$= 23.01 + 196.52 - 30 - 20$$
$$= 169.53 \, \text{dBf}$$
$$= 19.53 \, \text{dBW}$$

Notice in the last step, the power level in dBf was converted to dBW by subtracting 150 dB.

The power level in watts is then

$$P_T (\text{W}) = 10^{P_T (\text{dBW})/10} = 10^{19.53/10}$$
$$= 89.74 \, \text{W}$$

(16-38)

In Example 16-5, we worked with dBW, while in Example 16-6, we worked with dBf until the last step. It doesn't matter as long as consistency is maintained in the process, and in the final analysis, the results must be interpreted in the correct units.


16-4 Line of Sight Propagation

In the frequency range where direct wave propagation is feasible, there are numerous relatively short-range applications where this type of transmission can be achieved without the benefit of satellite relay systems. This includes microwave repeater stations, cell-phone transmission, public service and utility two-way systems, communication to airplanes, military applications, and many others. In general, this type of transmission is limited by two major factors: (a) curvature of the earth's surface and (b) natural obstacles such as buildings and mountains. The first effect is a natural one and can be circumvented partially by locating transmitting and receiving antennas as high above the earth as possible. The second effect can only be alleviated if the path between transmitter and receiver can be located above potential blocking objects.

Line of Sight Distance Along Smooth Earth

Because the natural properties of a perfect sphere can be described mathematically, it is possible to precisely predict the possible distance between a transmitting antenna and a receiving antenna in terms of their heights above the earth's surface. However, the situation is complicated by the fact that the mathematical equations derived by the perfect sphere model actually underestimate the distance somewhat. Apparently, early experimenters discovered that some bending of the radio waves occurs and that the earth acts as if it had a radius of about 4/3 of its actual radius in terms of the line-of-sight prediction. In fact, this phenomenon has been so well researched that special graph paper based on an assumption of 4/3 of the earth's actual radius is available to assist designers of microwave terrestrial systems.

Refer to Figure 16-3 for the discussion that follows. Based on the assumption of a reasonably smooth earth, and considering the 4/3 factor, the distance \( d \) in kilometers over which direct line-of-sight transmission is feasible can be approximated as

\[
d(km) = \sqrt{17h_T(m) + 17h_R(m)}
\]

where

\[ h_T(m) = \text{transmitter antenna height in meters} \]
\[ h_R(m) = \text{receiver antenna height in meters} \]

Note that there are mixed units in the equation; i.e., the antenna heights are in meters and the distance is in kilometers. Various constants in the development have taken care of the mixed units.

An alternate version in which the distance is measured in miles and the antenna heights are measured in feet is the following:

\[
d(\text{miles}) = \sqrt{2h_T(\text{feet}) + 2h_R(\text{feet})}
\]

where

\[ h_T(\text{feet}) = \text{transmitter antenna height in feet} \]
\[ h_R(\text{feet}) = \text{receiver antenna height in feet} \]

Both of these equations should be interpreted as reasonable approximations at best. In fact, the constants have been rounded somewhat, so if you calculate the distance from either formula and convert the units to the other, slightly different results are obtained. Again, these are based on a reasonably smooth earth and would not be very good approximations in a mountainous region or in a city, except in the latter case, they could be useful if the antennas were located at the tops of some of the tallest buildings.

13
Diffraction

A relatively complex subject in the study of wave theory is that of *diffraction*. Without attempting to get into the mathematical form of the subject, it is a phenomenon that results in the scattering of electromagnetic energy when a portion of a wave encounters an object. It has been determined that the effects of diffraction are minimized if the electromagnetic path clears any object by a distance equal to 60% of the distance to the *first Fresnel zone*. Fresnel zones are a result of a theory known as the Huygens-Fresnel Principle, which states that an object that diffracts waves acts as it were a second source for the waves. The direct and refracted waves combine, but they either reinforce each other or partially cancel each other. This phenomenon is illustrated in Figure 16-4.

Clearance Required

The distance in meters \( r(\text{m}) \) to about 60% of the first Fresnel zone, i.e., the distance above which the direct ray between transmitter and receiver should pass, is approximated by the following formula:

\[
\begin{equation}
    r(\text{m}) = 10.4 \sqrt{\frac{d_1(\text{km})d_2(\text{km})}{f(\text{GHz})[d_1(\text{km}) + d_2(\text{km})]}}
\end{equation}
\]

(16-41)

where

- \( d_1(\text{km}) \) = distance between object and transmitter in km
- \( d_2(\text{km}) \) = distance between object and receiver in km
- \( f(\text{GHz}) \) = frequency in GHz
Example 16-7

A utility company has its dispatching antenna located at a height of 50 m. The trucks have antennas located at heights of 5 m. Based on reasonably smooth terrain, determine the approximate range for transmission.

Solution

The applicable formula is (16-39). We have

\[
d(\text{km}) = \sqrt{17h_T(\text{m})} + \sqrt{17h_R(\text{m})}
\]

\[
= \sqrt{17 \times 50} + \sqrt{17 \times 5}
\]

\[
= 29.15 + 9.22
\]

\[
= 38.37 \text{ km}
\]
16-5 Radar Link Equation

The term radar is a contraction for radio detection and ranging. In this section, a development and discussion of the radar link transmission formula will be presented. This form of the link equation is a "two-way" version in that power is transmitted along a direct path in one direction, but the ultimate power level of interest is the amount that is returned along the same path in the opposite direction.

