A Distributed Triangulation Algorithm for Wireless Sensor Networks on 2D and 3D Surface

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Outline

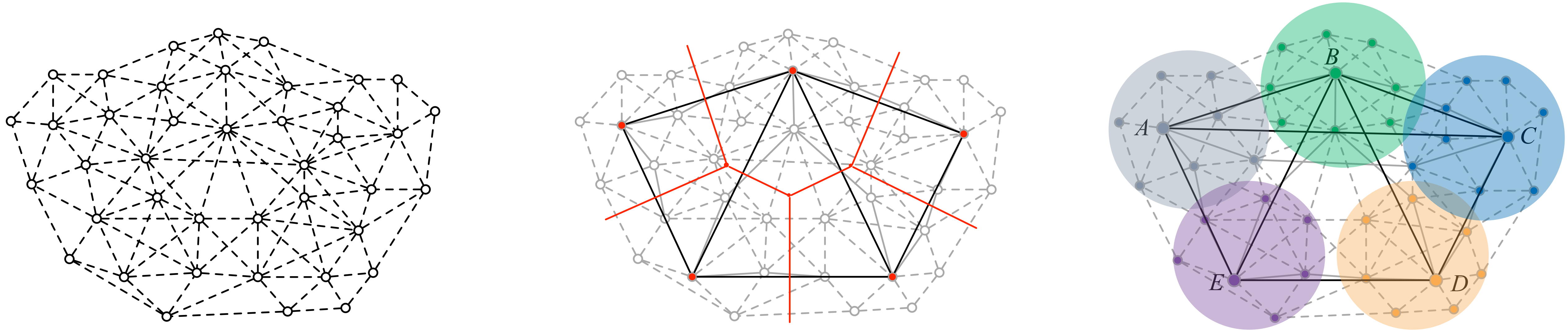
- Introduction
- Triangulation Algorithm
- Extension to 3D Surface
- Application and Simulation Results
- Conclusion

INTRODUCTION

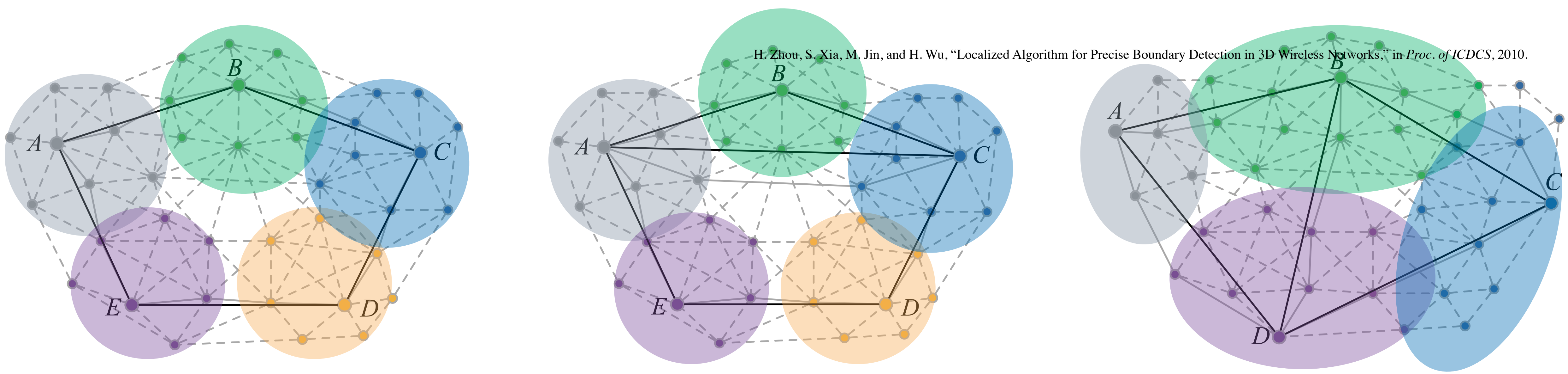
- Wireless sensor network exhibits randomness
- Wireless sensor network topology is a graph
- Triangulation is a subgraph of the topology
- Triangulation mesh is very important for many applications of sensor networks:
  geometry-based routing, localization, coverage, segmentation, data segmentation

Introduction

- Related work



(a) A 2D network graph. (b) Triangulation under ideal CDG. (c) CDG (k=1): not planar.



(d) CDM (k=1): planar but not triangulated. (e) Failed triangulation. (f) CDG (k=2): triangulated but coarse.

[2] R. Sarkar, X. Yin, J. Gao, F. Luo, and X. D. Gu, "Greedy Routing with Guaranteed Delivery Using Ricci Flows," in *Proc. of IPSN*, 2009.

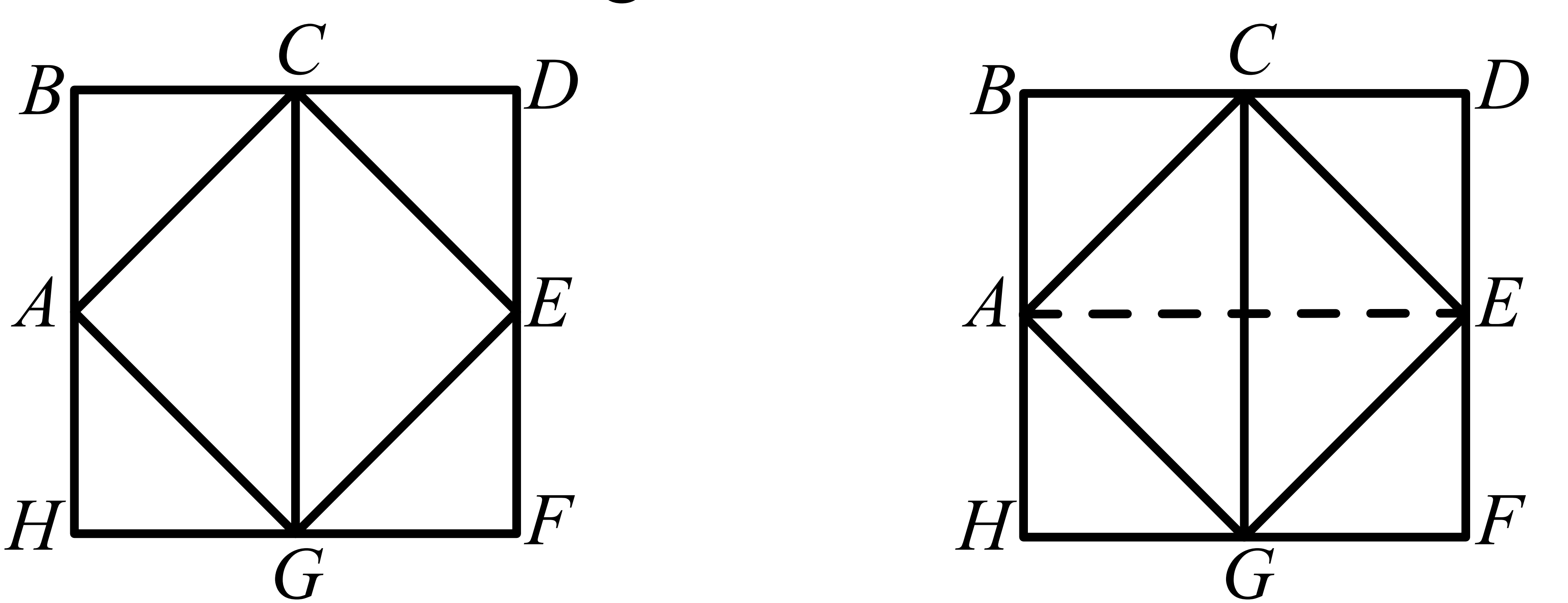
[3] H. Zhou, S. Xia, M. Jin, and H. Wu, "Localized Algorithm for Precise Boundary Detection in 3D Wireless Networks," in Proc. of ICDCS, 2010.

INTRODUCTION

- We proposed a distributed triangulation algorithm
  - works for any arbitrary 2D sensor networks without communication model constraint, and we prove the correctness of the algorithm in 2D sensor networks
  - tolerates some measurement errors
  - also works for 3D open or closed surfaces

Triangulation Algorithm

- Basic idea
  - Each triangulation mesh edge will associate with **two** triangles, except boundary edge
  - If we add extra edges into triangulation mesh, they will change this association
  - The association can be used to identify these extra edges.



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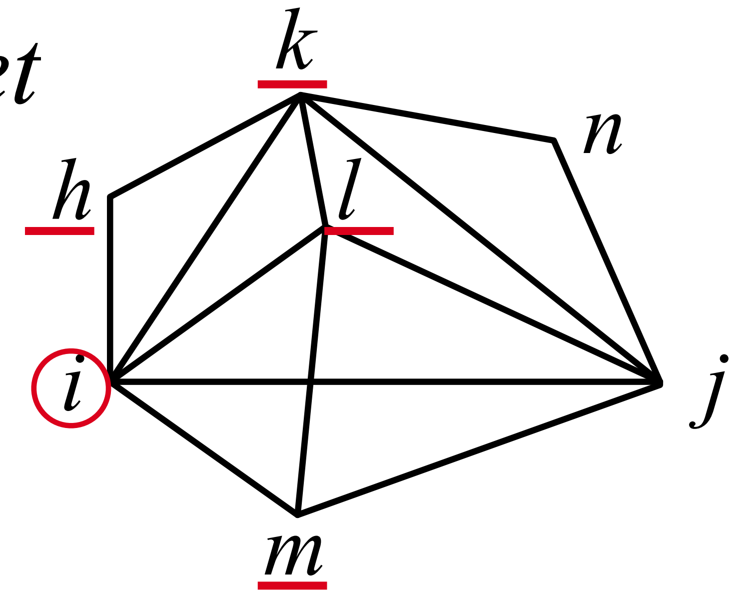
Triangulation Algorithm

- Definitions
  - ***node neighbor set* (NNS),** $N_v(i)=\{h,k,l,m\}$
  - *edge neighbor set* (ENS), $N_e(e_{ij})=N_v(i)\cap N_v(j)=\{k,l,m\}$
  - *refined edge neighbor set* (RNS), $R_e(e_{ij})=\{l,m\}$
  - *edge weight*, $W(e_{ij})=2$, $W(e_{hk})=2$
  - *associated edge neighbor set* (AENS), $A(e_{ij})=\{e_{il}, e_{im}, e_{jl}, e_{jm}\}$
  - *equivalent edges*, e_{ij}, e_{lm}
  - *critical edge*, e_{kj}



