A Distributed Delaunay Triangulation Algorithm Based oN Centroidal Voronoi Tessellation For Wireless Sensor Networks

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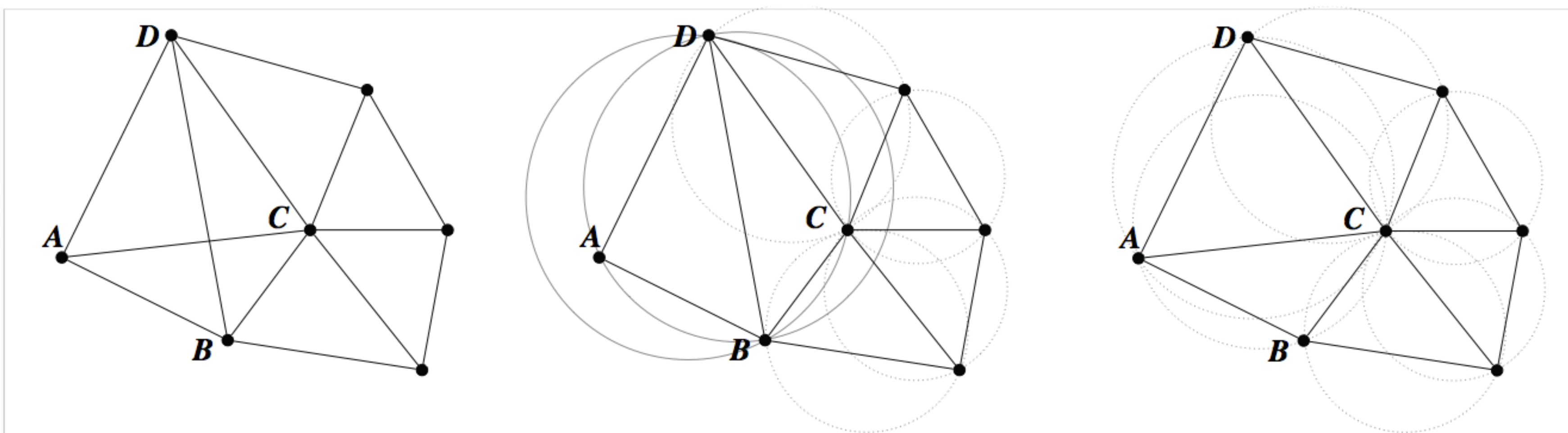
AUTONOMOUS LOCALIZATION IN WIRELESS SENSOR NETWORKS

- Sensor network graph often exhibits undesired randomness and irregularity
- Special treatment of the graph is required by a variety of network algorithms and protocols
- Many geometry-oriented algorithms depend on a type of subgraph called <u>triangulation</u>
  - Geometric routing
  - Autonomous localization
  - Sensor coverage
  - Network segmentation
  - Distributed data storage and processing
- Triangulation is a planar graph, with no crossing edges

AUTONOMOUS LOCALIZATION IN WIRELESS SENSOR NETWORKS

- Definition 1. A <u>Delaunay triangulation</u> for a set of points on a plane is a triangulation such that no point is inside the circumcircle of any triangle [12]
- Delaunay triangulation avoids skinny triangles, preferred
  - Greedy forwarding is guaranteed to succeed
  - Better localization can be achieved
- Delaunay triangulation with all edges equal is call an equilateral Delaunay triangulation



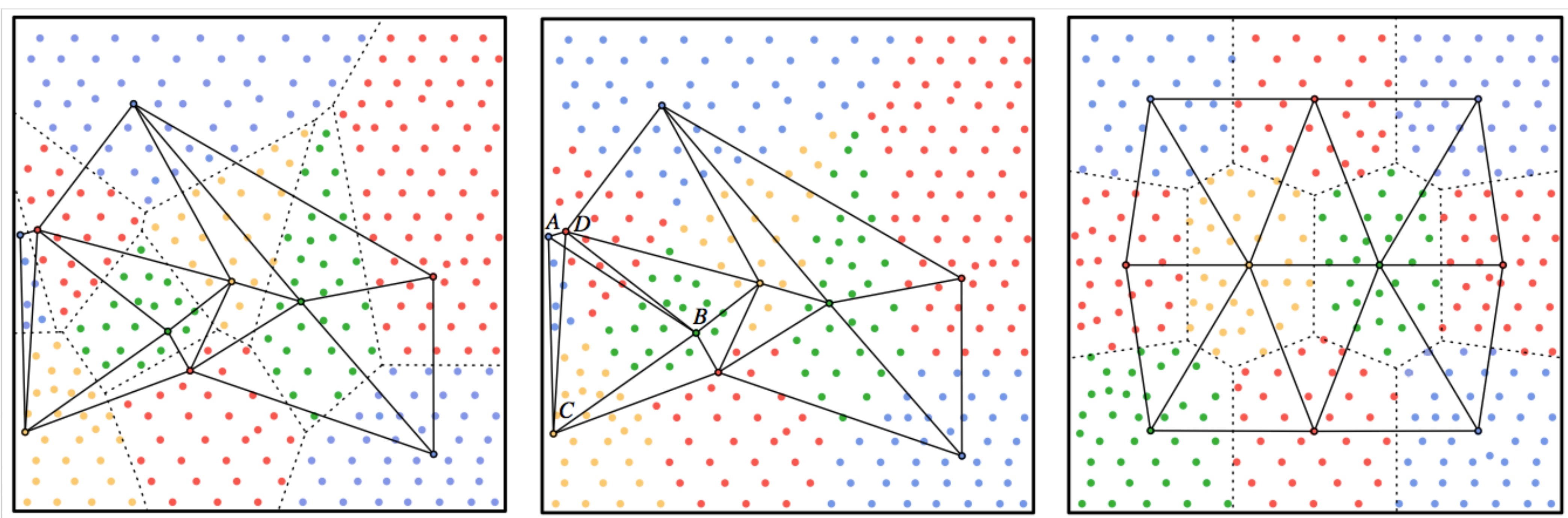
(a) A nonplanar network graph. (b) A triangulation of the graph. (c) A Delaunay triangulation of the graph.

