Localized Algorithm for Precise Boundary Detection in 3D Wireless Networks

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- 2. Boundary detection
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- 3. Triangular Boundary Surface Construction
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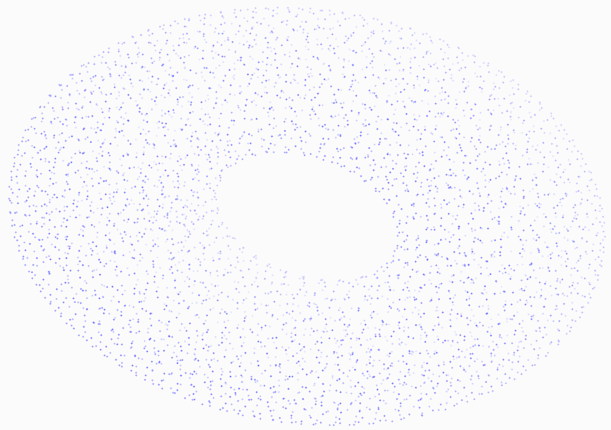
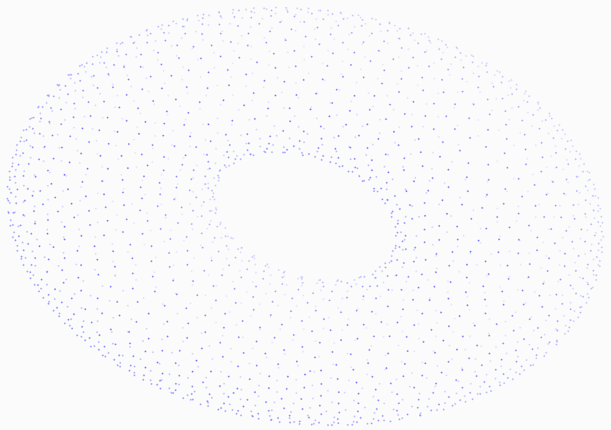
1. Introduction

- Motivation
  - Boundary nodes serve as a key attribute that characterizes the network, especially in geographic exploration tasks such as terrain and underwater reconnaissance.
  - Many wireless networks exhibit randomness
- Related works
  - All in 2D wireless networks

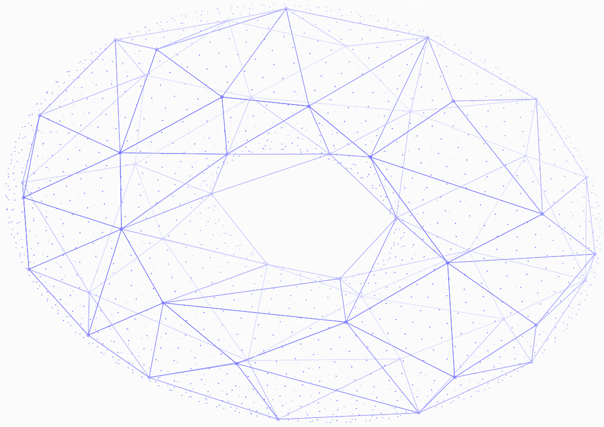
- Paper Contributions
  - Find a localized method that can precisely detect boundary nodes in 3D wireless networks;
  - Develop an algorithm to construct a 2-manifold planarized triangular mesh surface for 3D boundary