TRIANGULATION ALGORITHM

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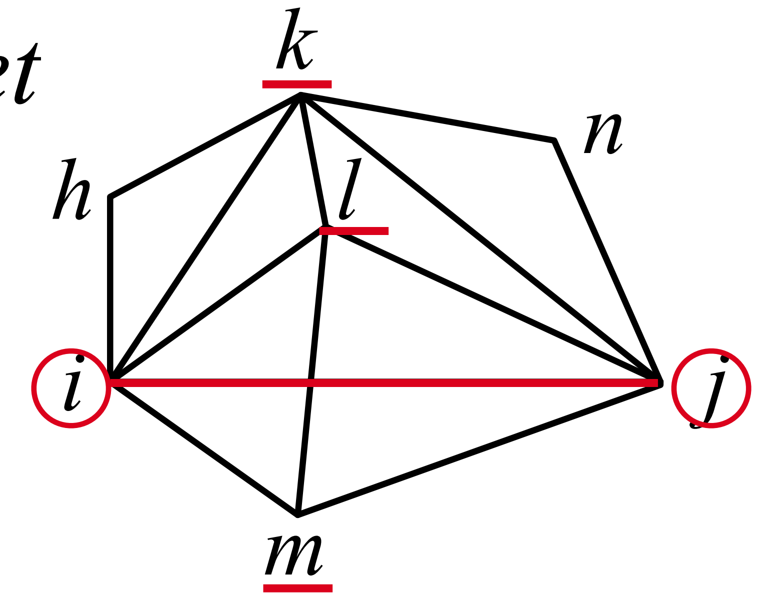

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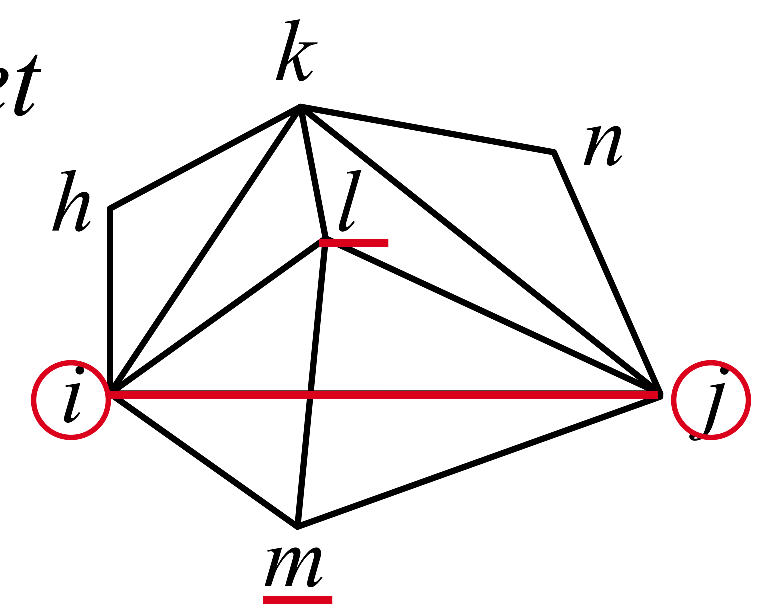

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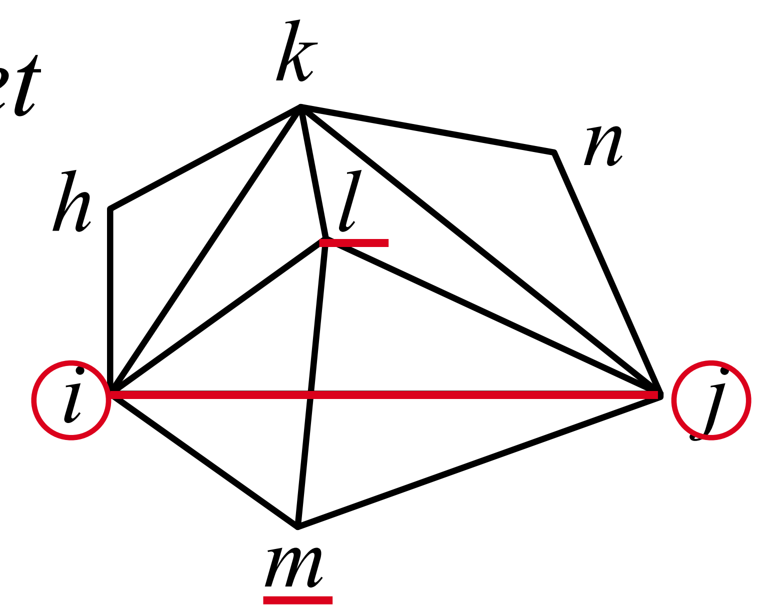

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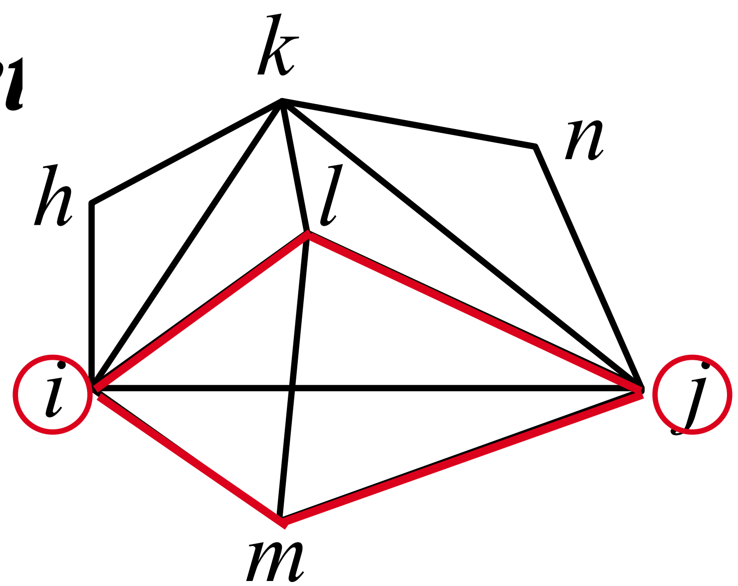

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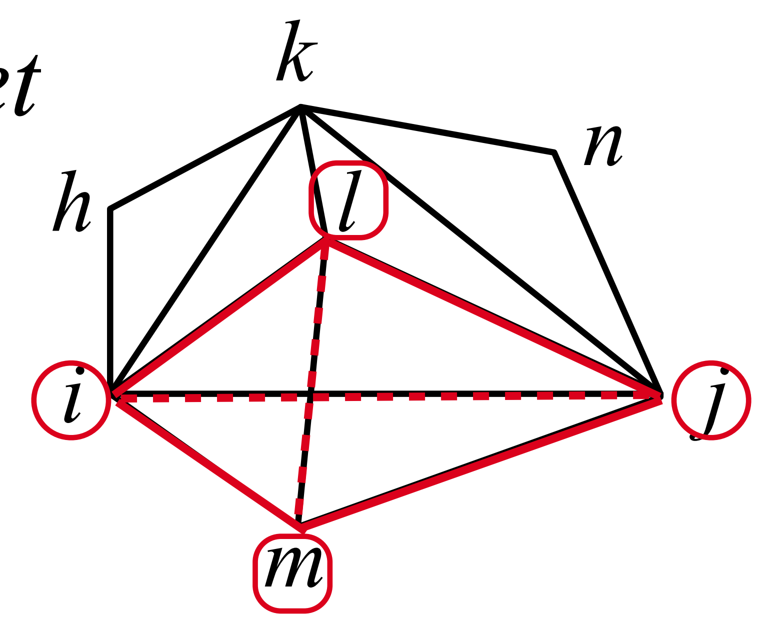

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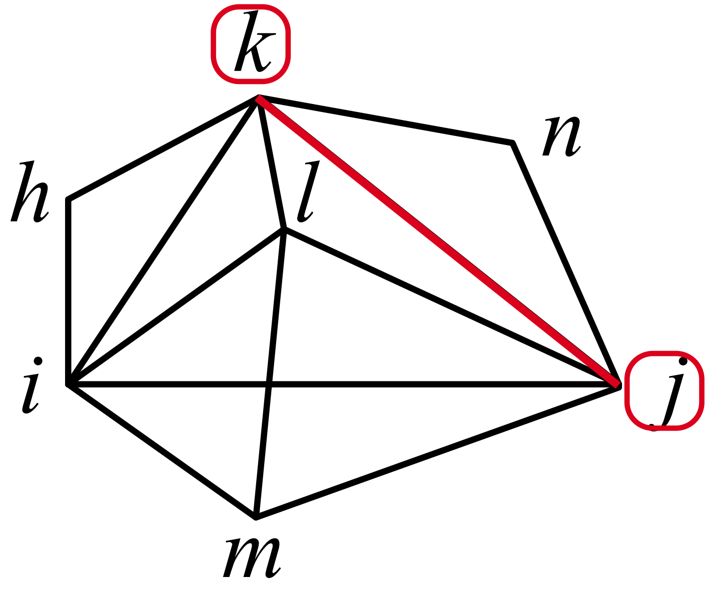
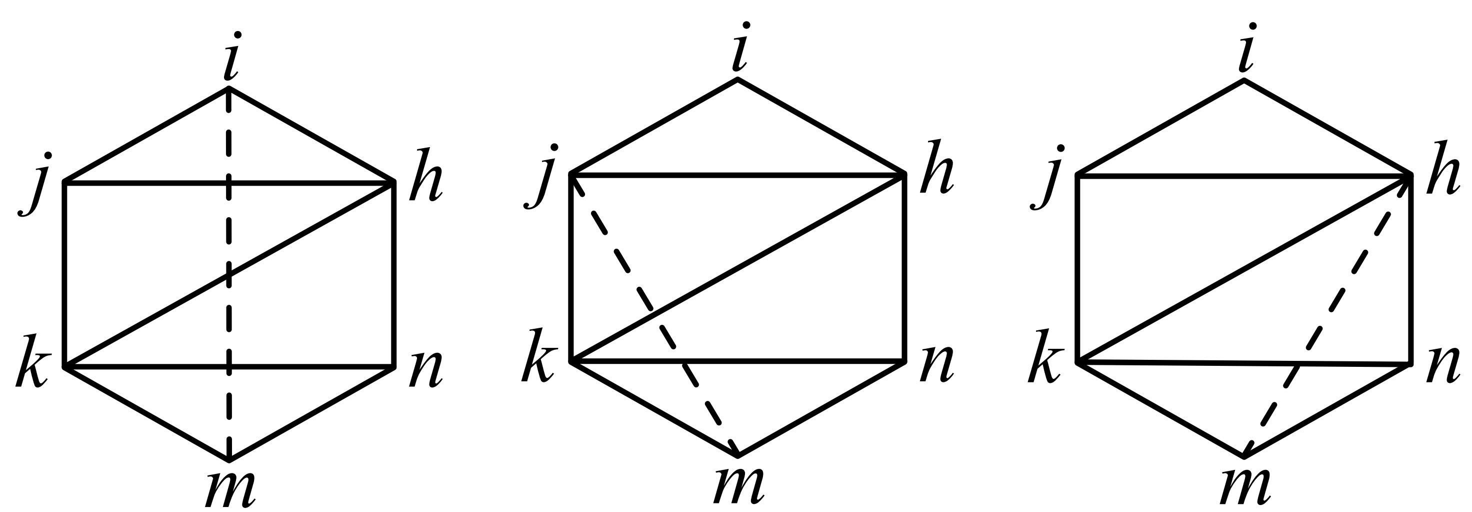