STATE-OF-THE-ART FOR CONNECTIVITY-BASED DELAUNAY TRIANGULATION

- If location or distance is available, Delaunay triangulation can be straightforwardly constructed [4]
- Nontrivial by using connectivity information only
  - Dual graph for a Voronoi diagram is a Delaunay triangulation
  - Establish approximate Voronoi diagram
    - A node is randomly chosen as a generating point
    - Claim its K-hop neighbors to form a cell
    - Repeat in the remaining network until every node is either a generating point or associated with a generating point
    - If a node is claimed by multiple generating points, it chooses the closest one (in term of hop count) and joins its cell
  - Dual graph, called combinatorial Delaunay graph (CDG)
    - If two cells are adjacent, i.e., have at least one pair of neighboring nodes (one in each cell), a virtual edge is established to connect the corresponding generating points

STATE-OF-THE-ART

- Although Dual graph of a precise Voronoi diagram is a Delaunay triangulation, CDG is not, not even planar
  - Approximation in constructing Voronoi and dual graph
  - Boundary of two adjacent approximate Voronoi cells can be shorter than one hop
  - Non-neighboring cells be mistakenly considered as neighbors



(a) (b) (c)

STATE-OF-THE-ART

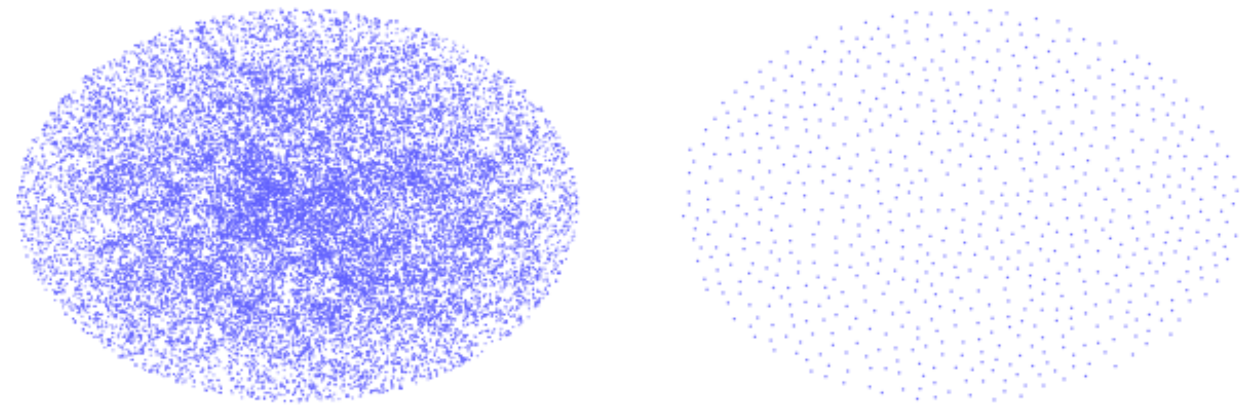
- Possible improvement
  - Larger K (cell size), less crossing edges
    - Coarse triangulation
    - No guarantee of crossing edges-free
  - Crossing edges in CDG can be removed [13]
    - Yielding a Combinatorial Delaunay Map (CDM)
    - Planar but with polygon holes
    - Therefore it is not a triangulation
  - The algorithms proposed in [1, 11] try to fill the holes
    - Form triangulation
    - No guarantee to hold the Delaunay property

MOTIVATION

- Problem of VD-based approaches
  - Fact: boundaries of Voronoi cells are nonuniform; some boundaries can be arbitrarily short
  - With connectivity information only, it is fundamentally unattainable to correctly judge neighboring cells when a Voronoi cell boundary is less than one hop
  - Generating points mistakenly connected, thus crossing edges
- Motivation: desired to build a Voronoi diagram with uniform cell boundaries, to maximize the minimum boundary length
- Proposed solution: triangulation based on centroidal Voronoi tessellation (CVT)
  - Proven result: always construct a Delaunay triangulation, if the CVT cell size is greater than a constant
  - The established Delaunay triangulation consists of close-to-equilateral triangles

CVT CONSTRUCTION

- Definition 2. A centroidal Voronoi tessellation (CVT) is a special Voronoi diagram, where the generating point of each Voronoi cell is also its mean (i.e., center of mass) [14]
- In order to build uniform cells, which induce a Delaunay triangulation with equilateral triangles, the network must have a uniform density function
- Network with non-uniform density
  - Single-hop Voronoi sampling



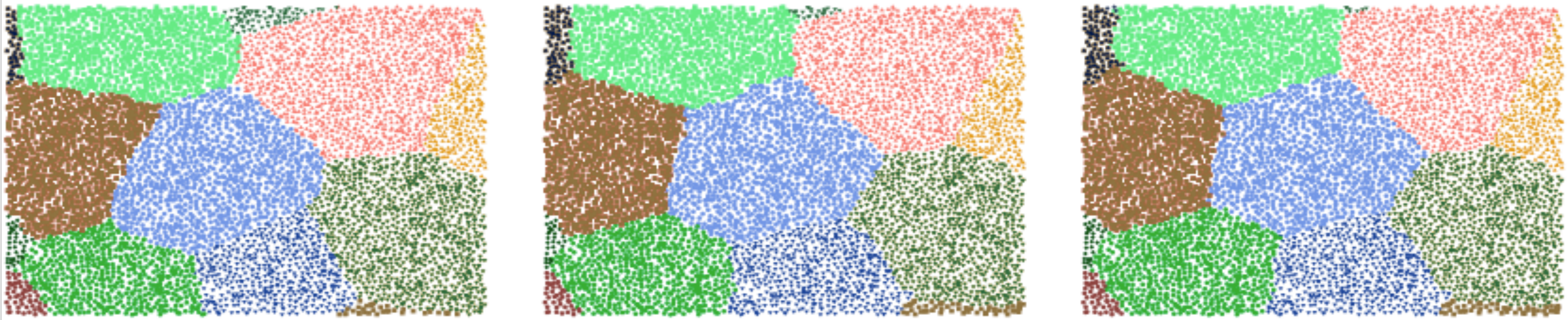
(a) Non-uniformed topology. (b) Sampled subgraph.