Model for Analysis

The model for developing the radar equation is shown in Figure 16-5. The radar receiver is located at essentially the same position as the radar transmitter and may share some of the same circuits. In the figure, the signal is transmitted to the right, and a portion of the energy that encounters the target is backscattered to the left toward the radar receiver. The reader may be tempted to use the term "reflection" to describe this process, and the term is commonly used in casual conversation. Strictly speaking, however, a reflected wave has a precise meaning in terms of a coherent relationship with the incident wave. The term backscattered wave represents energy that is a combination of both coherent and noncoherent components, which is the case with most radar signals. Consequently, the term backscatter will be used as the basis for discussing the radar return phenomena. The term echo will also be used.

The signal returning to the source location is processed by the radar receiver. Some means must be provided to avoid overloading the low-level receiver by leakage energy from the transmitter. In the simplest classical radar system, the transmitter is turned off during the "listen" interval, but other means of isolation are available. In this development, we will assume that the same antenna, by means of appropriate switching or isolation, is used for both transmitting and receiving.

Power Density at Target

The first part of the development parallels that of the one-way link equation. The power density \( P_d \) of the transmitted signal in the vicinity of the target is of the same form as (16-2) and can be expressed as

\[
P_d = \frac{G P_t}{4\pi d^2}
\]

where \( G \) is the common antenna gain. (No subscript is required since there is only one antenna.)

Backscatter Cross Section

We now introduce a term \( \sigma \) called the radar backscatter cross section, which is defined as

\[
\sigma = \frac{\text{power backscattered in direction of source (W)}}{\text{incident power density (W/m}^2\text{)}}
\]

Observe that \( \sigma \) has the units of meters\(^2\) so it is an "area" parameter. The backscattered power \( P_{bs} \) is then determined as

\[
P_{bs} = \sigma P_d = \sigma \frac{G P_t}{4\pi d^2}
\]

In some references, the backscatter cross section is denoted as RCS.

Back at the receiver, the power density \( P_{d1} \) at the antenna is
The total power $P_R$ captured by the antenna is

$$P_R = p_d A_e = \frac{\sigma GP_T A_e}{(4\pi)^2 d^4}$$  \hspace{1cm} (16-47)

We may then express $A_e$ as

$$A_e = \frac{\lambda^2 G}{4\pi}$$  \hspace{1cm} (16-48)

Substitution of (16-48) in (16-47) yields

$$P_R = \frac{\sigma \lambda^2 G^2 P_T}{(4\pi)^3 d^4}$$  \hspace{1cm} (16-49)

This result is the radar transmission formula, and it provides an insight into the tradeoffs possible with radar systems. Observe that the power is proportional to the square of the power gain of the common antenna and is inversely proportional to the fourth power of the distance between the source and the target.

The most nebulous quantity in the radar equation is the radar cross section $\sigma$. While this quantity has the dimensions of area and is treated as such, it is actually a complex function of the surface roughness, surface composition, angle of incidence, and many other factors. Much research has been conducted through the years to establish reasonable estimates of the radar cross section for many different types of surfaces and several books devoted to this topic have been written. Anyone using the radar equation for system design would likely need to utilize such information to assist in estimating power return levels.

In military applications, stealth airplanes have been designed for use. Such planes have utilized designs that minimize the cross-section area and maximize the absorption of the wave. This minimizes the probability of detection from enemy radar systems.

**Decibel Form of Radar Equation**

In the same spirit as for the one-way link equation, decibel forms for the two-way radar equation can be readily developed. Without going through all the details in this case, the two-way path loss in decibels will be denoted as $\alpha_z$ (dB) and it may be expressed as

$$\alpha_z (dB) = 20\log f (GHz) + 40\log d (km) + 163.43 - 10\log \sigma (m^2)$$  \hspace{1cm} (16-50)

The two-way link equation is then

$$P_R (dBx) = P_T (dBx) + 2G_T (dB) - \alpha_z (dB)$$  \hspace{1cm} (16-51)

Note in (16-50) that 40 (rather than 20 as in the one-way link equation) is the multiplier in the decibel distance term in the two-way radar equation since it is based on an inverse fourth power variation. Note also that the common antenna gain in decibels is multiplied by 2 in the radar case since it is used in both directions.
**Example 16-8**

A radar system observing a target is characterized by the following parameters:

- Transmitted Power = 10 kW
- Antenna Gain = 25 dB
- Frequency = 3 GHz
- Distance to target = 50 km
- Radar cross section = 20 m²

Determine the received power.