TRIANGULATION ALGORITHM

- **Lemma 1**: A subgraph of ***G*** is triangulated *if and only if* every edge of the subgraph has a weight of **two**.
- Given a graph G and a *triangulation subgraph T, extra edges are G-T*
- **Objective:** remove all extra edges
- There are three type of *extra edges, e_0, e_1, e_2*



(a) e_0 (b) e_1 (c) e_2

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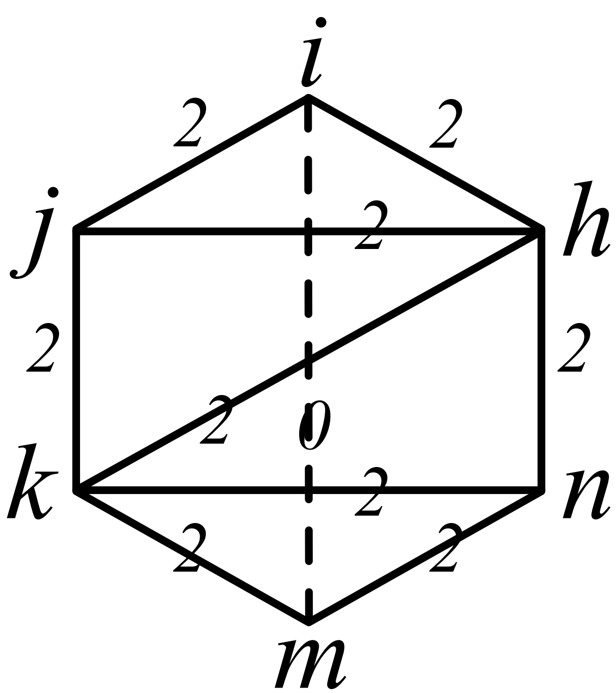
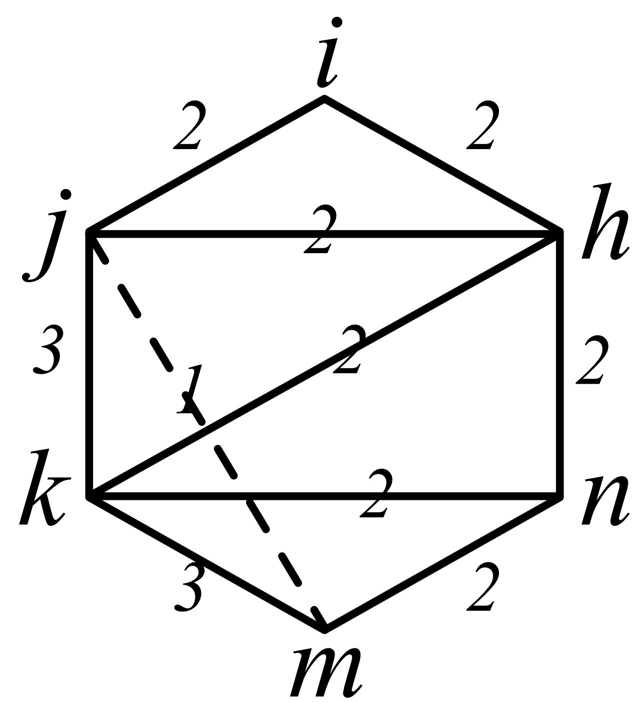
TRIANGULATION ALGORITHM

- **Theorem 1**: An edge with a weight *less than* **two** must be removed in order to produce a triangulated subgraph.



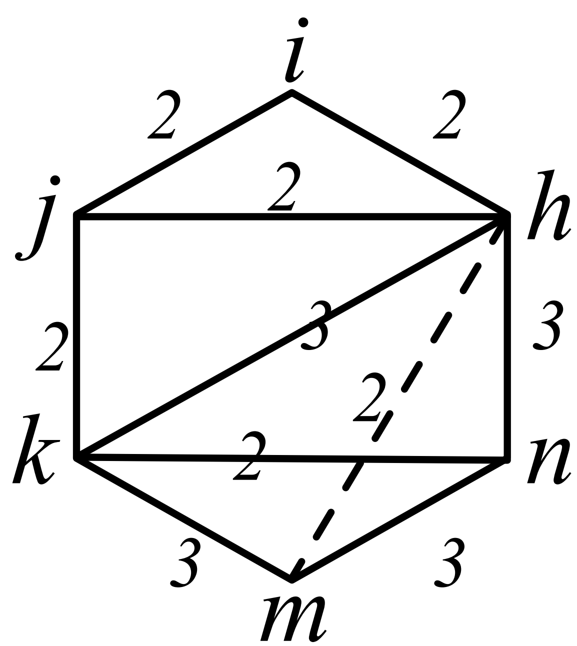
(a) e_0



(b) e_1

TRIANGULATION ALGORITHM

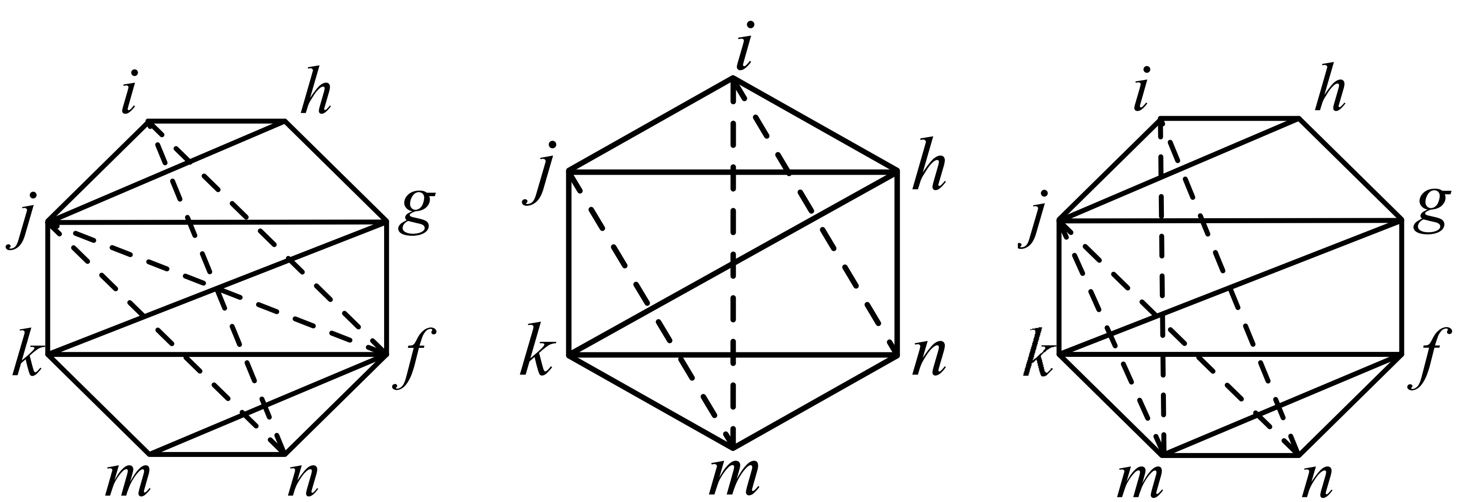
- **Theorem 2**:An *non-critical edge* can be recognized as an e_2 edge and safely removed if all edges in its *AENS* have their weight greater than **two**.



(c) e_2

TRIANGULATION ALGORITHM

- **Lemma 2**: An *e_0 extra edge* can exist in G'', only if it depends on at least two other *extra edges*.
- **Lemma 3**: An e_1 extra edge can exist in G'', only if it at least depends on another extra edge.



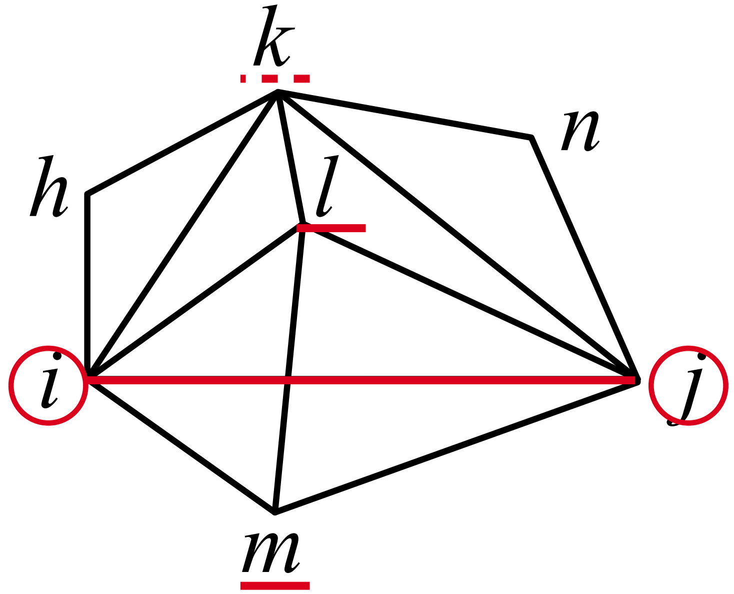
(a) Loop chain. (b) Non-loop chain. (c) Same head and tail.