A 3D network



Boundary nodes



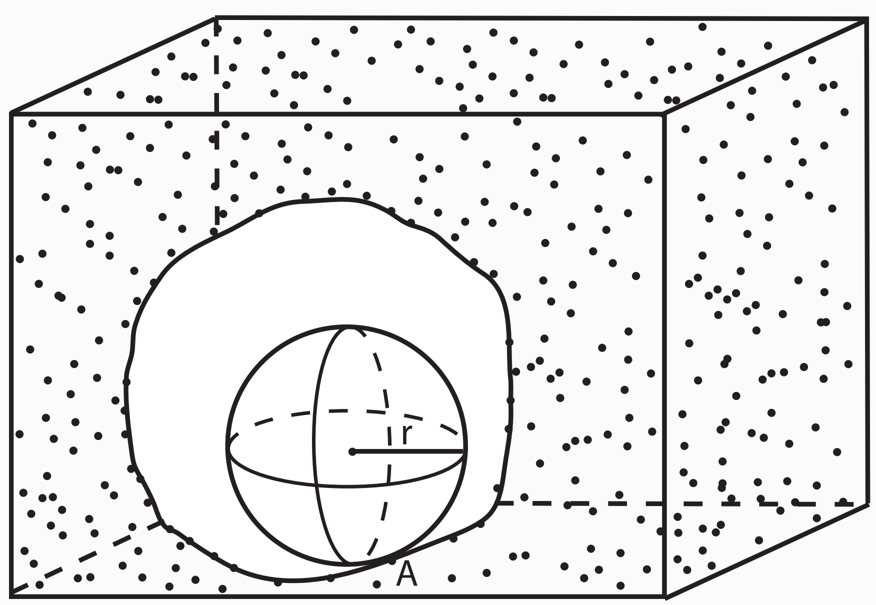
Triangular Mesh

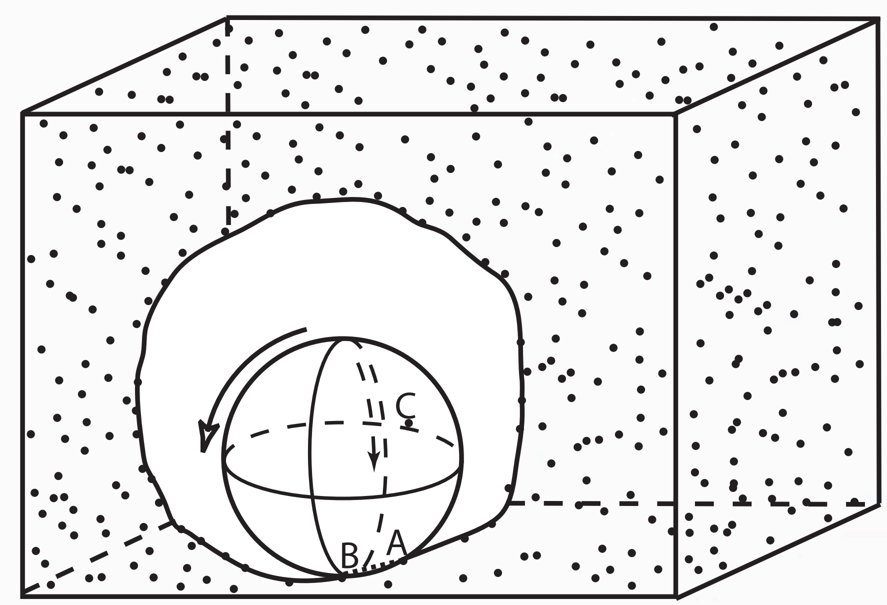
2. Boundary Node Identification

- 2.1 Unit Ball Fitting (UBF)
  - Definition 1: arbitrary radio transmission model with a maximum radio transmission range of 1;
  - Definition 2: the nodal density, denoted by ρ, is the average number of nodes in a unit volume;
  - Definition 3: networks are well connected, (1) no nodes are isolated; (2) no degenerated line segment;

- Definition 4: A unit ball is a ball with a radius of $r = 1+\delta$, δ is an arbitrarily small constant;
- Definition 5: An empty unit ball is a unit ball with no nodes inside;
- Definition 6: A unit ball *touch*es a node if the node is on the surface of the ball;
- Definition 7: A hole is an empty space that is greater than a unit ball. The space outside the network is treated as a special hole.

- **Lemma 1**: Node A can construct an empty unit ball that touches itself if and only if there exists an empty unit ball touching Node A and its two neighbors.
  - Sufficient condition: If a unit ball touched by Node A and its two neighbors is empty, this empty unit ball always touched by Node A.
  - Necessary condition: If there exits an empty unit ball with Node A on its surface
    - Fix node A and rotate the ball until it touches another node within $2r$, denoted by B. If node B does not exist, node A must be an isolated node.
    - Then rotating the ball with Line AB as an axis, until it touches another node, denoted by node C. And node C must exist, otherwise Line AB is degenerated.





- **Theorem 1**: Node A can determine if it can construct an empty unit ball that touches itself by testing $\Theta(\rho^2)$ unit balls and $\Theta(\rho)$ nodes for each ball.
  - Proof: According to Lemma1, Node A can exhaustively test all unit balls determined by Node A and its two distinct neighbors.
  - Node A has $\frac{4}{3}\pi(2r)^3\rho$, or $\Theta(\rho)$ neighbor nodes within $2r$, it needs to test up to $\Theta(2 \times \binom{\rho}{2}) = \Theta(\rho^2)$ unit balls;
  - For each unit ball, about $\frac{4}{3}\pi r^3\rho$, or $\Theta(\rho)$ nodes must be tested to judge if it is empty.
  - Therefore, the overall computing complexity is $\Theta(\rho^3)$.

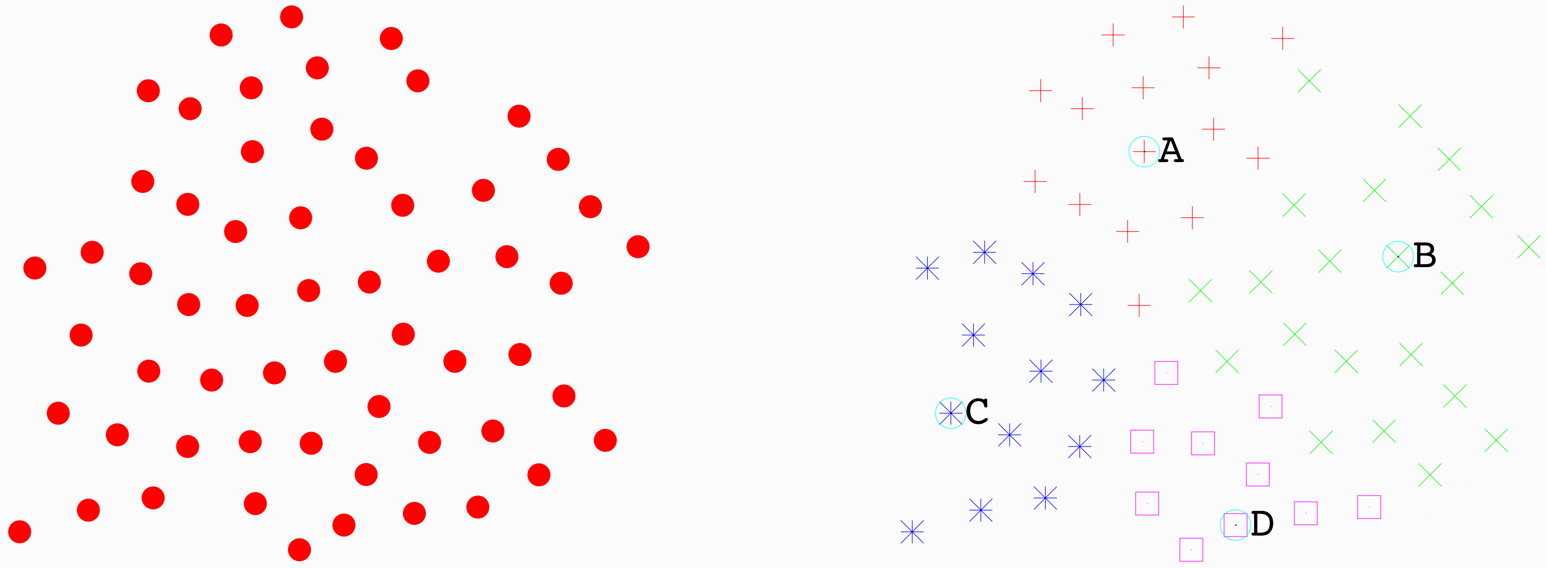
- Unit Ball Fitting (UBF) Algorithm Description
  - Step1: Local coordinates establishment; If all nodes already know their coordinates, this step can be skipped;
  - Step2: Unit Ball Identification; Calculate the center of the unit ball(s);
  - Step3: Empty unit ball checking.