CVT CONSTRUCTION

- Initialization: an arbitrary Voronoi diagram with a cell size of K-hops and a set of generating points: $L = \{L_1, L_2, \ldots, L_m\}$
- Iterative CVT construction
  - Cells construction: nodes are associated with their closest generating points to form tessellations
  - Generating points update: to the current centroid of the cell
    - Every node learns its hop distance to other nodes in the same cell via localized flooding (within its cell).
    - Each node calculates standard deviation of such distances
    - Node with minimal standard deviation is the approximate centroid of the tessellation and selected as the new generating point
    - Cells and generating points are updated repeatedly until the generating points remain unchanged

CVT CONVERGENCE

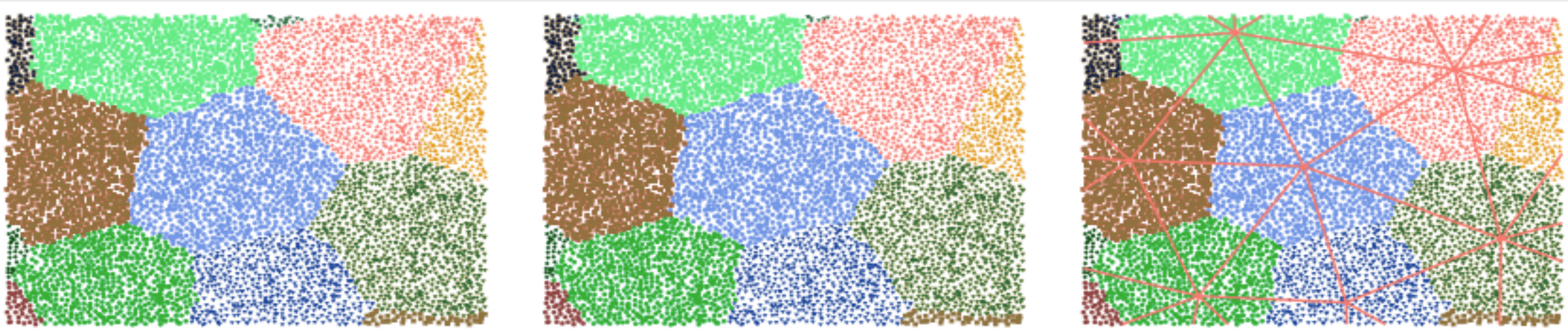
- Theorem 1. The proposed CVT construction algorithm converges
  - Proof: the algorithm monotonically decreases the average distance from a sensor to its generating point
- Speed of convergence
  - Difficult to derive a theoretic bound for the number iterations in order to reach the convergence
  - Converges very fast in practice (4-5 iterations in simulation)



(a) Initial Voronoi diagram. (b) Result after two iterations. (c) Result after four iterations.

DELAUNAY TRIANGULATION

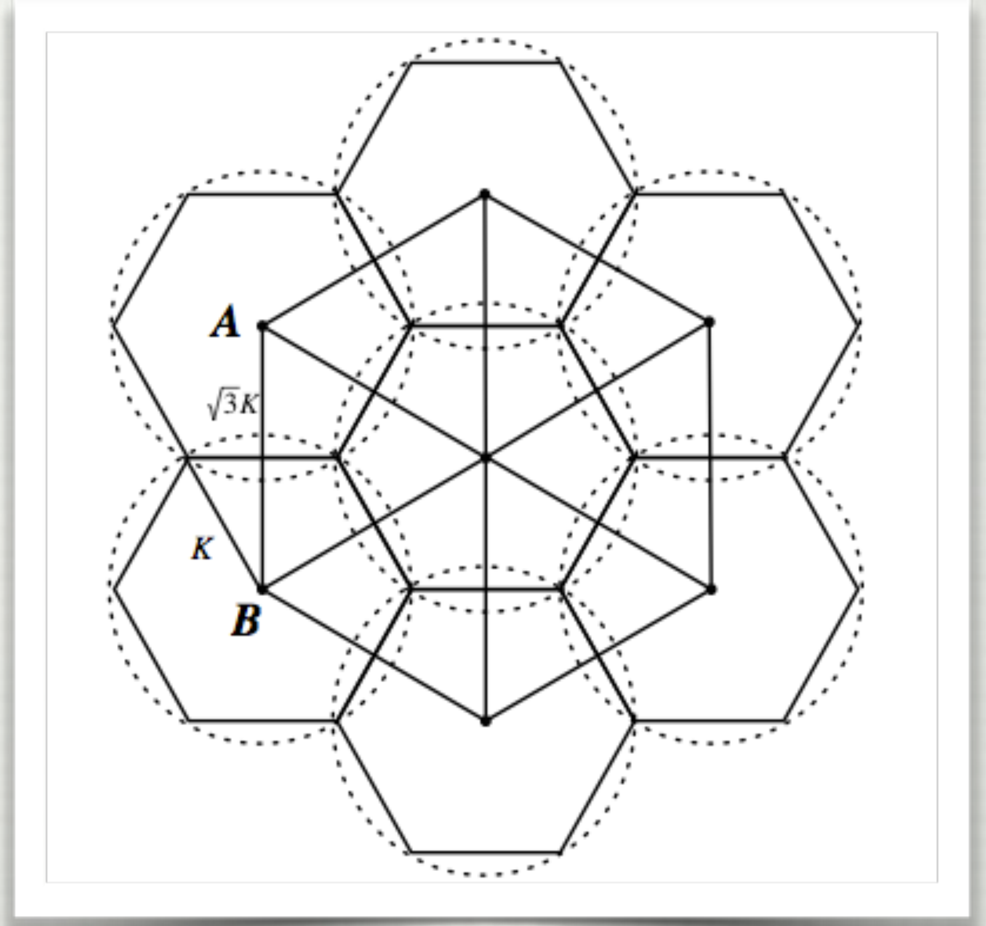
- Triangulation
  - Dual graph is established by connecting every two adjacent CVT generating points with a virtual edge
  - Two generating points are adjacent if at least one pair of their cell members (one in each cell) are neighbors



(b) Result after two iterations. (c) Result after four iterations. (d) Final result.