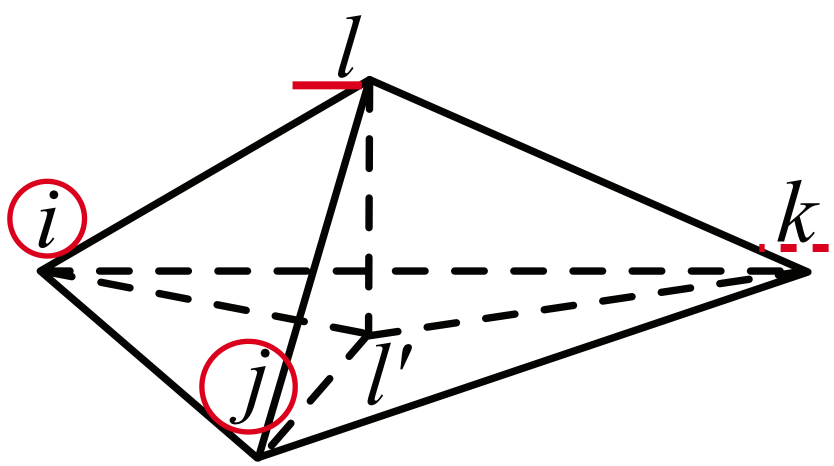
TRIANGULATION ALGORITHM

- Step1: Initialization.
  each edge calculate *NNS, ENS, RENS, AENS*, and *edge weight*;
- Step2: Iterative edge removal according to **Theorems** 1 and 2;
- Step3: Removal of e_0 and e_1 chains.

EXTENSION TO 3D SURFACE

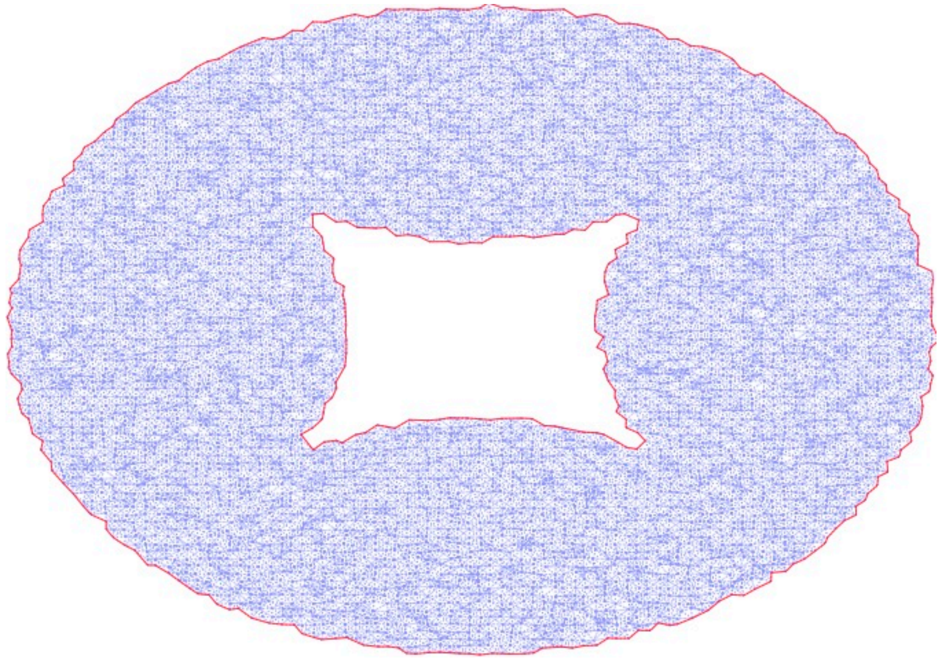
- Similar algorithm can be applied in a sensor network on 3D surface with minor modification in determining *RNS* and *edge weight*



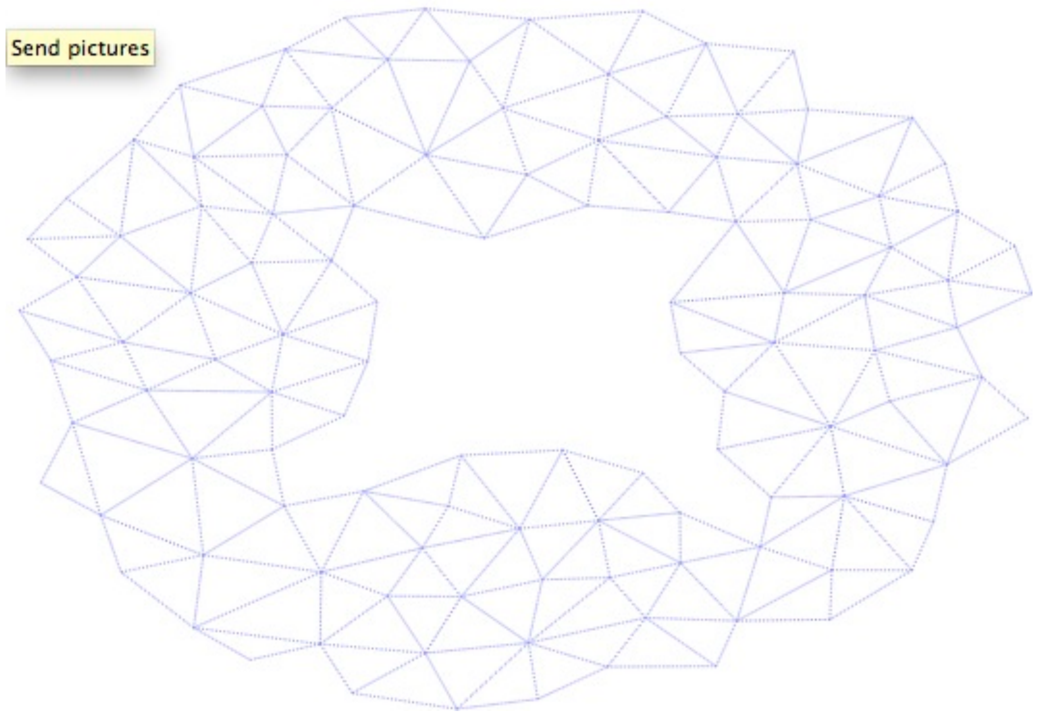


APPLICATION AND SIMULATION RESULTS

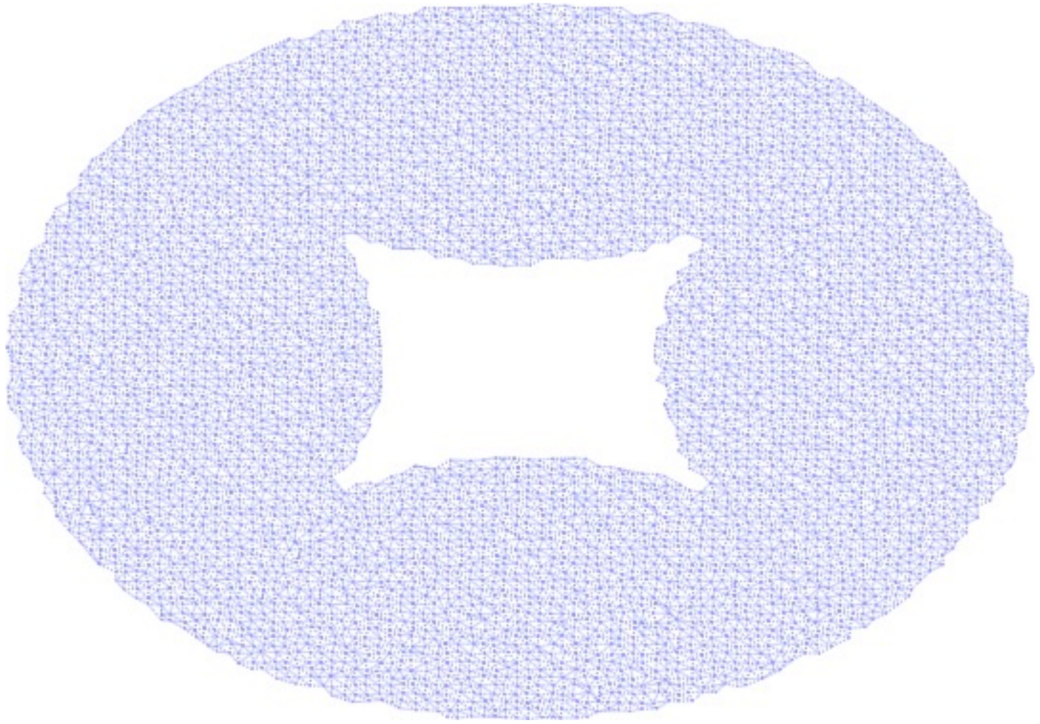
2D network



(a) Sensor network topology



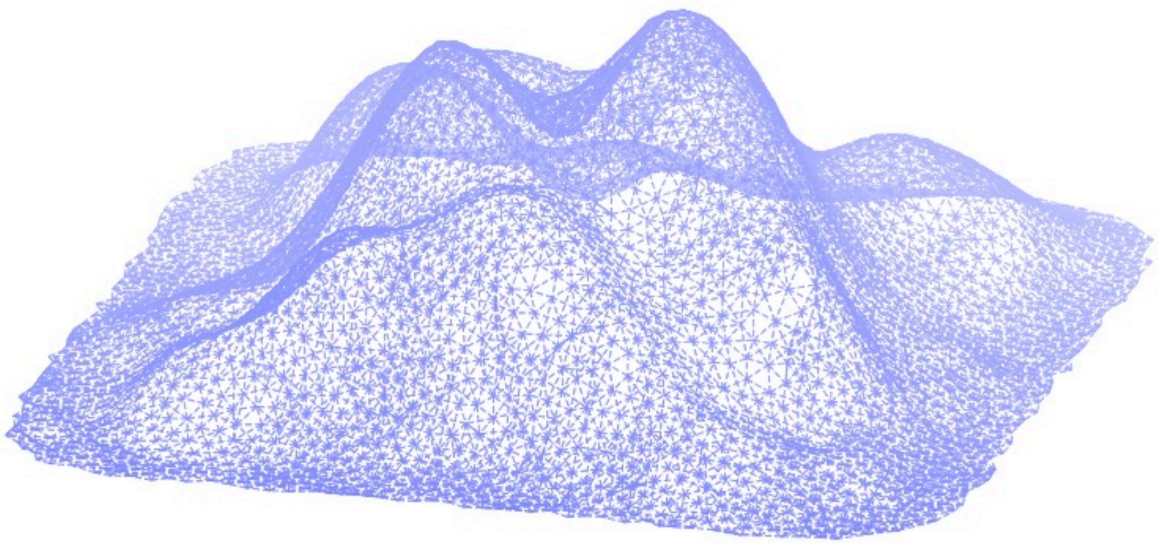
(b) Finest triangulation by [2]