**Solution**

First, the two-way path loss will be computed using (16-50).

\[
\alpha_2 (\text{dB}) = 20\log f (\text{GHz}) + 40\log d (\text{km}) + 163.43 - 10\log \sigma (\text{m}^2)
\]

\[
= 20\log 3 + 40\log 50 + 163.43 - 10\log 20
\]

\[
= 9.54 + 67.96 + 163.43 - 13.01
\]

\[
= 227.92 \text{ dB}
\]

The transmitted power will be expressed in dBW.

\[
P_T (\text{dBW}) = 10\log \frac{P_T (W)}{1 \text{ W}} = 10\log 10^4 = 40 \text{ dBW}
\]

(16-53)

The received power is then determined as

\[
P_R (\text{dBW}) = P_T (\text{dBW}) + 2G_T (\text{dB}) - \alpha_2 (\text{dB})
\]

\[
= 40 + 2(25) - 227.92
\]

\[
= -137.92 \text{ dBW}
\]

(16-54)

The absolute power is then

\[
P_R = 10^{P_R(\text{dBW})/10} = 10^{-137.92/10}
\]

\[
= 16.1 \times 10^{-15} \text{ W} = 16.1 \text{ fW}
\]

(16-55)
16-6 Pulse Radar

The oldest and most basic form of radar is called pulse radar and is used primarily for the measurement of distance. The power is transmitted in short bursts, and the time delay is measured for the return energy to be detected back at the source location. From a measurement of the time delay $T_d$, the distance $d$ to the object may be computed as

$$d = \frac{cT_d}{2} = 1.5 \times 10^8 T_d$$  (16-56)

The factor of 2 in the denominator of the equation results from the fact that the actual round-trip distance of the signal is $2d$, and this net distance must be divided by 2 to yield the actual distance to the object. In radar terminology, the object is often referred to as the "target", but the meaning does not necessarily imply any type of military significance.

There are several important parameters that are significant in determining the performance of pulse radar. Consider the periodic waveform shown in Figure 16-6. This baseband waveform may be considered as the modulating waveform for a high-frequency carrier, usually in the microwave region. Since it would be impossible to show more than a few cycles on the figure, we will skip the RF form altogether and just deal with this gating signal. Just remember that a high frequency RF transmitter is turned on during the time the pulse is on and it is off for the remainder of the period.

Let $\tau$ represent the width of the RF pulse and let $T$ represent the period. Thus, the RF transmitter is turned on for a relatively short time $\tau$ and is turned off for an interval $T - \tau$. During this latter time interval, the receiver "listens" for an echo, in which the time can be used to directly determine the distance according to (16-56). In any modern system, the various waveforms will be used with calibrated instrumentation to produce an output reading directly in distance units. Moreover, the antenna may be changing its orientation at a rate that would permit the observation of a wide field of view.

Pulse Repetition Frequency

Let $f_p$ represent the frequency of the baseband pulse train. It is given by

$$f_p = \frac{1}{T}$$  (16-57)

Like most specialty areas, radar has its own terminology. This frequency is often referred to as the pulse repetition frequency (prf) or the pulse repetition rate (prr).

Maximum Unambiguous Range

Any echo observed at the receiver may have originated from the pulse transmitted at the beginning of the particular cycle or it may have originated from a pulse transmitted during a previous cycle. The maximum unambiguous range $d_{\text{max}}$ is the greatest distance away from the radar that can be measured within one cycle. This value is

$$d_{\text{max}} = \frac{cT}{2} = \frac{c}{2f_p}$$  (16-58)

The design and expected performance of the radar must work around this potential ambiguity. Note that the maximum unambiguous range increases with the period of the baseband gating waveform.
Resolution

Another parameter is the smallest target that can be measured based on the pulse width. If the pulse is too wide, it will fail to distinguish two targets that are very close together and will give a distorted view of the size of the target. This value can also represent the closest distance to a target that can be measured and will be denoted as $d_{\text{min}}$. This value is

$$d_{\text{min}} = \frac{c\tau}{2} \quad (16-59)$$

This value is directly proportional to the width $\tau$ of the gating pulse.

Tradeoffs

Like most engineering systems, there are tradeoffs that must be considered. A greater unambiguous range calls for an increasing value of the period. Likewise, finer resolution calls for a shorter pulse width. However, decreasing the pulse width means that the transmission bandwidth must be increased, which would decrease the signal-to-noise ratio. Moreover, increasing the period and/or decreasing the pulse width will decrease the average transmitted power and this will degrade the signal processing. Therefore, compromises are always made, depending on the constraints that are most significant in a particular application.

Clutter

Clutter is any superfluous returns that are not part of the desired target being measured. There are sophisticated techniques for removing or minimizing clutter while maximizing the desired signal return.
Example 16-9

A pulse radar system operating at 10 GHz measures an echo 400 is after the pulse is transmitted. Determine the distance to the target.

Solution

The basic relationship of (16-56) is applicable. We have

\[
d = \frac{cT_d}{2} = 1.5 \times 10^8 T_d = 1.5 \times 10^8 \times 400 \times 10^{-6} = 60 \text{ km}
\]  

Example 16-10

A pulse radar system operates at a frequency of 10 GHz with a pulse repetition frequency of 2 kHz and a pulse width of 6 is. Determine (a) the maximum unambiguous range and (b) the resolution or minimum range.

Solution

(a) The maximum unambiguous range is determined from (16-58).

\[
d_{\text{max}} = \frac{cT}{2} = \frac{c}{2f_p} = \frac{3 \times 10^8}{2 \times 2 \times 10^3} = 75 \text{ km}
\]  

(b) The resolution is determined from (16-59).

\[
d_{\text{min}} = \frac{c\tau}{2} = \frac{3 \times 10^8 \times 6 \times 10^{-6}}{2} = 900 \text{ m}
\]
16-7 Doppler Radar

Another important type of radar is that of Doppler radar. While pulse radar is primarily used to measure distance to a target, Doppler radar can be used to measure the speed of a moving object, a likely unpleasant experience of readers who have been caught for speeding. The concept will be developed in some detail in this section.