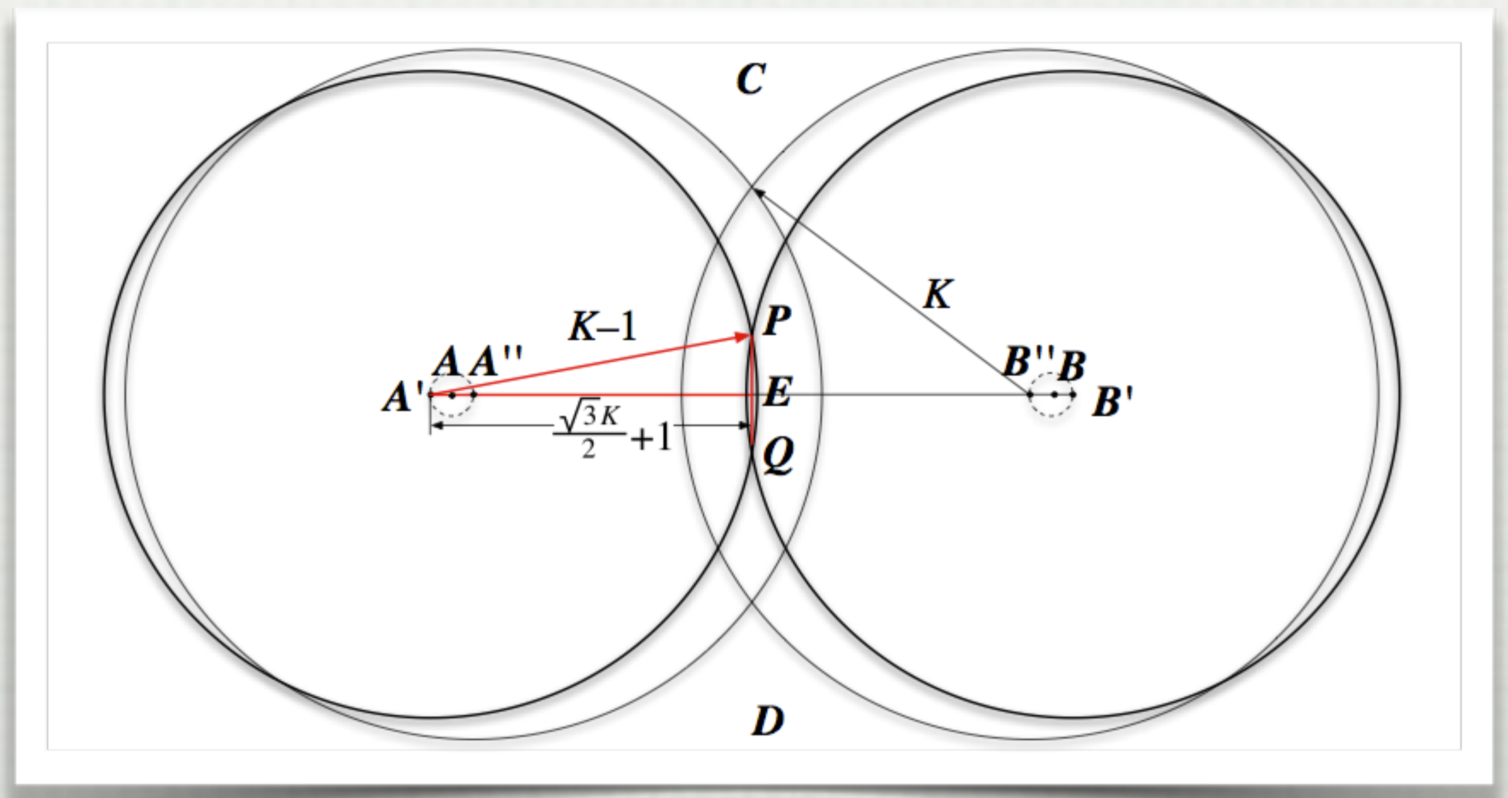
DELAUNAY TRIANGULATION

- Theorem 2. Given an asymptotically deployed wireless sensor network, the centroidal Voronoi tessellation (CVT) with a cell size greater than 15 always yields a Delaunay triangulation.
- Proof
  - Ideal CVT: if the nodal density is uniform and infinity and network is asymptotically deployed, a cell of CVT is a regular hexagon
  - The distance between any two neighboring centroids is √3K
  - Ideal triangulation: the dual of the CVT is a Delaunay triangulation with equilateral triangles