(c) Proposed triangulation

Application and Simulation Results

3D network with open surface



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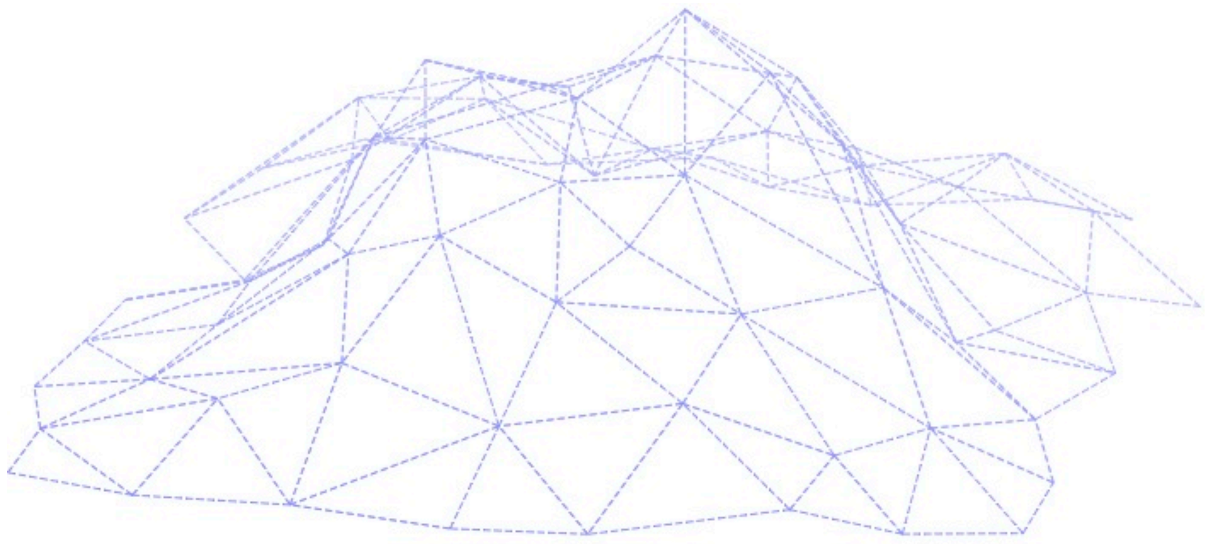
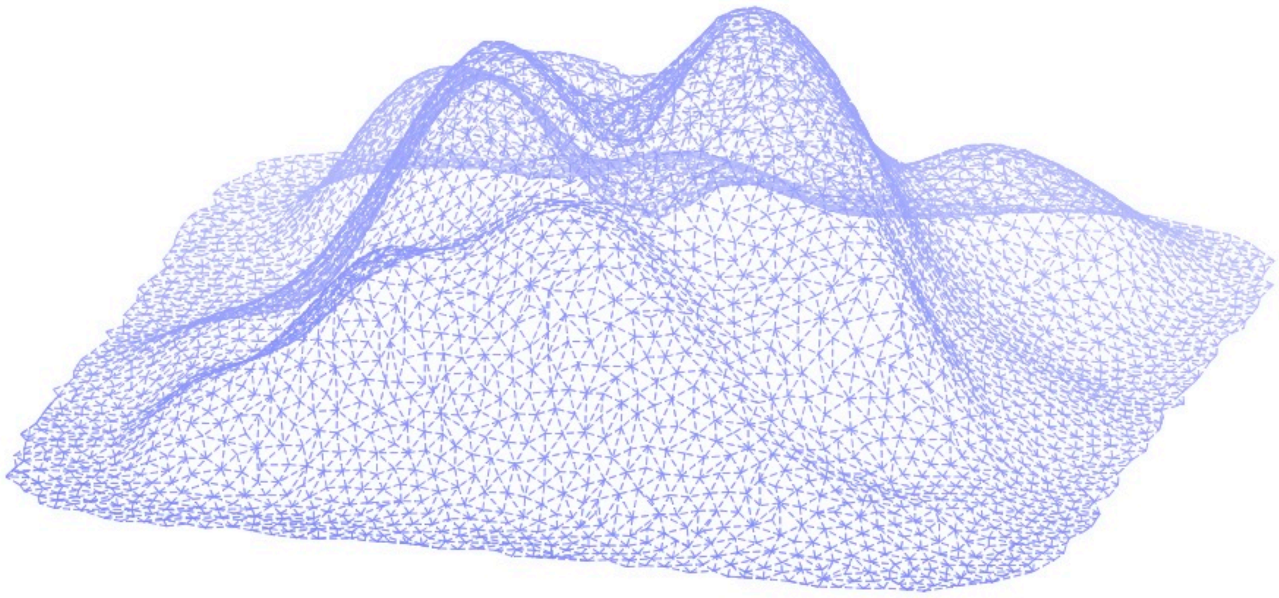


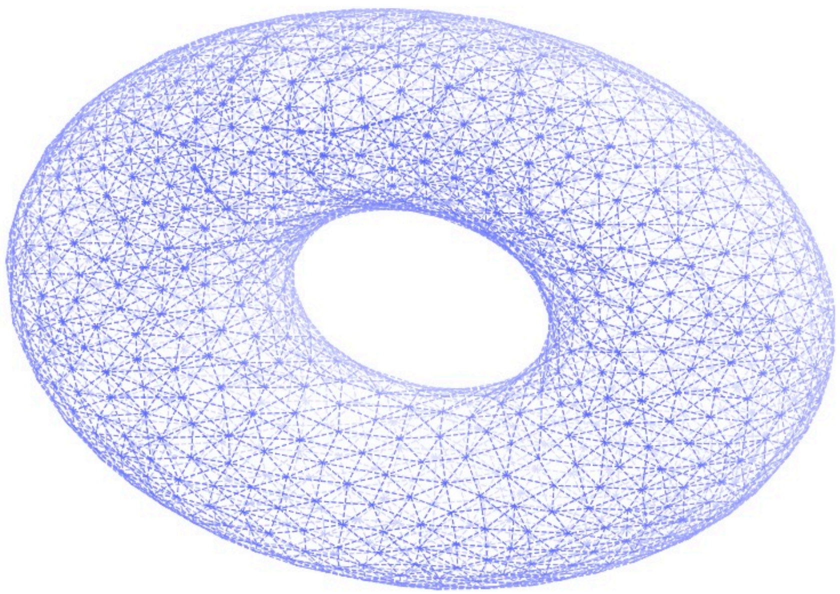
(b) Finest triangulation by [3]



(c) Proposed triangulation

APPLICATION AND SIMULATION RESULTS

3D network with
closed surface



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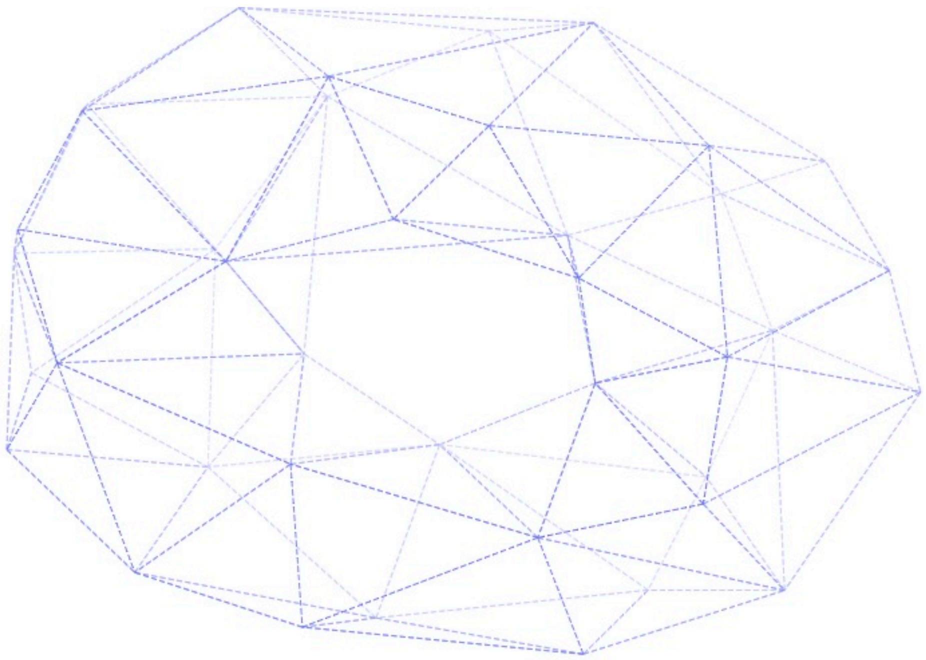
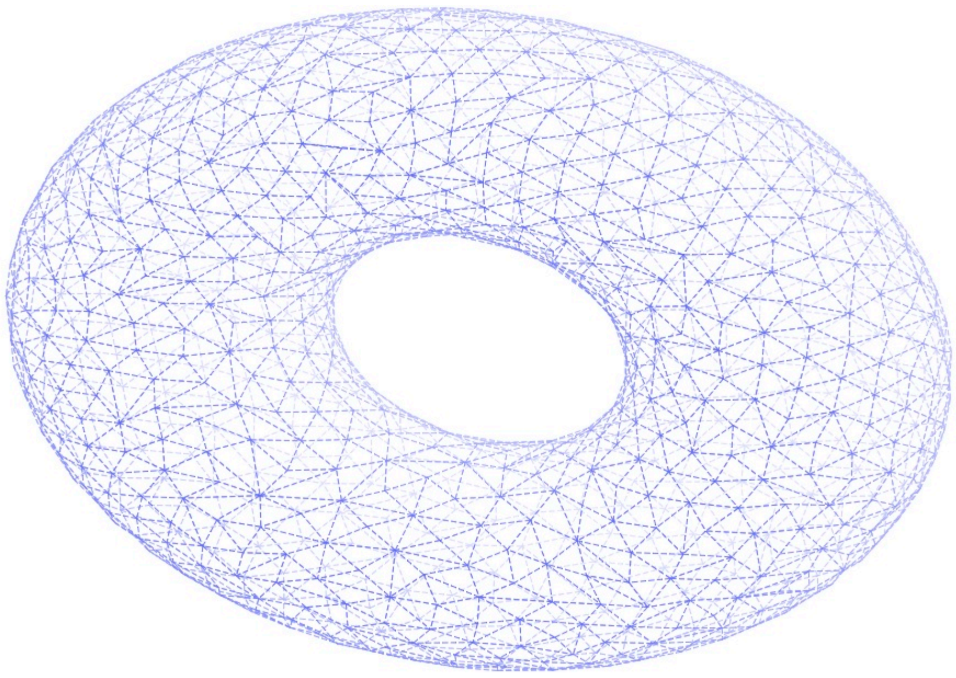