Consider the airplane in Figure 16-7, which is moving at a velocity $V$ in the direction toward the receiver on the right. (We will assume, of course, that the plane is far enough away that it poses no danger of collision with the receiver!) Assume that the plane is transmitting a sinusoidal signal of cyclic frequency $f_c$ and radian frequency $\omega_c = 2\pi f_c$. If the plane were not moving, the signal received by the observer would have the form

$$v(t) = A \sin \omega_c t = A \sin 2\pi f_c t$$  \hspace{1cm} (16-63)

(Don't confuse the symbol $V$ representing velocity with the symbol $v(t)$ representing voltage.) The plane is actually moving with velocity $V$ toward the receiver and this causes the phase of the transmitted sinusoidal function to advance at a greater rate than if it were stationary. Indeed, in the distance of one wavelength $\lambda$, the phase shift due to the plane movement alone will have advanced by $2\pi$ radians. This means that the signal may actually be expressed as

$$v(t) = A \cos(2\pi f_c t + \frac{2\pi}{\lambda} x)$$  \hspace{1cm} (16-64)

where $x$ is the distance traveled from some arbitrary beginning reference point.

Instantaneous Frequency

Let us momentarily return to the concept of instantaneous frequency as considered in Chapter 7 on FM. The instantaneous radian frequency $\omega_i(t)$ associated with the argument $\Theta_i(t)$ of the cosine function in (16-64) can be expressed as

$$\omega_i(t) = \frac{d\Theta_i(t)}{dt} = 2\pi f_c + \frac{2\pi}{\lambda} \frac{dx}{dt} = 2\pi f_c + \frac{2\pi}{\lambda} V$$  \hspace{1cm} (16-65)

where $V = dx / dt$ is the velocity of the airplane.

The instantaneous cyclic frequency is

$$f_i(t) = \frac{\omega_i(t)}{2\pi} = f_c + \frac{V}{\lambda} = f_c + f_D$$  \hspace{1cm} (16-66)

where $f_D$ is the Doppler shift. In effect the frequency appears at the receiver to be higher than the transmitted frequency by the value $f_D$. Of course, if the plane were moving away from the observer, the Doppler shift would be considered negative and the net effect would be to decrease the received frequency.

In the form of (16-66), the doppler shift is $f_D = V / \lambda$. A more convenient form can be obtained by setting $\lambda = c / f_c$ and substituting that value in the expression for the Doppler shift. The result for the one-way Doppler shift then becomes
This form clearly shows that the Doppler shift is directly proportional to the velocity of the moving object and the frequency being transmitted. Note, however, that this is a **one-way Doppler shift** since the signal being transmitted is received at a stationary point and is not returned to the transmitter.

### Two-Way Doppler Shift

Now consider the situation depicted in Figure 16-8. This situation differs from the previous one in that the signal is transmitted from the Doppler radar on the right and is backscattered from the plane on the left back to the Doppler radar receiver. The frequency is measured at the receiver on the right. Since the phase is being advanced on both the signal being transmitted and the return echo from the target, the result is that the Doppler shift is *twice as great* as before. Thus, the **two-way Doppler shift** is given by

\[
f_D = \frac{2v}{c} f_c \tag{16-68}
\]

This is the form that would be measured at a stationary Doppler transmitter-receiver on the right based on the airplane moving in a straight line toward the Doppler radar. If both the radar and the target were moving, the appropriate velocity in (16-68) would be the *relative velocity* between them. If they are moving toward each other, the Doppler shift is positive; whereas, if they are moving away from each other, the Doppler shift is negative.

### Two-Way Doppler Shift at an Angle

Next, consider the situation shown in Figure 16-9. The target is moving at a velocity \(v\), but it is moving at an angle \(\theta\) with respect to a straight-line between the plane and the radar. The component of velocity in the direction of the radar is now \(v \cos \theta\). The Doppler shift is then given by

\[
f_D = \frac{2v}{c} f_c \cos \theta \tag{16-69}
\]

If \(\theta = 0^\circ\), (16-69) reduces back to (16-68), and the Doppler shift is maximum. Conversely, if \(\theta = 90^\circ\), there is no Doppler shift.
Example 16-11

A Doppler radar operating at 15 GHz is viewing a target moving directly toward it at a speed of 25 m/s. Determine the Doppler shift.

Solution

\[ f_D = \frac{2v}{c} f_c = \frac{2 \times 25}{3 \times 10^8} \times 15 \times 10^9 = 2500 \text{ Hz} = 2.5 \text{ kHz} \]  

(16-70)

The frequency shift is typically determined by multiplying in a mixer the return signal with a signal proportional to the signal being transmitted. One of the output components of the mixer is directly proportional to the difference frequency and, after filtering, can be calibrated directly in terms of the velocity of the target.

Example 16-12

A Doppler radar operating at 10 GHz is being used to measure the speed of an automobile moving directly toward it. The frequency shift is 2 kHz. Determine the speed of the automobile in miles per hour.