2.2 Isolated Fragment Filtering (IFF)

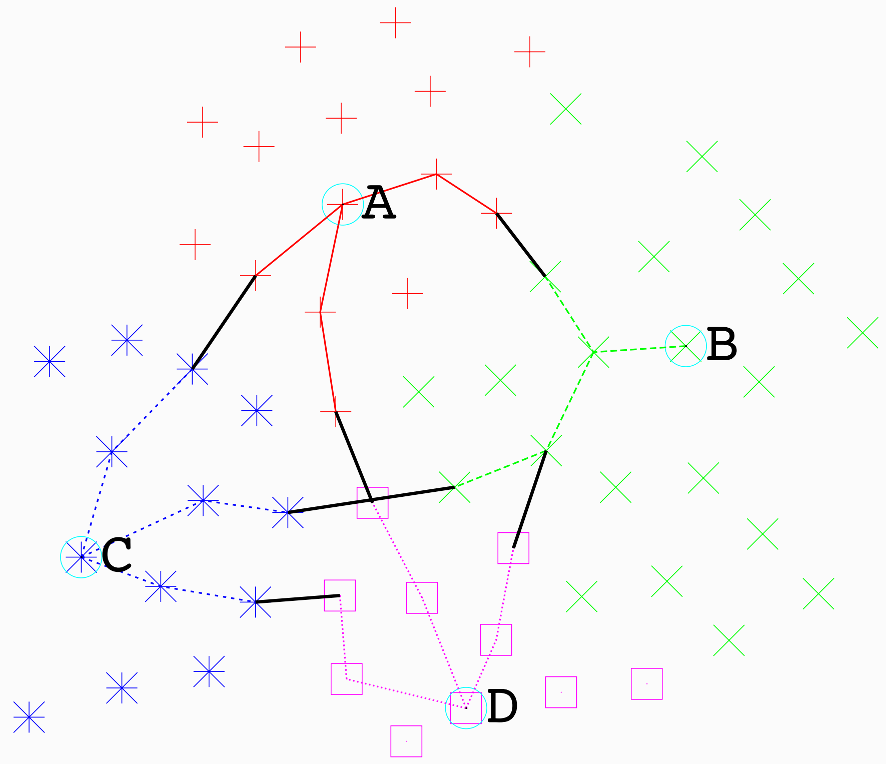
- Observation:
  - A small number of interior nodes may be interpreted by UBF as boundary nodes due to inaccurate nodal coordinates;
- Property of IFF:
  - The nodes on a boundary should form a well connected closed surface;
  - Set a threshold γ. Any fragment that consists of less than γ nodes is not considered as a boundary;
  - IFF can also be used to group boundary nodes, e.g, inner boundaries, outer boundary.

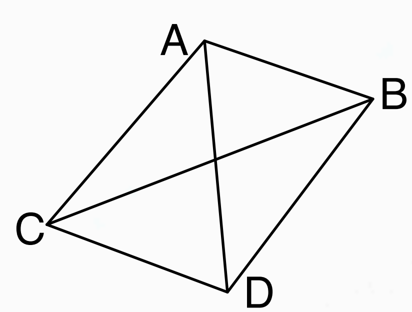
3. Triangular Boundary Surface Construction

- Step1: Landmark Selection
  - Select a subset of boundary nodes as “landmark”;
  - Any two landmarks must be k-hops apart;
  - Every other nodes will associate with the nearest landmark.