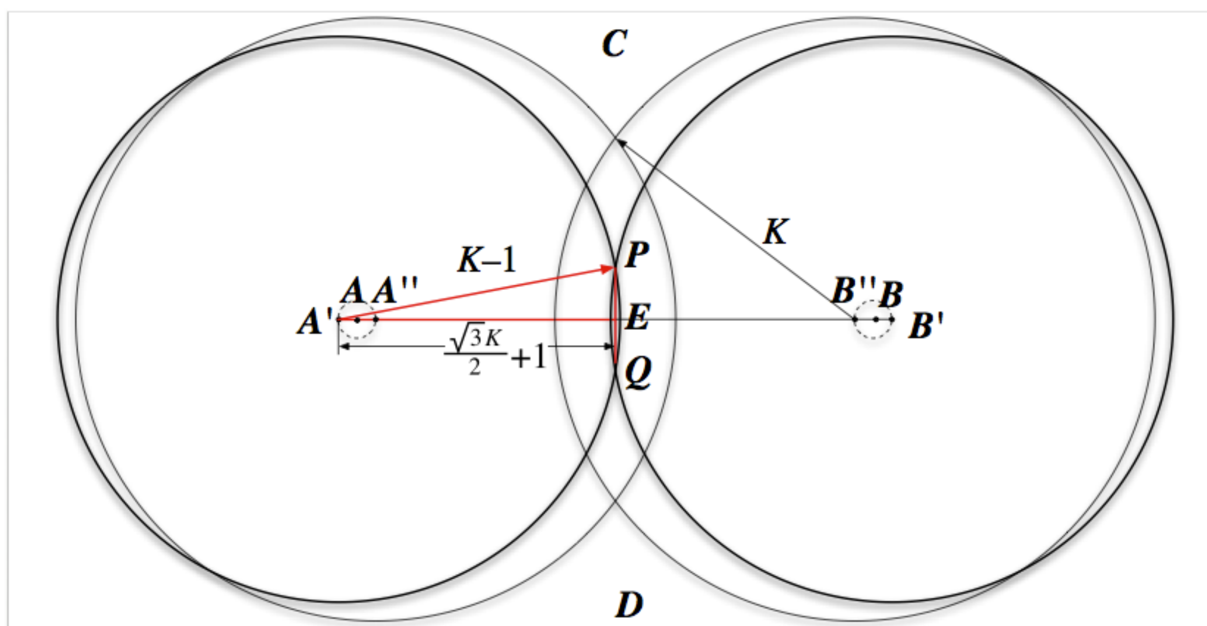
DELAUNAY TRIANGULATION

- Approximation under a discrete setting
  - An approximate CVT, where the tessellation is performed according to hop counts
  - The cell size is the range of (K-1, K]
  - A node is not always available at its real centroid



DELAUNAY TRIANGULATION

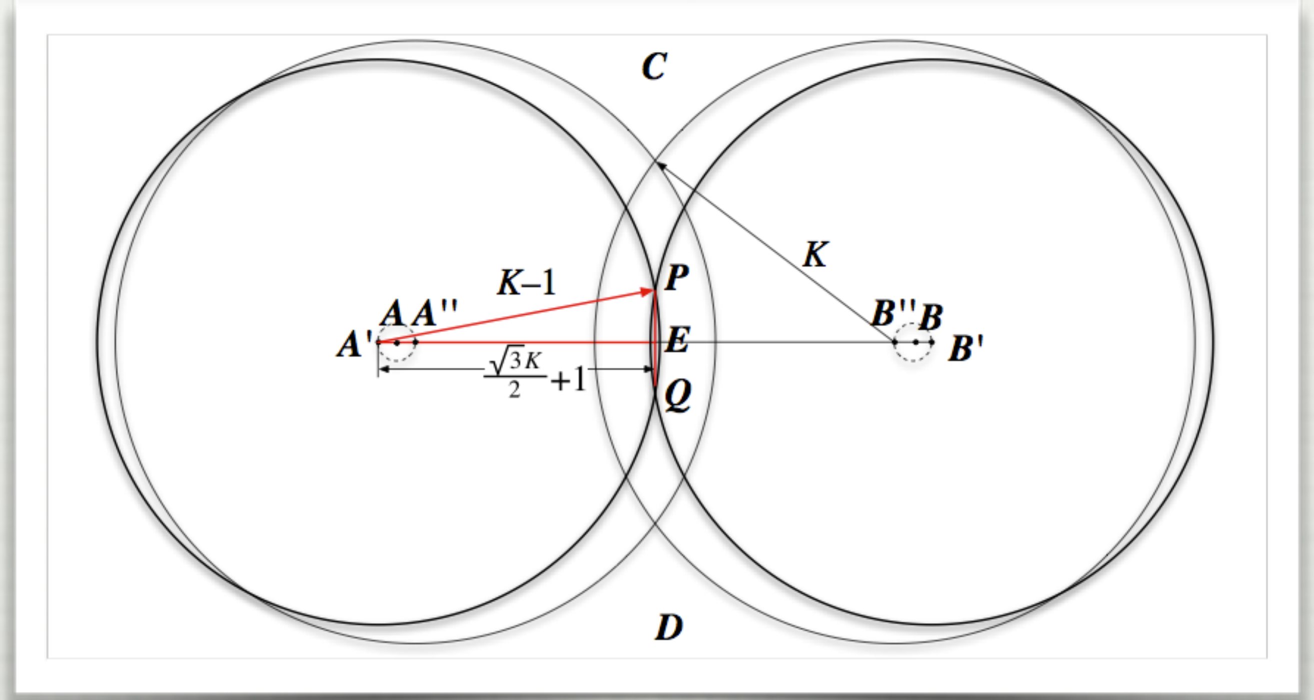
- Show: when K greater than a constant, approximate CVT induces the same ideal triangulation, with neither extra nor missing edges
  - If extra edge is added, it must result in a crossing edge
  - To ensure free of crossing edges, the boundary between the two cells must be greater than the radio range
  - Examine the worst case: two generating points are farthest away from each other and the cell size reaches minimum (k-1)



To ensure PQ>1,K must be >15

DELAUNAY TRIANGULATION

- Since PQ is greater than one, Cells C and D not connected,
- If Cells A and B are not connected either, there must exist a void (i.e., a hole) in the middle of the four cells
- The hole forms a network boundary
- Contradicts to the assumption of asymptotic sensor deployment