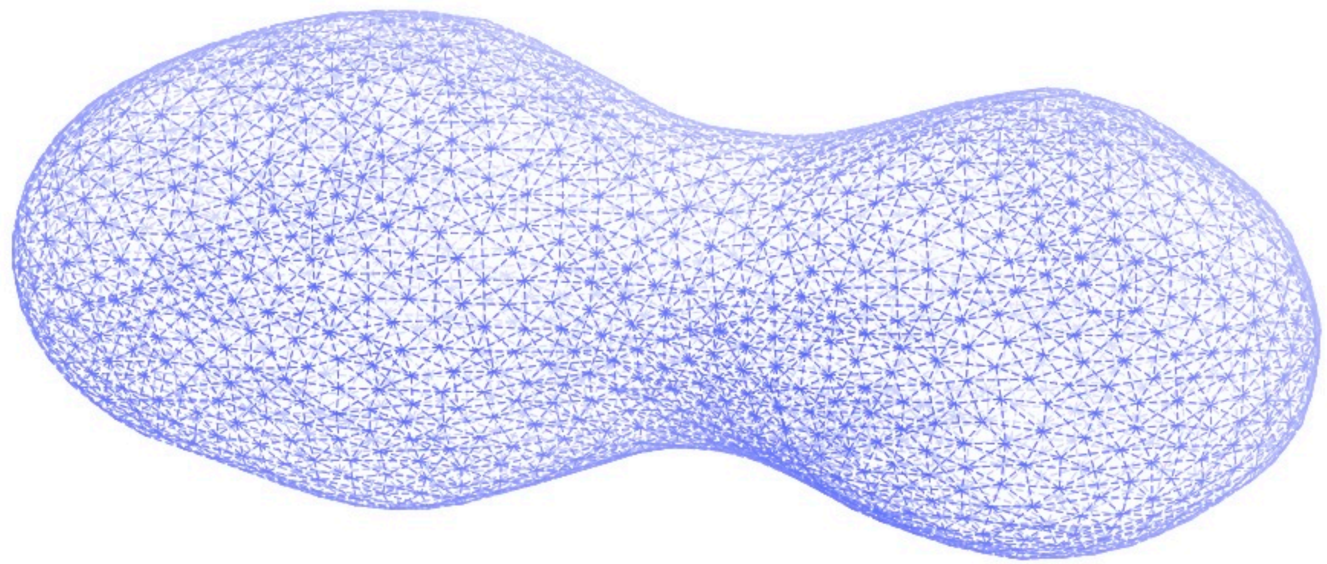
(b) Finest triangulation by [3]



(c) Proposed triangulation

Application and Simulation Results

3D network with
closed surface



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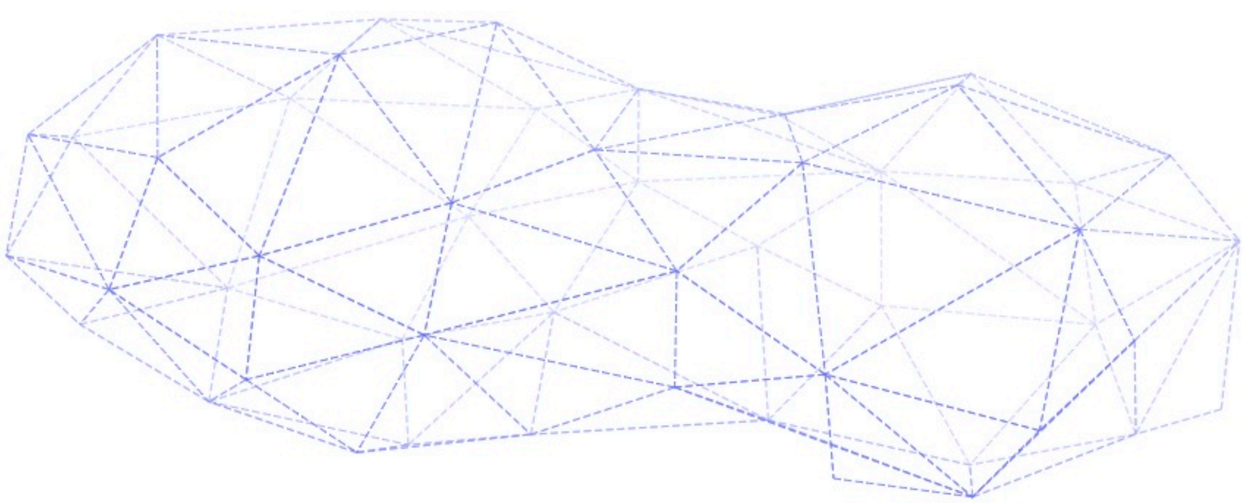

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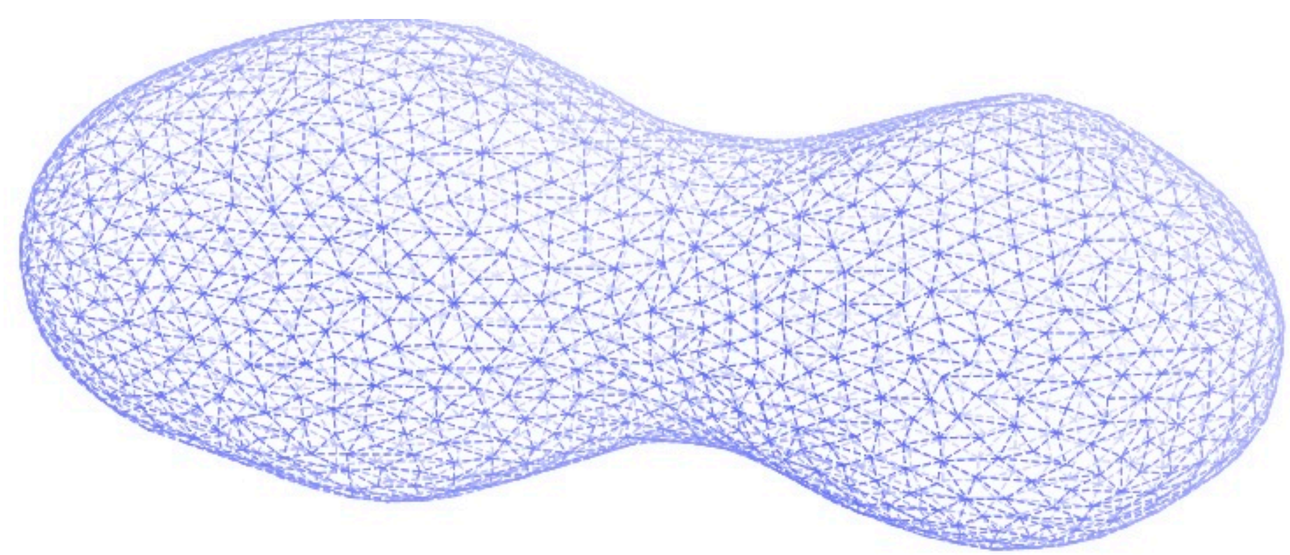