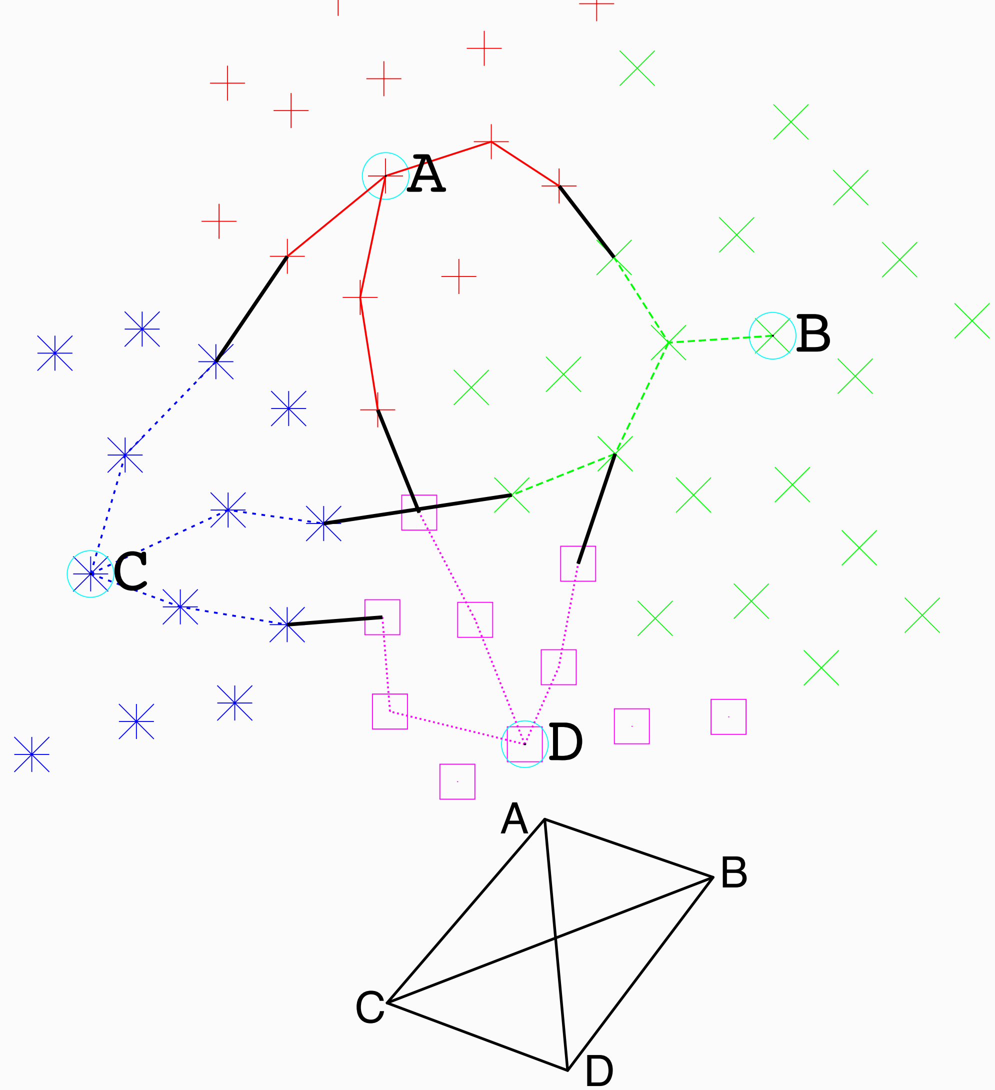


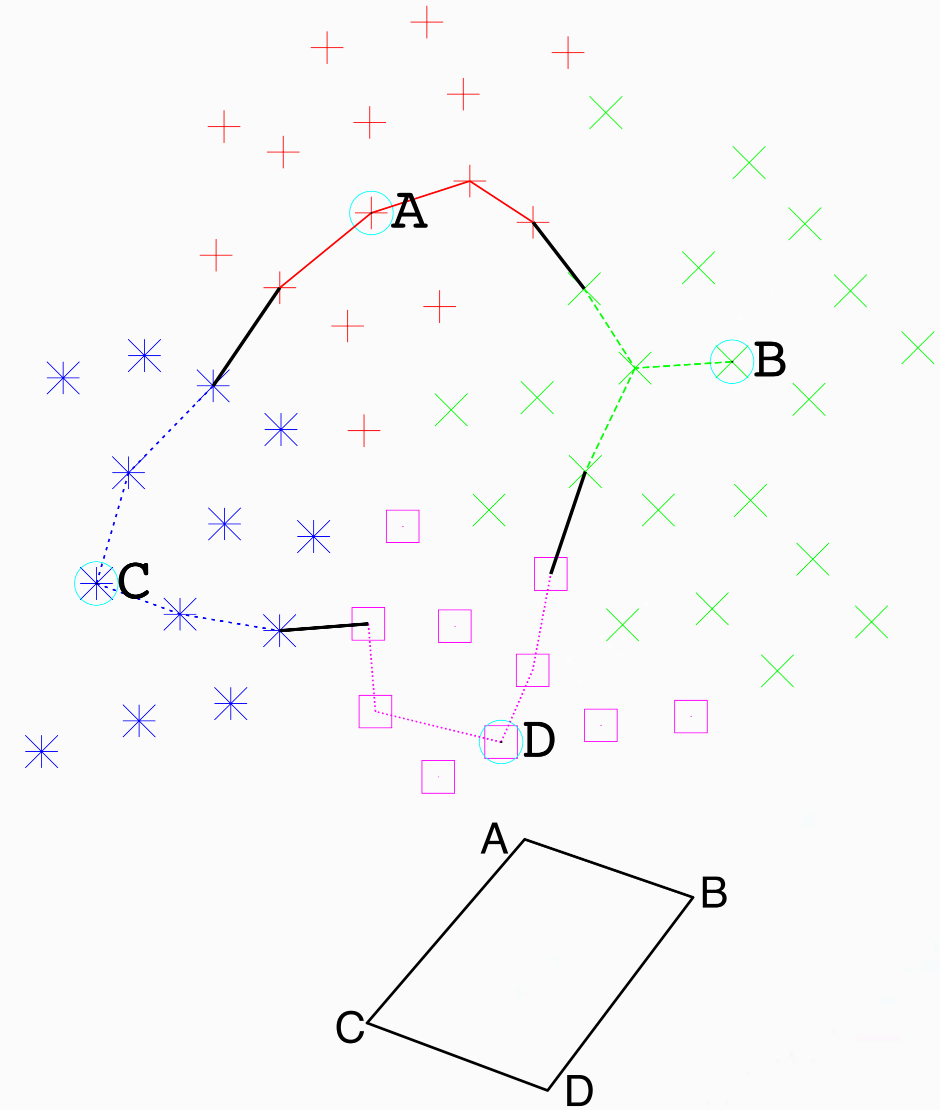
- Step2: Construction of *Combinatorial Delaunay Graph* (CDG)
  - Landmarks serve as CDG vertices;
  - If node A and node B are connected in CDG, there must exist a path between landmark A and landmark B and all the nodes on the path are associated with either A or B;
  - CDG is not a planar graph;