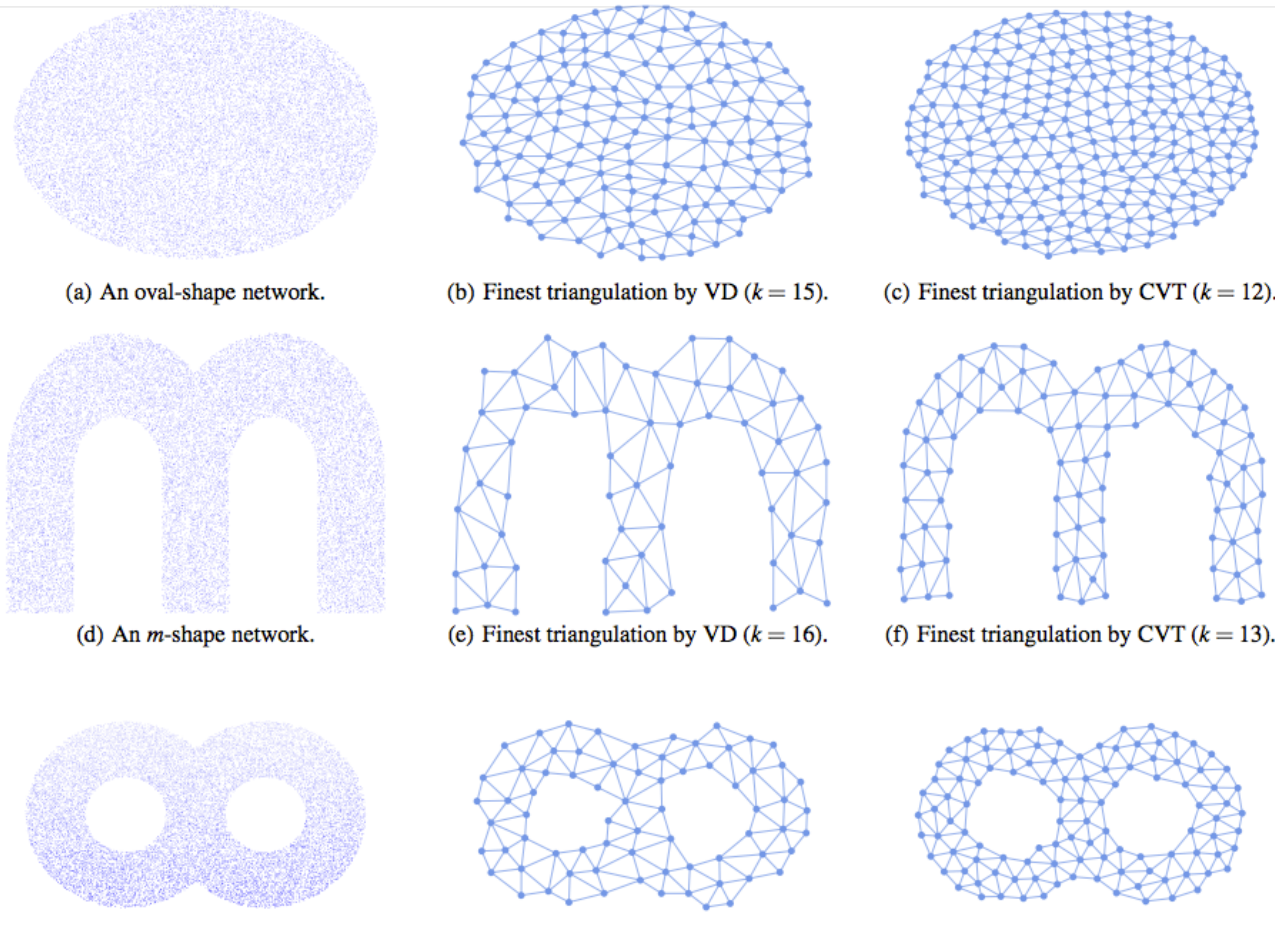
DELAUNAY TRIANGULATION

- Summary: when K>15, there is neither extra nor missing edges in comparison with ideal triangulation. The theorem is proven.
  - Theorem 2 only shows a provable sufficient condition
  - Not always necessary to have K > 15 for Delaunay triangulation
  - When K ≤ 15, although without a proof, it is intuitively better than its VD-based counterpart
    - High success rate
    - Close-to-equilateral triangles
- Time complexity and communication cost
  - Both linear, O(n)

SIMULATION RESULTS

- Sensor nodes are randomly deployed
- Proposed algorithm does not depend on any specific communication model
- General communication model, with merely a constraint on maximum radio transmission range, normalized to one.
  - Two nodes are disconnected if they are separated by a distance greater than one
  - Connected if their distance is less than α
  - Connected with a probability if their distance is between α and one
  - In this simulation, α is set to 0

TRIANGULATION GRANULARITY

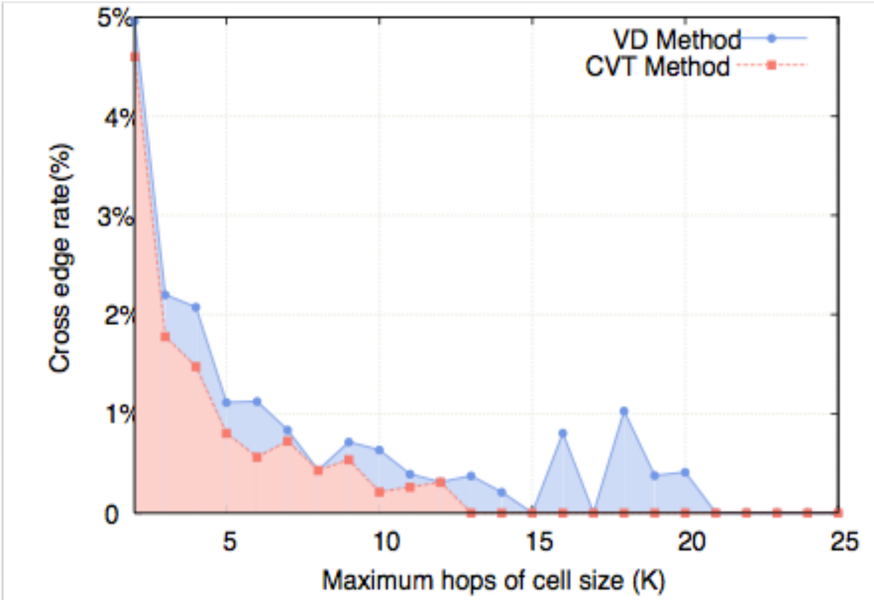


(a) An oval-shape network. (b) Finest triangulation by VD ($k = 15$). (c) Finest triangulation by CVT ($k = 12$).