(c) Proposed triangulation

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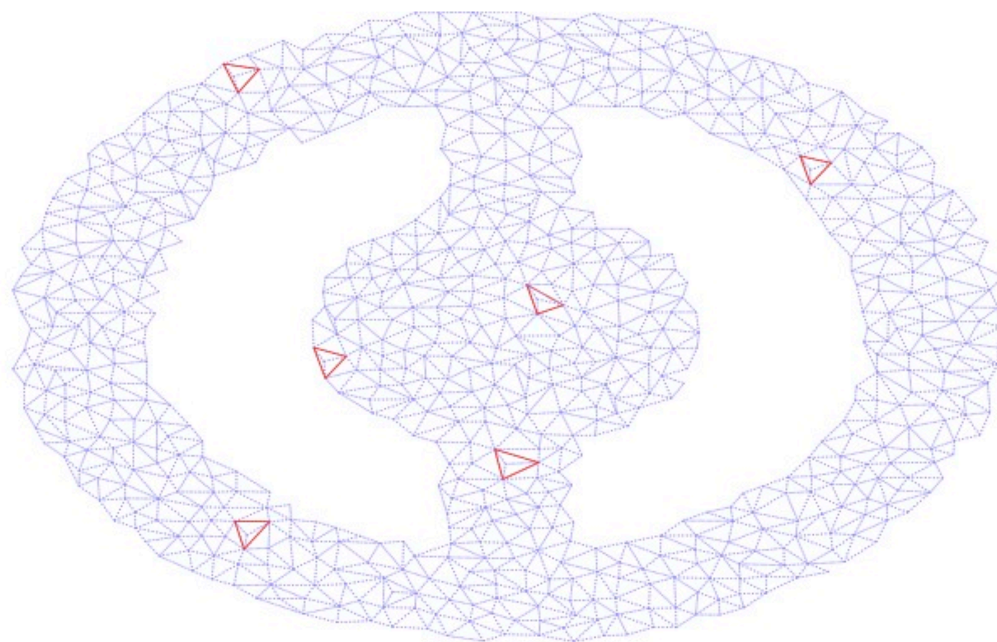
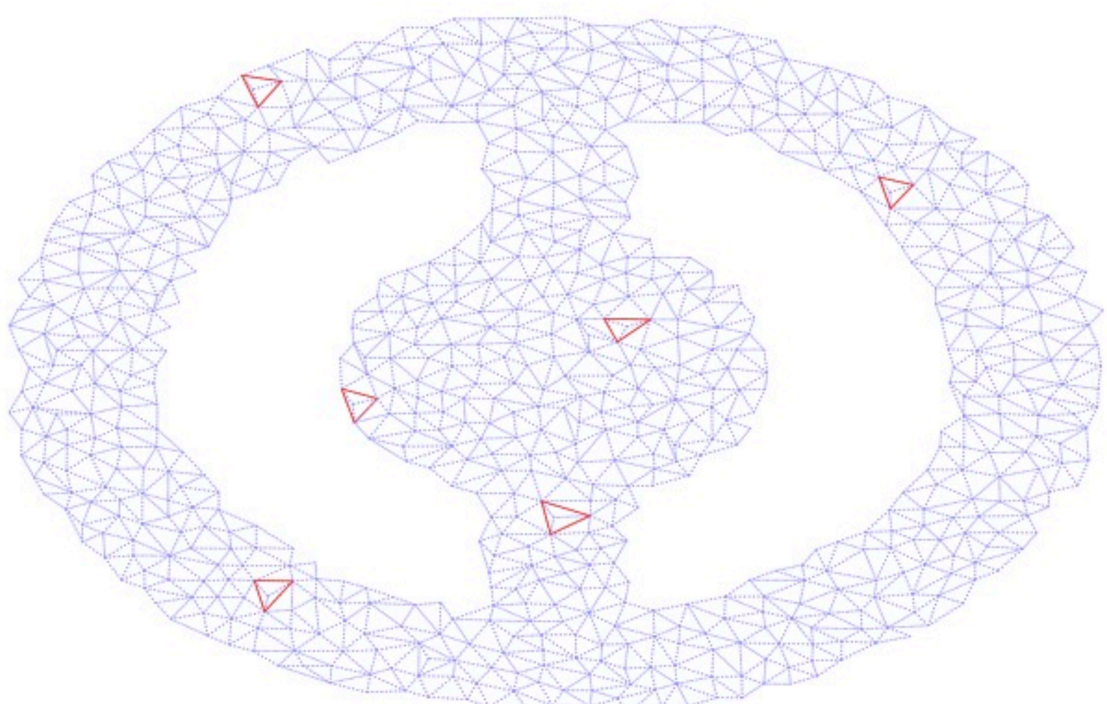
APPLICATION AND SIMULATION RESULTS

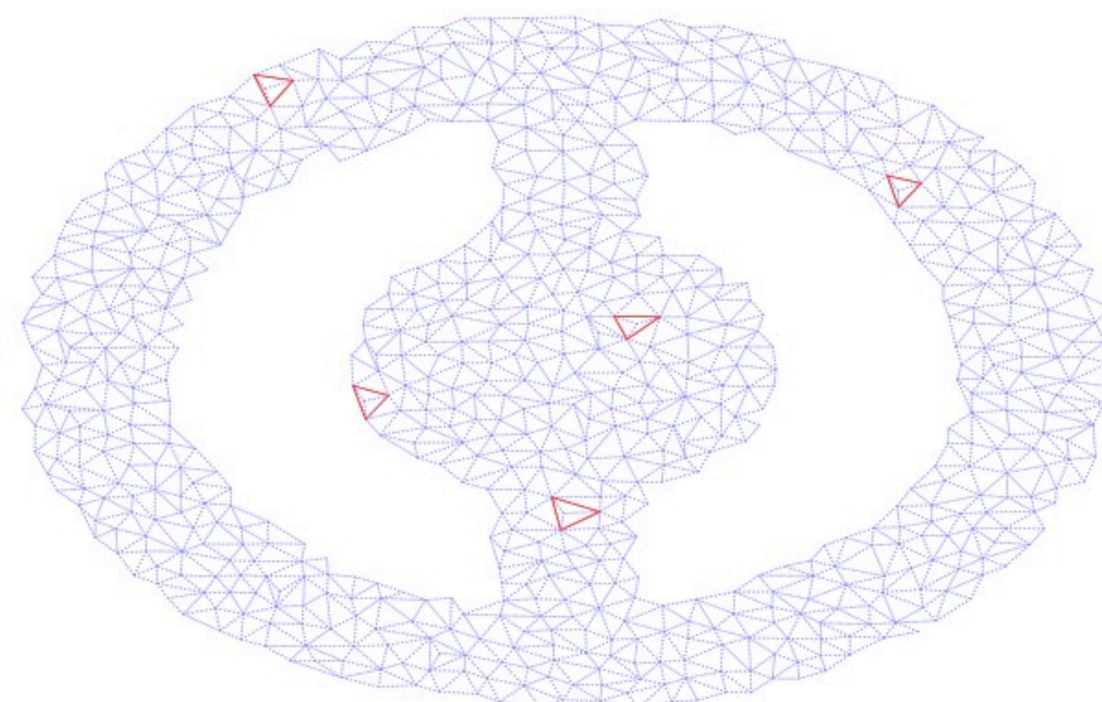
2D network with distance errors



(a) Triangulation under 10% distance errors.



(b) Triangulation under 20% distance errors.



(c) Triangulation under 30% distance errors.

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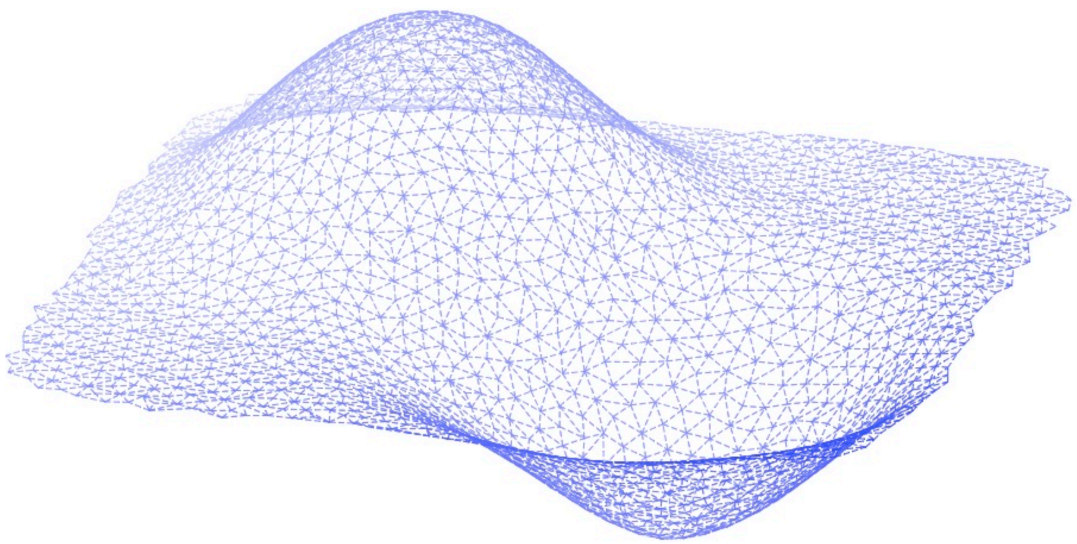
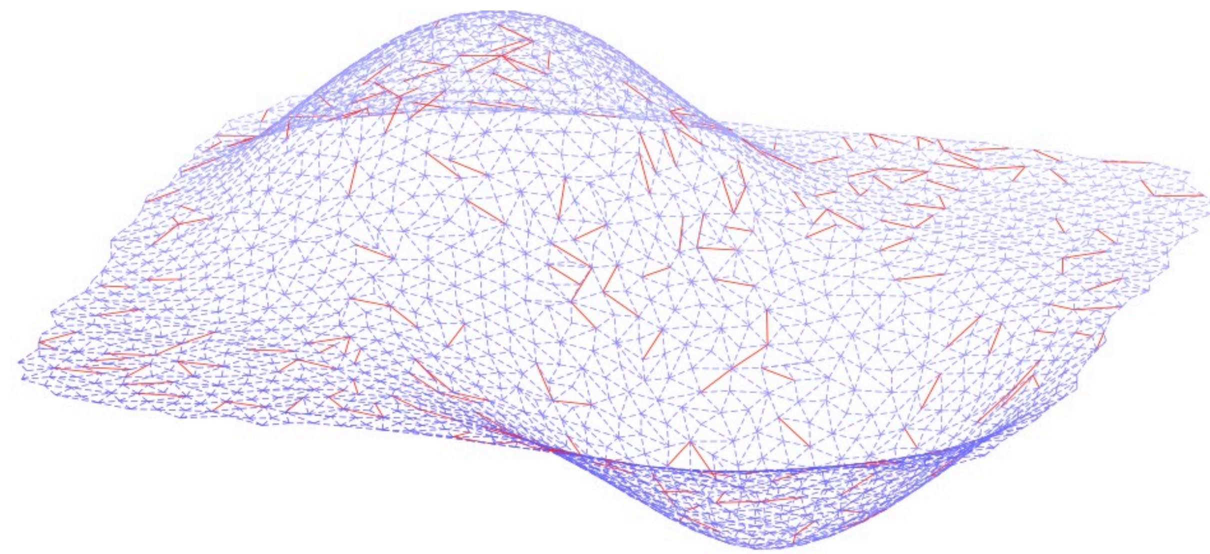
Application and Simulation Results