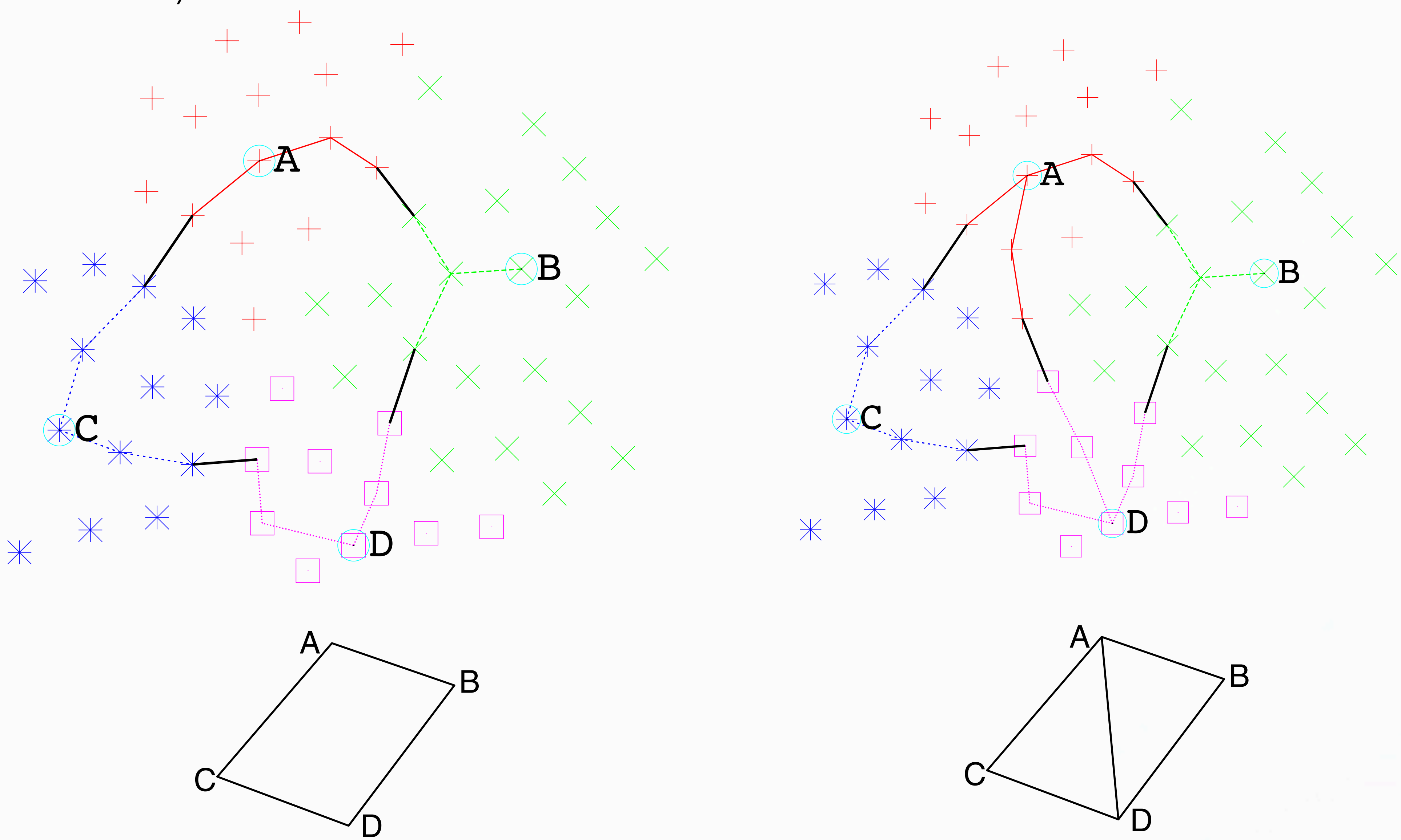


- Step3: Construction of *Combinatorial Delaunay Map* (CDM)
  - CDM is a subgraph of CDG and it is a planar graph;
  - If landmarks A and B are connected in CDM, besides all the nodes on the path between them are associated with either landmark A or B, all nodes in the1-hop neighborhood of the path also need to be associated with landmark A or B.