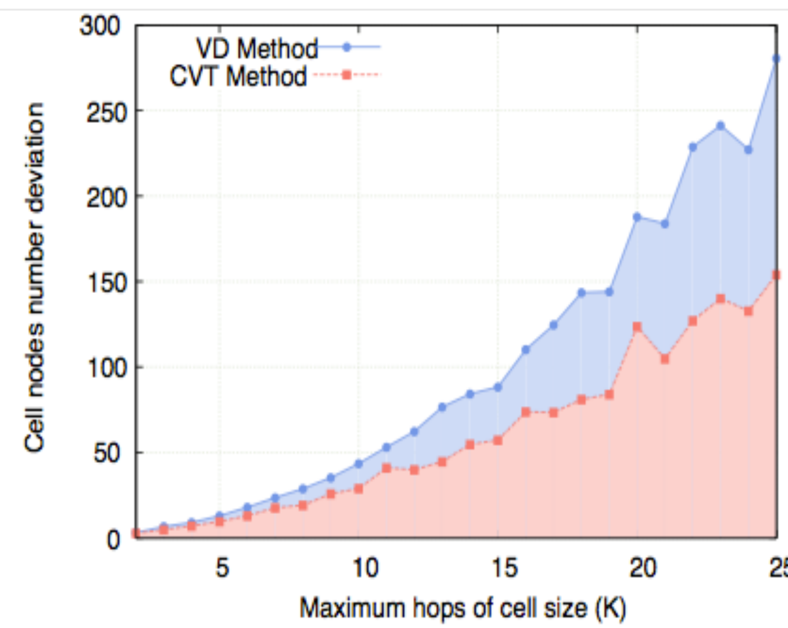
(d) An m-shape network. (e) Finest triangulation by VD ($k = 16$). (f) Finest triangulation by CVT ($k = 13$).

(g) An 8-shape network (with 2 holes). (h) Finest triangulation by VD ($k = 15$). (i) Finest triangulation by CVT($k = 12$).

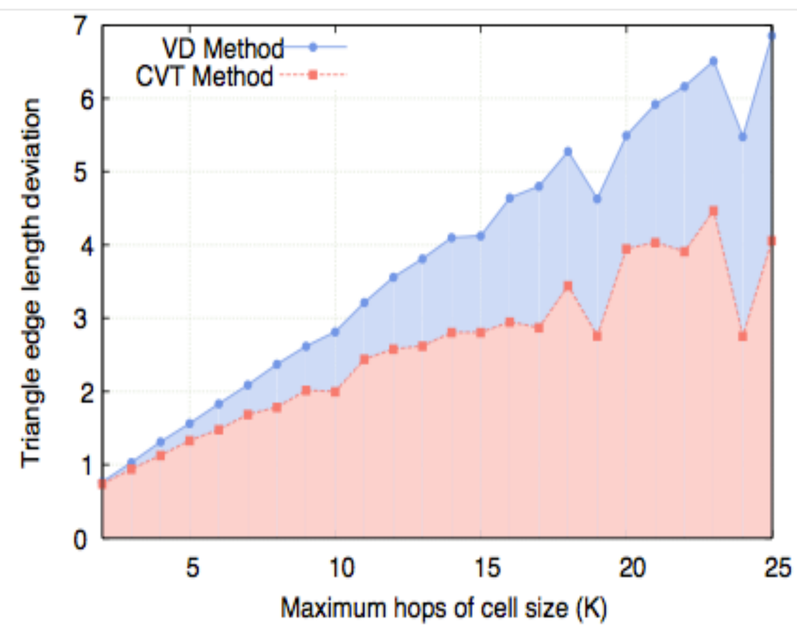
TRIANGULATION REGULARITY



(a) Crossing edge rate.



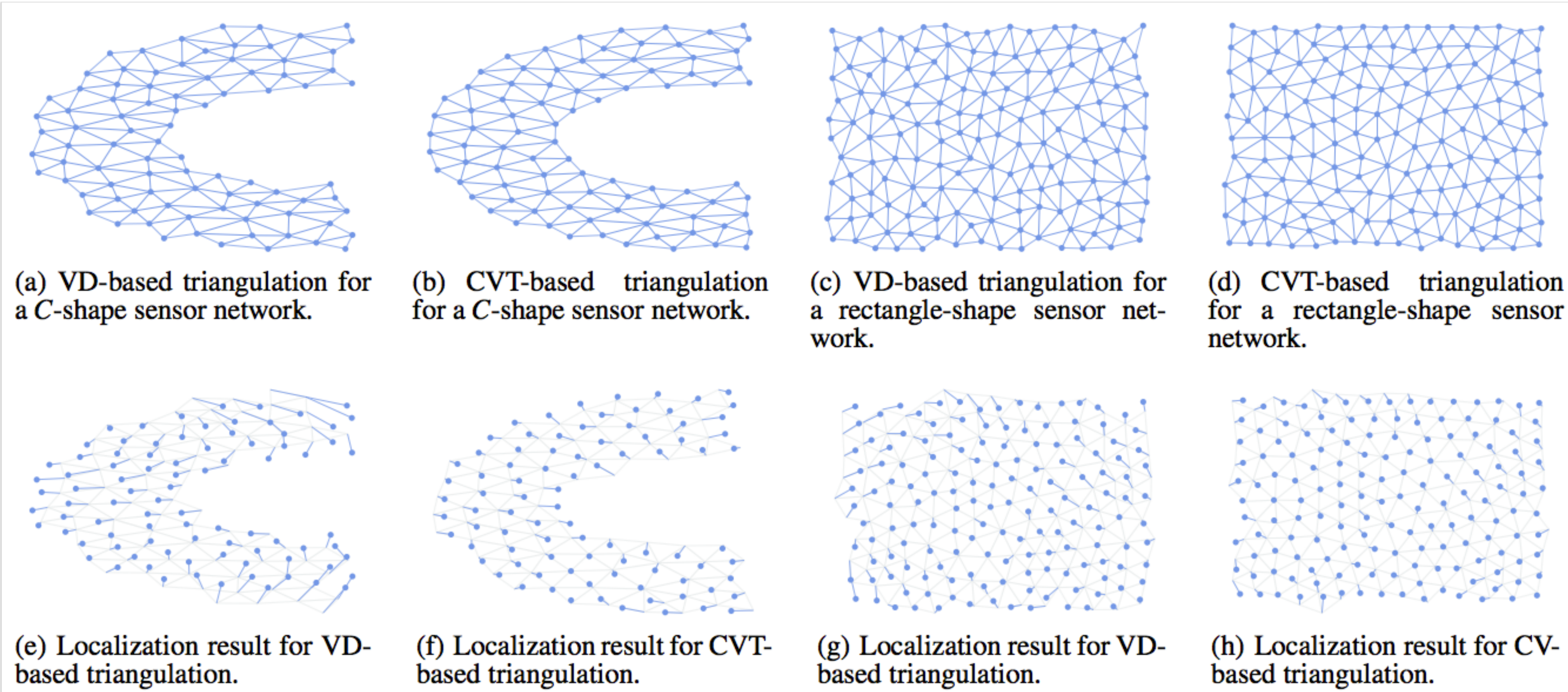
(b) Deviation of cell size.