Solution

We start with (16-68) and solve for velocity. We have

\[ v = \frac{c f_D}{2 f_c} \]  

(16-71)

We could take several different paths at this point. One way would be to express the speed of light in meters/second, determine the velocity in the same units and then convert to miles per hour. An alternate approach is to express the speed of light as 186,000 miles/second, which would make the result appear in miles per second. By an additional multiplication of 3600 seconds/hour, the result will be in miles per hour. We will take the second approach. First the velocity in miles/second is

\[ v = \frac{186,000 \times 2 \times 10^3}{2 \times 10^9} = 18.6 \times 10^{-3} \text{ mi/s} \]  

(16-72)

Conversion to miles/hour yields

\[ v = 18.6 \times 10^{-3} \text{ mi/s} \times 3600 \text{ s/hr} = 67.0 \text{ mi/hr} \]  

(16-73)

Watch out for a traffic ticket!
16-8 Reflection and Refraction

When an electromagnetic wave encounters a boundary between two media, some of the energy is reflected by the boundary while the remaining portion is transmitted into the second medium. This situation is similar to the transmission line case where the traveling waves on the transmission line arrive at an impedance mismatch. A portion of the wave is reflected by the mismatch, and the remaining portion is transmitted past the impedance mismatch.

The direction of propagation for the plane wave can approach the boundary at any angle. The direction of propagation for the reflected plane wave is a function of the angle of the incident wave and the smoothness of the surface of the boundary between the two media in terms of the wavelength of the electromagnetic propagation.

Perfect Conductor Reflection

The reflection from a conductor with a smooth surface is illustrated in Figure 16-10. An incident electromagnetic wave in medium 1 is represented by $p_i$. The angle between the incident wave and the normal line from the surface is called the angle of incidence and is denoted by $\theta_i$. The smooth conducting surface acts like a mirror and the reflected wave is denoted by $p_r$. The angle between the reflected wave and the normal line from the surface is called the angle of reflection and is denoted by $\theta_r$. For the perfect conductor with a smooth surface, the angle of reflection is equal to the angle of incidence; i.e.,

$$\theta_r = \theta_i \quad (16-74)$$

If medium 2 had been a dielectric, some of the energy in the incident wave would have been transmitted into medium 2, and the magnitude of the reflected wave would have been reduced. The angle of reflection, however, would remain the same for a smooth boundary surface.

Refraction

Consider now the situation depicted in Figure 16-11 containing a smooth interface between two different dielectric media. Medium 1 contains an incident electromagnetic wave indicated by $p_i$. Some of the energy will be reflected, and the angle of reflection will equal the angle of incidence as indicated earlier, but that component is not shown here. Instead, the emphasis in this figure will be directed toward the component that enters medium 2 and it is called the refracted wave.

As has been the case all along, assume that the permeability in medium 2 is the same as that of medium 1 and that both have the value of the free-space permeability $\mu_0$. However, assume that the permittivity in medium 1 is $\varepsilon_1$, and the permittivity in medium 2 is $\varepsilon_2$. This means that the respective velocities of propagation in the two media are

$$v_1 = \frac{1}{\sqrt{\mu_0 \varepsilon_1}} \quad (16-75)$$

and

$$v_2 = \frac{1}{\sqrt{\mu_0 \varepsilon_2}} \quad (16-76)$$
It can be shown with geometric optics that the wave will change directions as it passes through the boundary. Let $\theta$ represent the angle of refraction as illustrated in Figure 16-11. (Think of the subscript "r" as representing "transmitted" wave.) If the velocity in medium 2 is less than the velocity in medium 1, the wave will bend in the direction of the normal and that is the situation depicted on the figure.

**Snell's Law**

Snell's law provides the relationship between the angle of incidence $\theta_i$ and the angle of refraction (or transmission) $\theta_r$. It can be stated as

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{v_2}{v_1}$$  \hspace{1cm} (16-77)

Substituting expressions for the velocities, (16-74) can be expressed as

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_2}} = \frac{\sqrt{\varepsilon_{r1}}}{\sqrt{\varepsilon_{r2}}}$$  \hspace{1cm} (16-78)

where the permittivity values have been expressed in terms of the dielectric constant values and the common value of $\varepsilon_0$ has been cancelled in the last form.

**Index of Refraction**

A term used extensively in optics is the index of refraction. The index of refraction $n$ is defined as the ratio of the velocity of propagation in a vacuum to the velocity of propagation in a particular medium. The values in this case are given by

$$n_1 = \sqrt{\varepsilon_{r1}}$$  \hspace{1cm} (16-79)

$$n_2 = \sqrt{\varepsilon_{r2}}$$  \hspace{1cm} (16-80)

Snell's law can then be expressed as

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{n_1}{n_2}$$  \hspace{1cm} (16-81)

Because of these various forms, it is easy to get mixed up in interpreting the formulas. A worthwhile point to remember is that the side having the highest velocity will have the largest angle measured from the normal. The equations can then be arranged in the appropriate form.

The bending of electromagnetic waves plays a major role in sky wave radio propagation, as will be discussed in Section 16-10. It is also important in fiber optic transmission systems.
Example 16-13

An electromagnetic wave propagating in air encounters a boundary with a material having a dielectric constant of 4. The angle of incidence in air is 50°. Determine the angle of refraction.