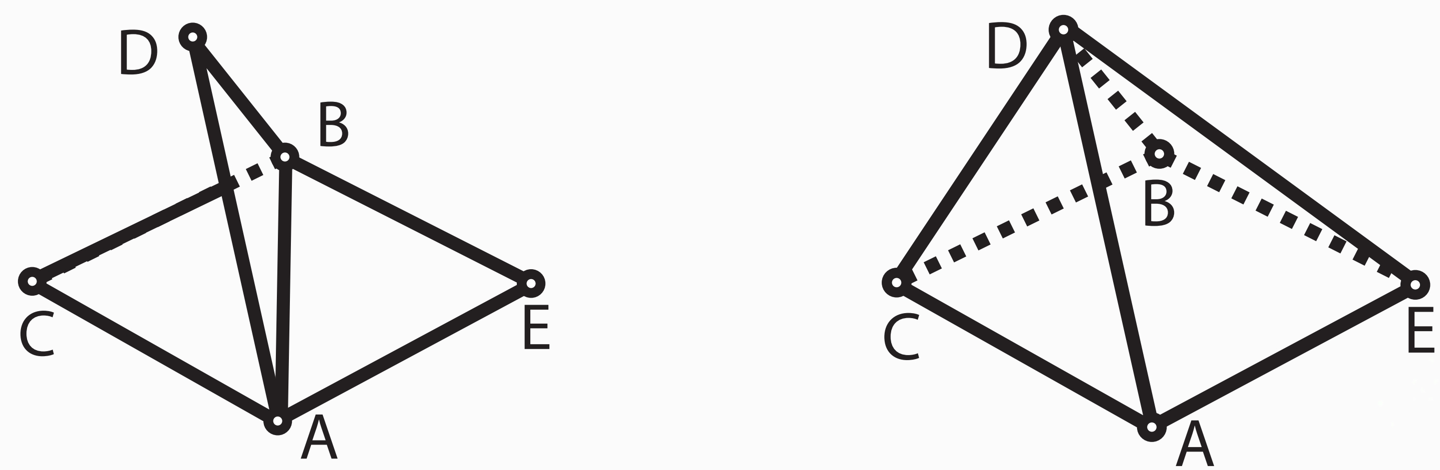


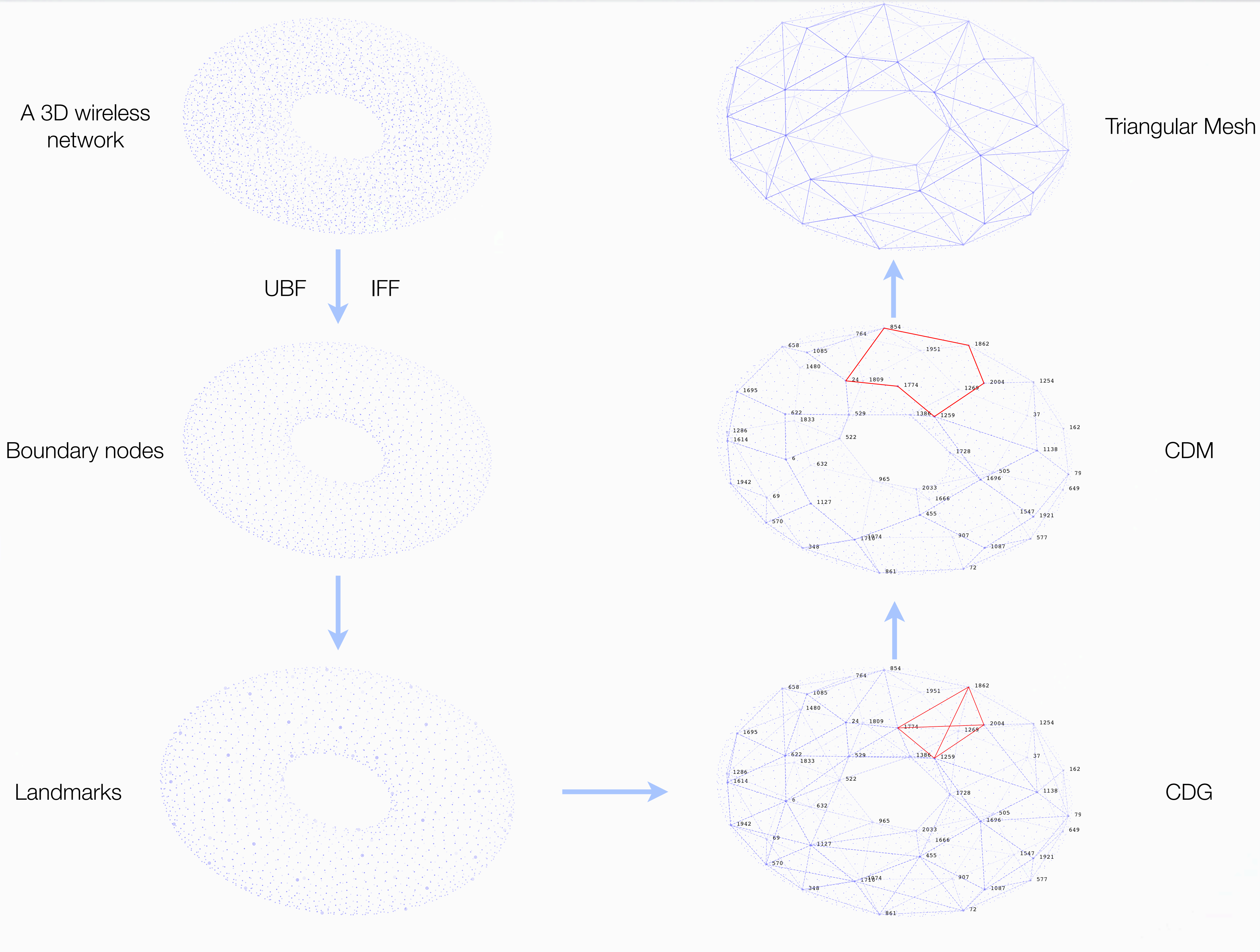


- Step 4: Construction of Triangular Mesh
  - CDM is a planar graph, but not always a triangular mesh;
  - Adding virtual edges in polygons by sending connection packet between landmarks(shortest path based on the identified boundary nodes).