(c) Deviation of triangle edge length.

IMPROVE LOCALIZATION

- The regularity of triangulation benefits a range of applications



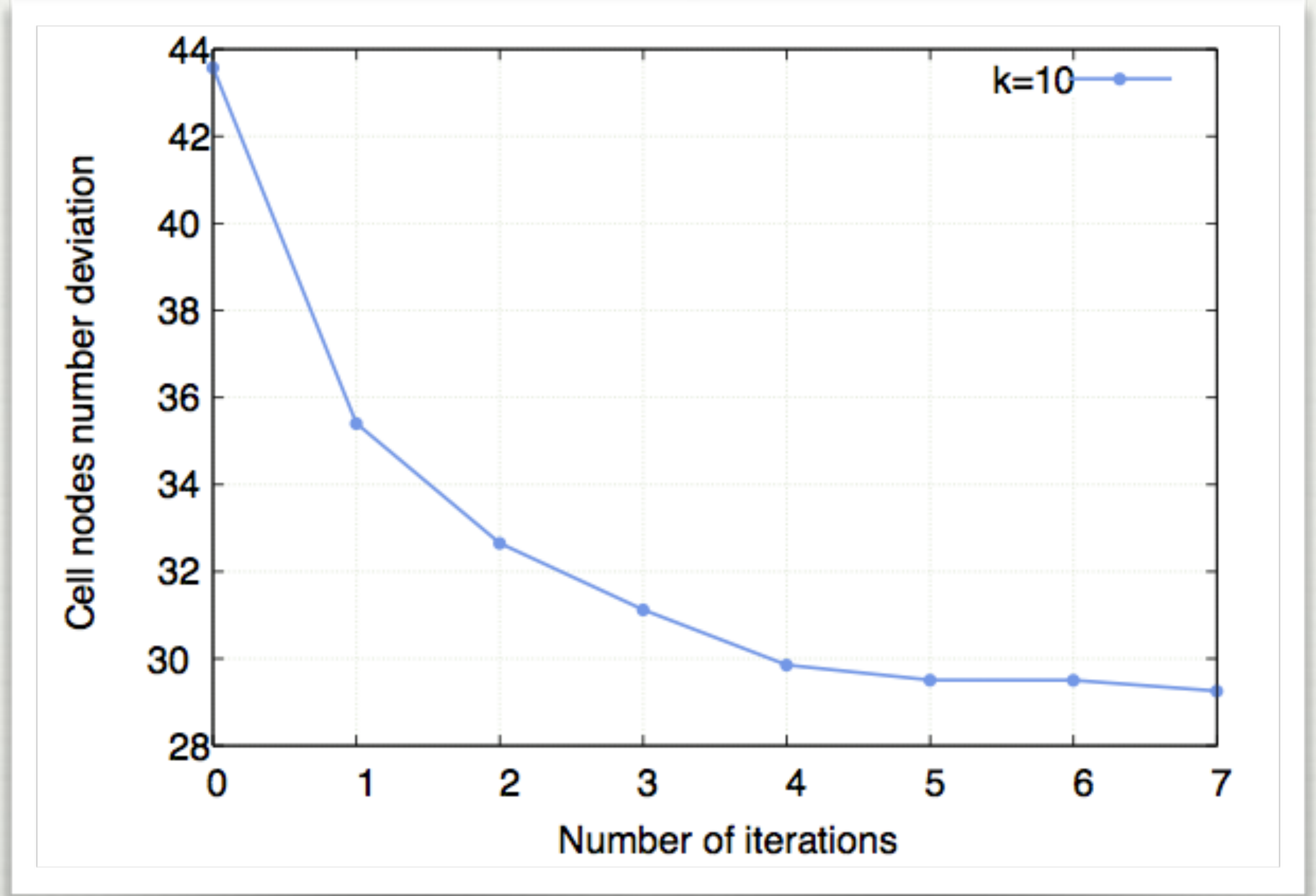
(a) VD-based triangulation for a *C*-shape sensor network.
(b) CVT-based triangulation for a *C*-shape sensor network.
(c) VD-based triangulation for a rectangle-shape sensor network.
(d) CVT-based triangulation for a rectangle-shape sensor network.
(e) Localization result for VD-based triangulation.
(f) Localization result for CVT-based triangulation.
(g) Localization result for VD-based triangulation.
(h) Localization result for CV-based triangulation.

Results of connectivity-based localization method is introduced in [6]

IMPROVE LOCALIZATION

	C-shape Model (see Fig. 9(a)-9(b))	Rectangle Model (see Fig. 9(c)-9(d))
VD	0.46	0.24
CVT	0.21	0.18

CONVERGENCE



CONCLUSIONS

- Delaunay triangulation is desired in many geometry-oriented algorithms
- Nontrivial to achieve based on connectivity information
- Propose a distributed algorithm for Delaunay triangulation
  - Performs centroidal Voronoi tessellation
  - Constructs dual graph to yield Delaunay triangulation
- Proven convergence of centroidal Voronoi tessellation in sensor networks
- Proven to succeed in constructing a Delaunay triangulation, if the CVT cell size is greater than a constant
- Evaluate via simulations and demonstrate benefits in applications