Solution

We will assume that the index of refraction in air is $n_1 = 1$. The index of refraction in the second medium is

$$n_2 = \sqrt{\varepsilon_2} = \sqrt{4} = 2 \quad (16-82)$$

The angle of refraction is determined as follows:

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{n_1}{n_2} = \frac{1}{2} \quad (16-83)$$

This leads to

$$\sin \theta_r = \frac{1}{2} \sin \theta_i = \frac{1}{2} \times \sin 60^\circ = 0.4330 \quad (16-84)$$

The angle of refraction is then

$$\theta_r = \sin^{-1} 0.4330 = 25.66^\circ \quad (16-85)$$

Note that the wave bends toward the normal in medium 2, a result of the higher index of refraction and lower velocity in that medium.
16-9 Ground Wave Propagation

Direct wave propagation as predicted by the Friis equation is very reliable at the higher frequencies used for space and satellite communications. However, at lower frequencies, and especially with vertical polarization, direct wave propagation is limited due to ground effects. In the discussion that follows, the **direct wave** as predicted by the Friis formula will be considered as one of the components of a **space wave**, due to common terminology and usage.

**Ground Wave**

The **ground wave** is commonly referred to as the electromagnetic propagation that travels from a transmitting antenna to a receiving antenna along or near the surface of the earth without leaving the lower portion of the earth's atmosphere. The ground wave can be divided into a **surface wave** and a **space wave**, which travels in the lower portion of the earth's atmosphere near the surface. The **space wave** consists of two components: the **direct wave** and the **indirect** or **ground-reflected wave**. The two waves are illustrated in Figure 16-12.

**Surface Wave**

The surface wave is used primarily for vertical polarization propagation below 3 MHz. The horizontally polarized signal is quickly attenuated since the electric field vector is parallel to the earth's surface. The vertical polarized surface wave is the propagation mode for the standard AM broadcast band. It can provide coverage up to about 100 miles in the AM band during the daytime. At 300 kHz, vertically polarized surface waves can provide coverage to about 500 miles over good conductivity earth and to 1000 miles over sea water. The use of the surface wave above 3 MHz is very limited due to its restricted coverage. At 30 MHz, the typical range is 10 miles or less.

**Space Wave**

The space wave is very poor for communications below 3 MHz. The reflected wave undergoes a 180° phase shift when it is reflected from the earth's surface. For antennas located near the earth's surface and operating at long wavelengths, the distances that the direct and indirect waves travel are essentially the same. Therefore, when arriving at the receiving site, they will be 180° out of phase and will cancel. The vertically polarized signal is reflected better than the horizontally polarized signal. Also, as the frequency is increased, the path difference in terms of wavelength is increased, and the cancellation between the direct and indirect waves is less significant.
Sky Wave Propagation

In addition to the ground wave discussed in the previous section, an antenna with a broad beam or one having an orientation toward space will radiate a *sky wave*. As the sky wave travels through the earth's atmosphere, it encounters a region of the atmosphere known as the *ionosphere*. The ionosphere consists of several layers of ionized gases that are located from 40 to 300 km above the earth's surface. The electromagnetic radiation is both refracted and attenuated by the ionosphere, depending on the frequency of the electromagnetic radiation, the radiation angle from the transmitting antenna, and the degree of ionization of the atmospheric gases at that level in the ionosphere. If the electromagnetic radiation is within a certain frequency range, typically 3 to 30 MHz, the signal will be refracted, i.e., bent back toward the earth's surface and received by a receiving site on the earth. If the frequency is below a certain frequency, typically 3 MHz, it will be totally absorbed by the ionosphere. If the frequency is above a certain frequency, typically 30 MHz, it will pass through the ionosphere and will continue into outer space.

It should be noted that the ionosphere is between the earth's surface and the orbits of communication satellites. Therefore, satellite communication systems must use frequencies sufficiently high enough to be unaffected by the passage through the ionosphere. Satellite systems operate in the microwave region at frequencies typically above 3 GHz.

**Skip Effect**

The behavior at a typical frequency that can be refracted back to the earth's surface is illustrated in Figure 16-13. Six different rays, numbered 1 through 6, are shown as a function of radiation angle. Rays 1, 2, and 3 are radiated at too high an angle and therefore are not bent enough to return to the earth's surface. Ray 4 is transmitted at an angle, known as the *critical angle*, such that it will be returned to the earth's surface. The *skip distance* is the distance along the earth's surface from the transmitting site to the point on the surface where ray 4 returns. Since ray 4 is at the critical angle, this is the closest distance, outside of the area covered by the ground wave, that a receiving station could receive propagation from the transmitter. The range between the ground wave and the skip distance is the *quiet zone*, where no transmission can be received.

Let ray 6 represent the lowest radiation angle that the transmitting angle can radiate a sky wave. The distance from where ray 4 returns and ray 6 returns is the range over which reception of propagation from the transmitter can be received. This region is the *reception zone* and ray 5 represents a typical radiation angle in this range. The maximum range for one hop of this type of propagation is typically 4000 km or about 2500 miles.

Actually, greater ranges can be obtained since the propagation can be reflected from the earth's surface back toward the ionosphere and refracted back to the earth a second time, as illustrated by ray 4 in Figure 16-7. Multiple hops up to 5 to 7 times have been achieved and this has resulted in propagation over very large ranges around the earth's surface.