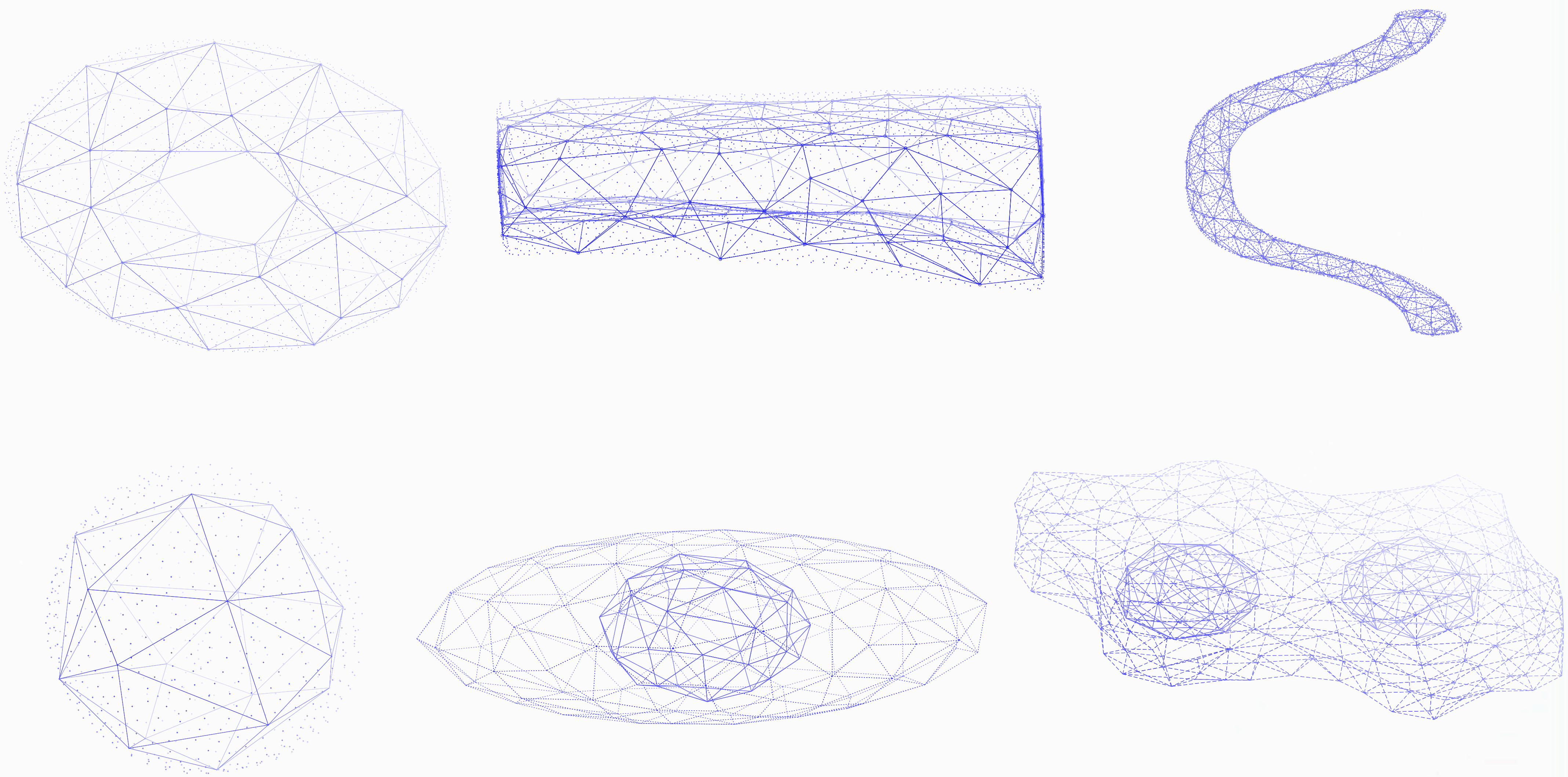
- Step 5: Edge Flip
  - To ensure the triangular mesh is 2-manifold, each virtual edge must be associated with two triangles. After above 4 steps, there still possibly exist edges with three triangular faces.

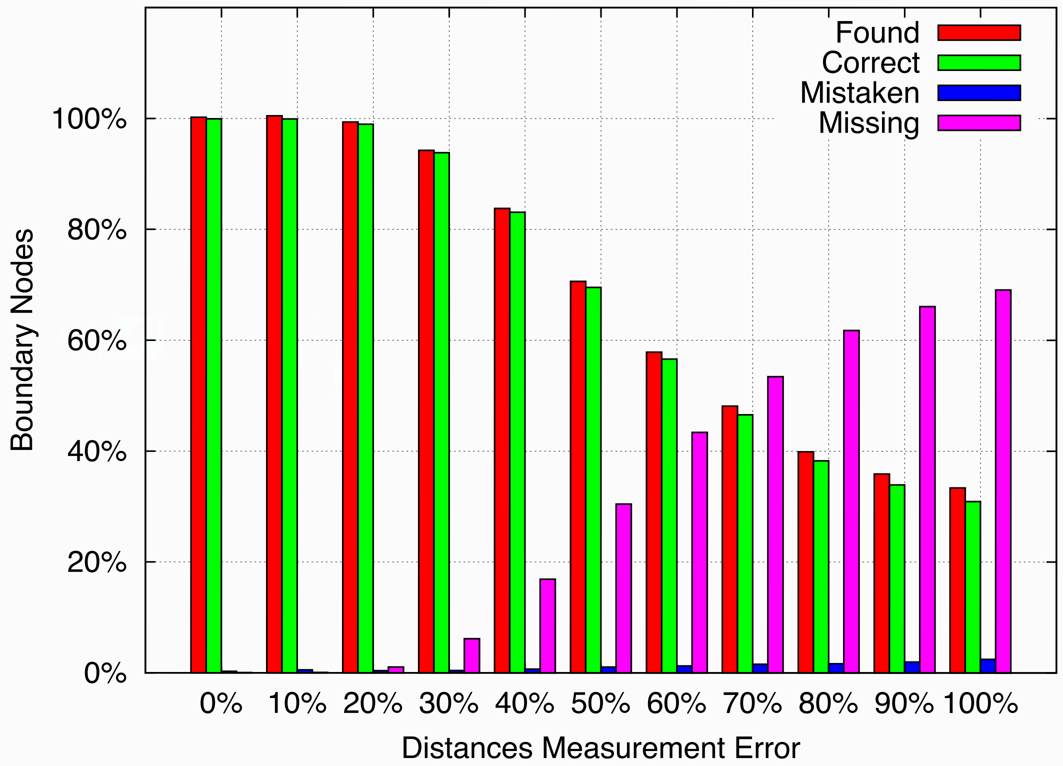


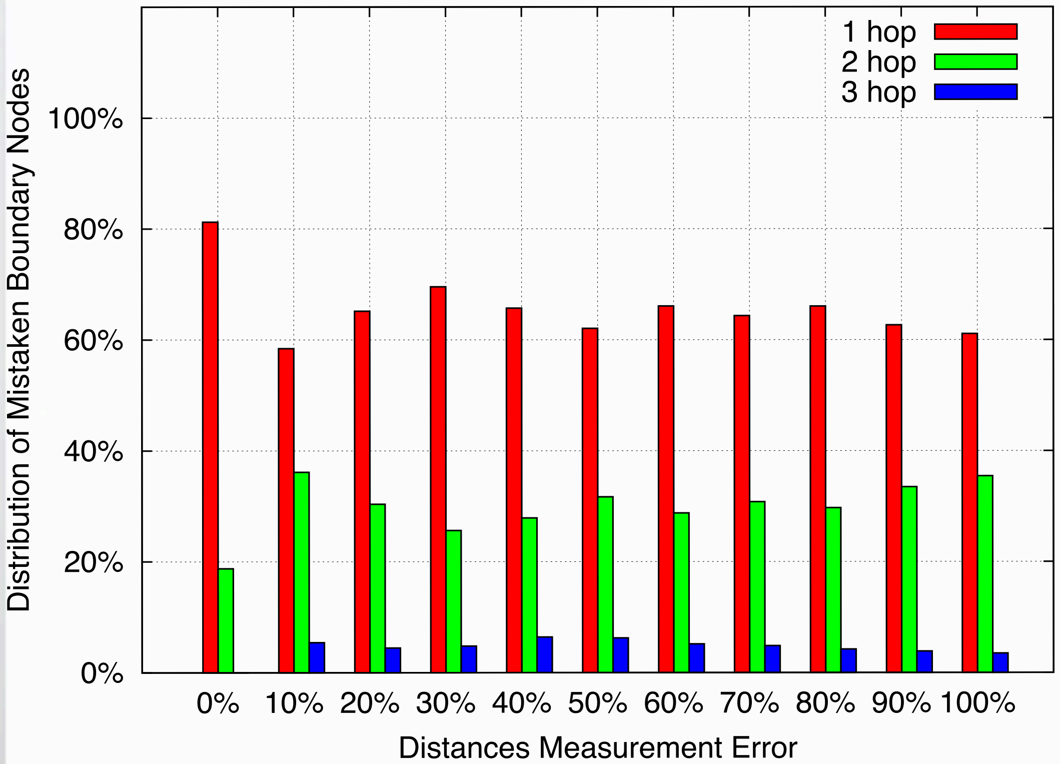

A 3D wireless network
UBF
IFF
Boundary nodes
Landmarks
Triangular Mesh
CDM
CDG

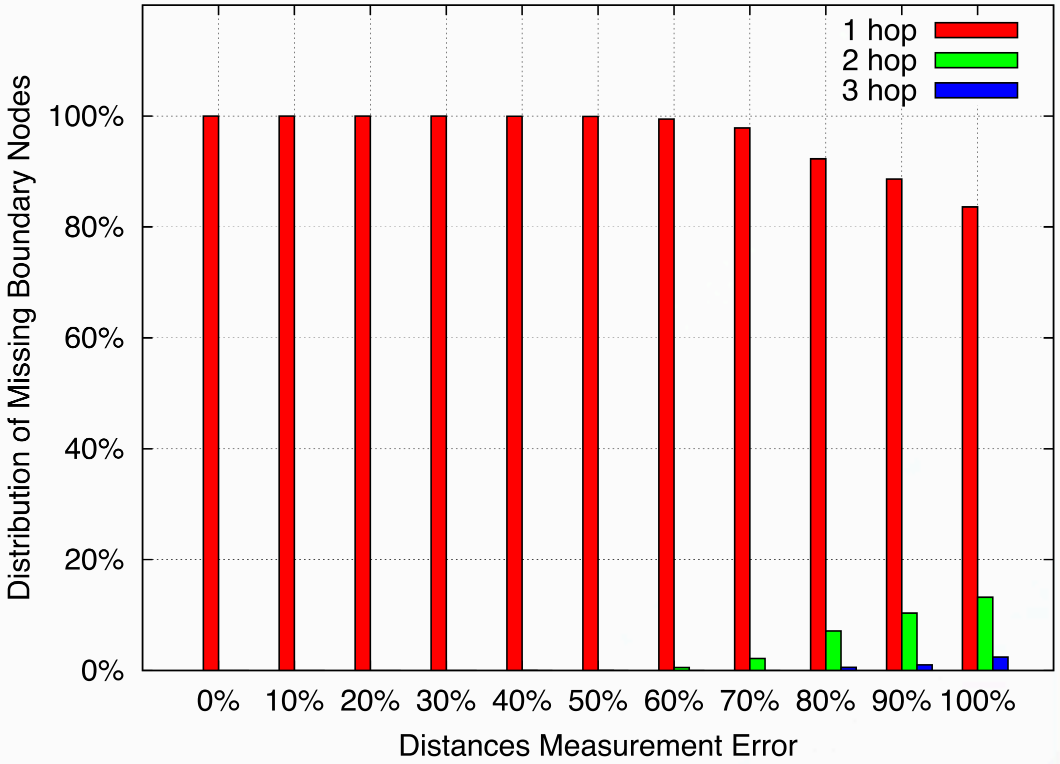
4. Simulation

- Six 3D wireless networks, over 10,000 nodes, node degree from 5 to 45, average degree 18.5. Random errors from 0 to 100% are introduced in the distance measurement.




Found
Correct
Mistaken
Missing
Boundary Nodes
0%
20%
40%
60%
80%
100%
0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
Distances Measurement Error


1 hop
2 hop
3 hop
Distribution of Mistaken Boundary Nodes
0%
20%
40%
60%
80%
100%
0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
Distances Measurement Error


1 hop
2 hop
3 hop
Distribution of Missing Boundary Nodes
0%
20%
40%
60%
80%
100%
0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
Distances Measurement Error

5. Conclusion

- We have proposed distributed and localized algorithms for precise boundary detection in 3D wireless network:
  1) Identify the the boundaries nodes of a 3D network;
  2) Construct planarized 2-manifold surfaces for inner and outer boundaries.
- As far as we know, this is the first work for discovering boundary nodes and constructing boundary surface in 3D wireless networks.

Thank you!