**Ionosphere and its Layers**

The ionosphere is a region where the pressures are so low that the constituents of the atmosphere are ionized; i.e., a molecule loses an electron and becomes a positively charged ion. The region, therefore, consists of both free electrons and ions. With the low pressure, the density of molecules is such that it takes a longer time for recombination to occur. The major source of energy for the ionization process comes from solar ultraviolet radiation, although solar X rays and meteor radiation also play a role. The major ionization process begins right after sunrise, peaks around local noon, and decays after sunset.

During the daytime, the ionization is in four distinct layers due to the ionization requirements of the different constituents of the atmosphere. The ionization peaks at about the midrange of each layer and tapers off both above and below the altitude of each layer's maximum ionization. The lowest ionized layer is the D layer, which lies between 60 and 92 km (about 37 to 57 miles) above the earth's surface. The next ionized layer is the E layer, which lies between 100 and 115 km (about 62 to 71 miles) above the surface.
The next ionized layer is the F layer, which lies between 160 and more than 500 km (about 100 to over 310 miles) above the surface.

During daytime hours, the F layer breaks into two distinct layers: the F1 layer and the F2 layer. After the sun sets, the ionization in the D and E layers ceases due to the fairly rapid recombination of electrons and ions, and these layers disappear. The F1 layer, which is lower and weaker than the F2 layer, also disappears at night, and the F2 layer drops in altitude. The F2 layer is slow to recombine and, therefore, lasts throughout the night, reaching a minimum sometime after midnight.

The use of the ionosphere to "bounce" electromagnetic radiation off the ionosphere and back to the earth is a technique that requires a great deal of information on a large number of factors. These factors include (a) location on the earth of both the transmitting and receiving stations, (b) time of day, (c) season of the year, and (d) degree of solar activity.

**Maximum Usable Frequency**

The proper choice of the frequency of the electromagnetic radiation is important to the successful completion of a communication link. There is a frequency that is defined as the *maximum usable frequency* (MUF). If a variable-frequency transmitter is used to radiate an electromagnetic wave straight up (at a 90° angle), the highest frequency that is reflected back to the earth for that layer in the ionosphere is the *vertical incidence critical frequency*. The MUF is the vertical incidence critical frequency for that layer. The MUF is a variable that changes significantly as a function of the time of day, solar activity, propagation path distance, and propagation path between the transmitting and receiving stations. For example, the MUF between the eastern United States and Europe has varied from 7 MHz to 70 MHz because of these variables. Predictions for the MUF as a function of the propagation path, time of year, and time of day are published periodically by several agencies worldwide.

The degree of refraction in the ionosphere is a function of the frequency. Therefore, since the transmitting antenna typically radiates at a constant radiation angle relative to the surface, the skip distance is a function of frequency. The shorter the distance between the transmitter and receiver, the lower the MUF will be for that propagation path.

**Lowest Usable Frequency**

A second important frequency is the *lowest usable frequency* (LUF). If the frequency of the electromagnetic radiation is lowered from the MUF, the losses due to absorption in the atmosphere will increase as the frequency decreases. The frequency at which these losses cause the signal strength to fall below the background noise is the LUF. Therefore, it is desirable to operate as close to the MUF as the available frequency allocation will allow. Since each type of communication system is restricted to operate in specified frequency allocation bands, sometimes the frequency range from the LUF to the MUF is so narrow that there will be no allocated frequency band for successful communications.
PROBLEMS

16-1 A communication system has the following parameters:

\[
P_T = 20 \text{ W} \\
G_T (\text{dB}) = 15 \text{ dB} \\
G_R (\text{dB}) = 22 \text{ dB} \\
d = 50 \text{ km} \\
f = 10 \text{ GHz}
\]

Determine the received power using the basic Friis transmission formula (not the decibel form)

16-2 A communication system has the following parameters:

\[
P_T = 120 \text{ W} \\
G_T (\text{dB}) = 30 \text{ dB} \\
G_R (\text{dB}) = 24 \text{ dB} \\
d = 40,000 \text{ km} \\
f = 15 \text{ GHz}
\]

Determine the received power using the basic Friis transmission formula (not the decibel form)

16-3 Repeat the analysis of Problem 16-1 using the decibel form of the one-way link equation.

16-4 Repeat the analysis of Problem 16-2 using the decibel form of the one-way link equation.

16-5 The closest distance between Earth and Mars is about 54.5\times 10^6 \text{ km}. Determine the decibel path loss between the two planets at (a) 1 \text{ GHz} and (b) 10 \text{ GHz}.

16-6 The closest distance between Earth and Jupiter is about 600\times 10^6 \text{ km}. Determine the decibel path loss between the two planets at (a) 1 \text{ GHz} and (b) 10 \text{ GHz}.

16-7 Assume that a radio signal is being sent from Earth to Mars based on the distance given in Problem 16-5. If there was an immediate response when it arrived, how long would it be from the time that you sent the message until you received an answer?

16-8 Assume that a radio signal is being sent from Earth to Jupiter based on the distance given in Problem 16-6. If there was an immediate response when it arrived, how long would it be from the time that you sent the message until you received an answer?

16-9 At a frequency of 15 \text{ GHz}, determine the path loss in decibels at the following distances: (a) 1 \text{ km}, (b) 100 \text{ km} and (c) 10 \text{ Mm}.

16-10 At a frequency of 6 \text{ GHz}, determine the path loss in decibels at the following distances: (a) 5 \text{ km}, (b) 50 \text{ km}, and (c) 500 \text{ km}.

16-11 A certain analog system requires an antenna signal power of 100 \text{ pW} to meet the required detected signal-to-noise ratio Other system parameters are given as follows:

\[ G_T (\text{dB}) = 4 \text{ dB} \]
\[ G_R (\text{dB}) = 6 \text{ dB} \]
\[ f = 900 \text{ MHz} \]
\[ d = 50 \text{ km} \]

Assuming direct ray propagation, determine the minimum value of the transmitter power required.

16-12 A certain analog system requires an antenna signal power of 50 dBf to meet the required detected signal-to-noise ratio. Other system parameters are given as follows:

\[ G_T (\text{dB}) = 12 \text{ dB} \]
\[ G_R (\text{dB}) = 20 \text{ dB} \]
\[ f = 5 \text{ GHz} \]
\[ d = 100 \text{ km} \]

Assuming direct ray propagation, determine the minimum value of the transmitter power required.

16-13 In a certain binary digital communications system, the bit energy required at the receiver input terminals is \(50\times10^{-18} \text{ J} \). Other system parameters are as follows:

\[ G_T (\text{dB}) = 24 \text{ dB} \]
\[ G_R (\text{dB}) = 26 \text{ dB} \]
\[ f = 10 \text{ GHz} \]
\[ d = 40,000 \text{ km} \]

Assuming direct ray propagation, determine the minimum value of the transmitter power required if the data rate is 5 kbits/s.

16-14 For the system of Problem 16-13, determine the minimum value of the transmitter power required if the data rate is increased to 40 kbits/s.

16-15 Based on a height of 200 m for both a transmitting and a receiving antenna, determine the approximate range in kilometers for transmission based on reasonably smooth terrain.

16-16 A company has its base antenna at a height of 60 ft. All vehicles have their antennas at heights of 8 ft. Based on reasonably smooth terrain, determine the approximate range in miles for transmission.

16-17 A radar system observing a target is characterized by the following parameters:

\[ \text{Transmitted Power} = 5 \text{ kW} \]
\[ \text{Antenna Gain} = 30 \text{ dB} \]
\[ \text{Frequency} = 6 \text{ GHz} \]
\[ \text{Distance to target} = 50 \text{ km} \]
\[ \text{Radar cross section} = 30 \text{ m}^2 \]

Determine the received power.
16-18 A radar system observing a target is characterized by the following parameters:

- Transmitted Power = 40 dBW
- Antenna Gain = 28 dB
- Frequency = 10 GHz
- Distance to target = 100 km
- Radar cross section = 50 m$^2$

Determine the received power.

16-19 Measurements are performed to determine the cross section of a certain target and the following parameters are measured:

- Transmitted Power = 25 dBW
- Antenna Gain = 26 dB
- Frequency = 5 GHz
- Distance to target = 2 km
- Received Power = 60 dBf

Determine the radar cross section in m$^2$.

16-20 Measurements are performed to determine the cross section of a certain target and the following parameters are measured:

- Transmitted Power = 50 dBm
- Antenna Gain = 30 dB
- Frequency = 10 GHz
- Distance to target = 5 km
- Received Power = 30 dBf

Determine the radar cross section in m$^2$.

16-21 A pulse radar system operates at a frequency of 12 GHz with a pulse repetition period of 2 ms and a pulse width of 3 $\mu$s. Determine (a) the maximum unambiguous range and (b) the resolution.

16-22 A target is located at a distance of 12 km from the radar. Determine the time from the beginning of the transmitted pulse to the received echo.

16-23 A Doppler radar operating at 9 GHz is viewing a target moving directly away from it at a speed of 40 m/s. Determine the doppler shift.

16-24 A Doppler radar operating at 12 GHz is viewing a target moving directly toward it at a speed of 72 km/hour. Determine the doppler shift.

16-25 A Doppler radar operating at 10 GHz is located on a police car moving at 50 mph and is pointed backwards toward a car overtaking it. The frequency shift is 500 Hz. Determine the speed of the approaching vehicle.
16-26 Consider the situation depicted in Problem 16-25 with the police car moving at 50 mph. If the Doppler shift is -300 Hz, determine the speed of the back vehicle.

16-27 Consider the situation shown in Figure P16-27 with the Doppler radar operating at 12 GHz. If the frequency shift is 2 kHz, determine the speed of the car in miles per hour.

16-28 Consider the situation shown in Figure P16-28 with the Doppler radar operating at 10 GHz. Determine the Doppler shift if the car is moving at 50 mph.

16-29 An electromagnetic wave propagating in a medium with a dielectric constant of 3 encounters a boundary with air. The angle of incidence is $30^\circ$. Determine the angle of refraction.

16-30 An electromagnetic wave propagating in a medium with a dielectric constant of 3 encounters a medium with an dielectric constant of 6. The angle of incidence is $60^\circ$. Determine the angle of refraction.