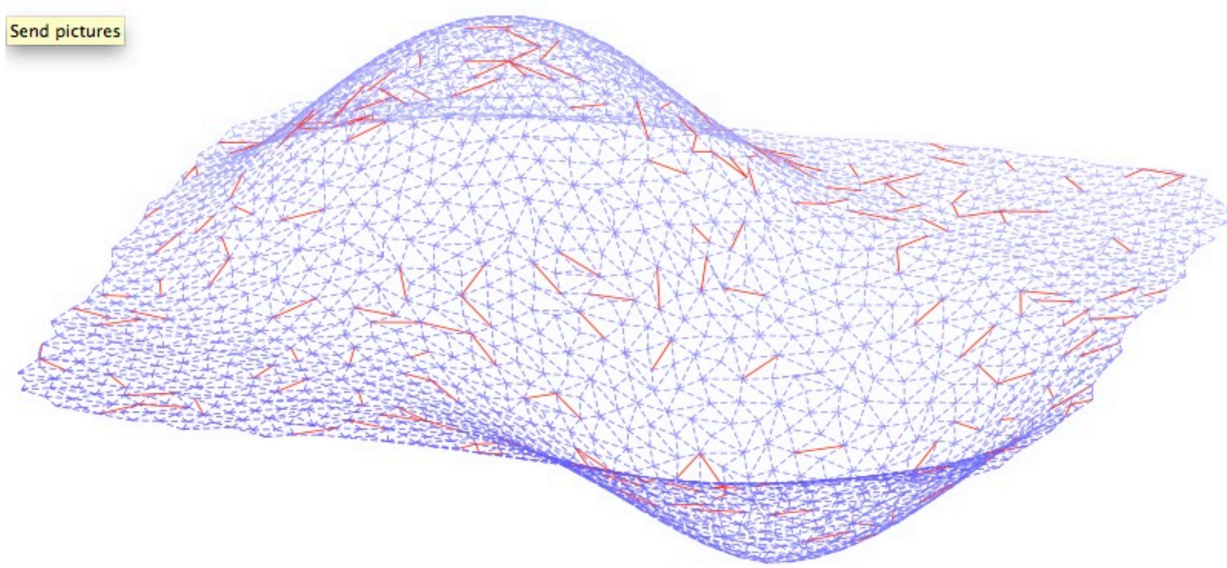
Difference Communication Models



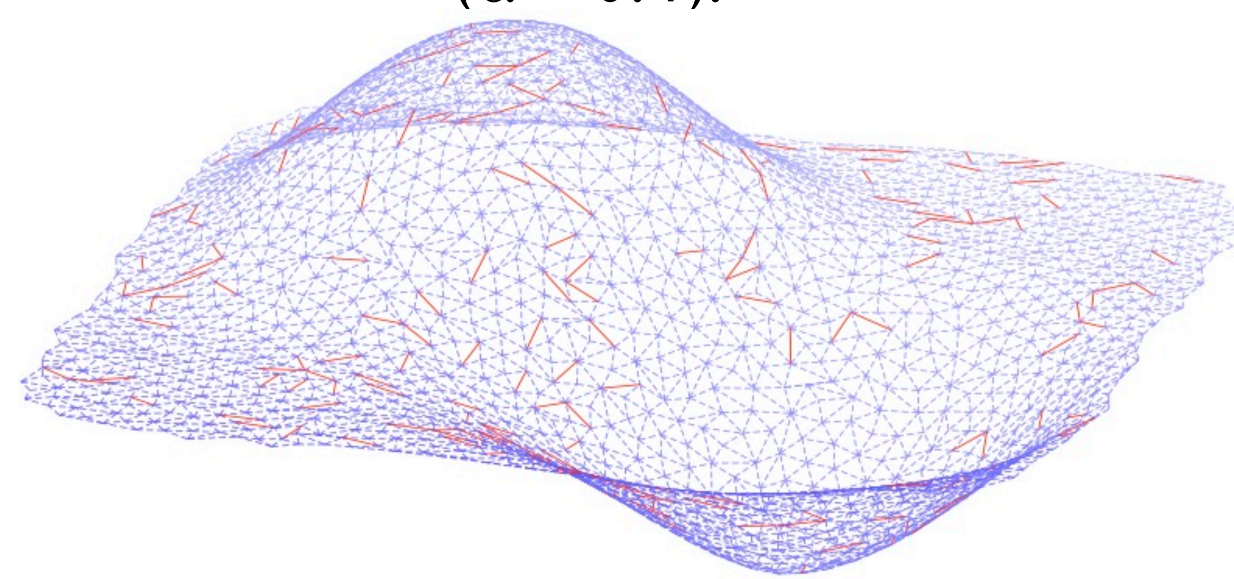
(a) UDG



(b) Triangulation under Quasi-UDG ($\alpha = 0.4$).



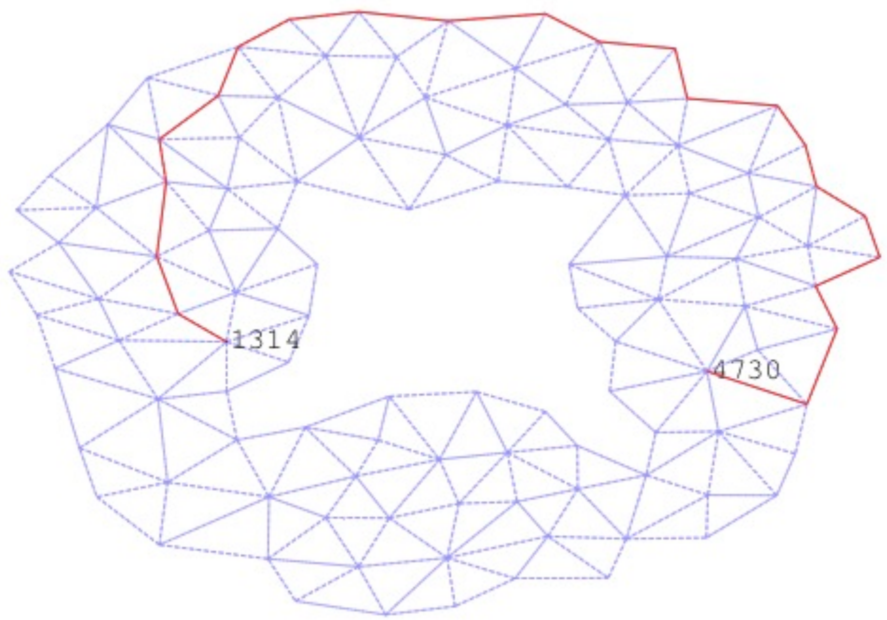
(c) Triangulation under Quasi-UDG ($\alpha = 0.6$)



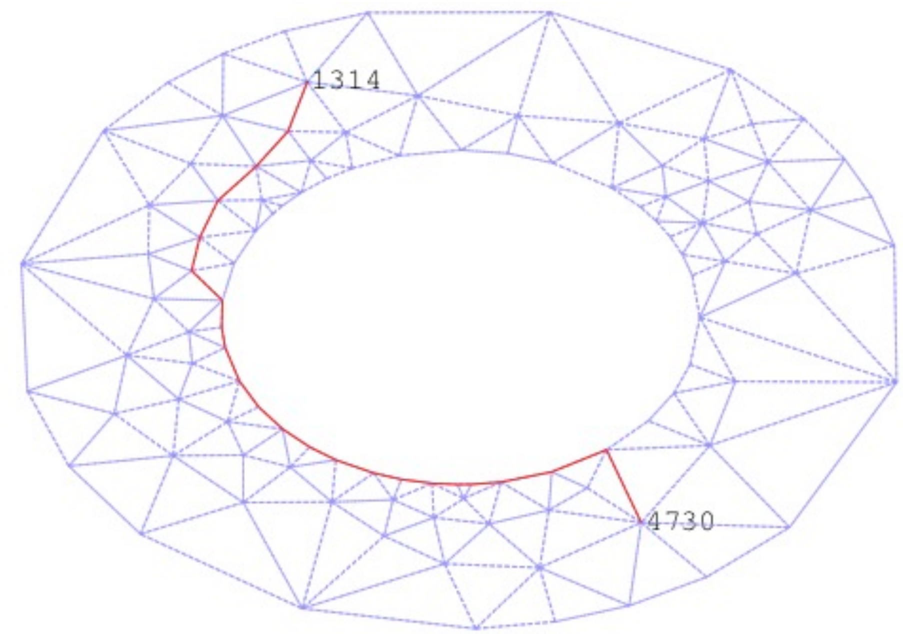
(d) Triangulation under Log-normal.

APPLICATION AND SIMULATION RESULTS

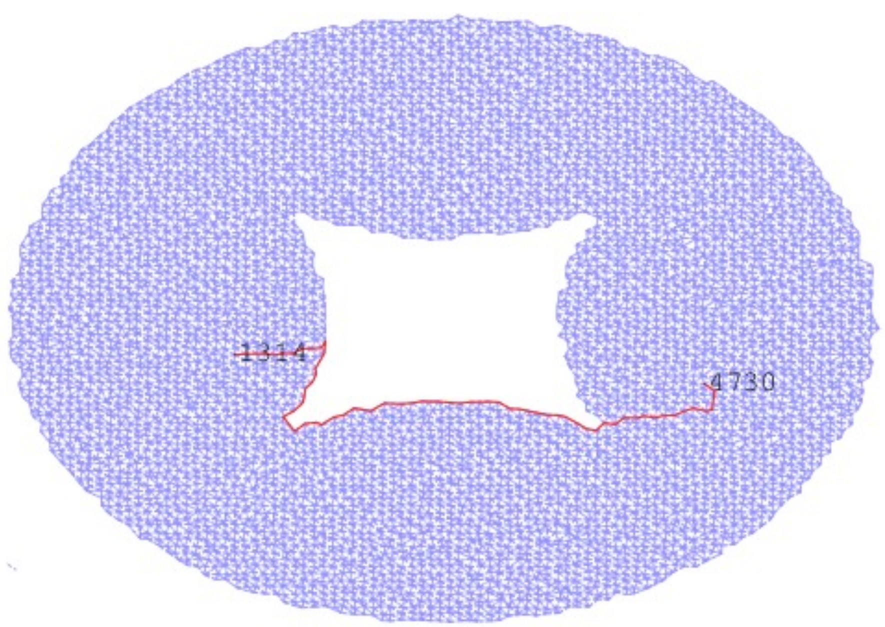
Greedy routing based on Ricci flow in 2D networks



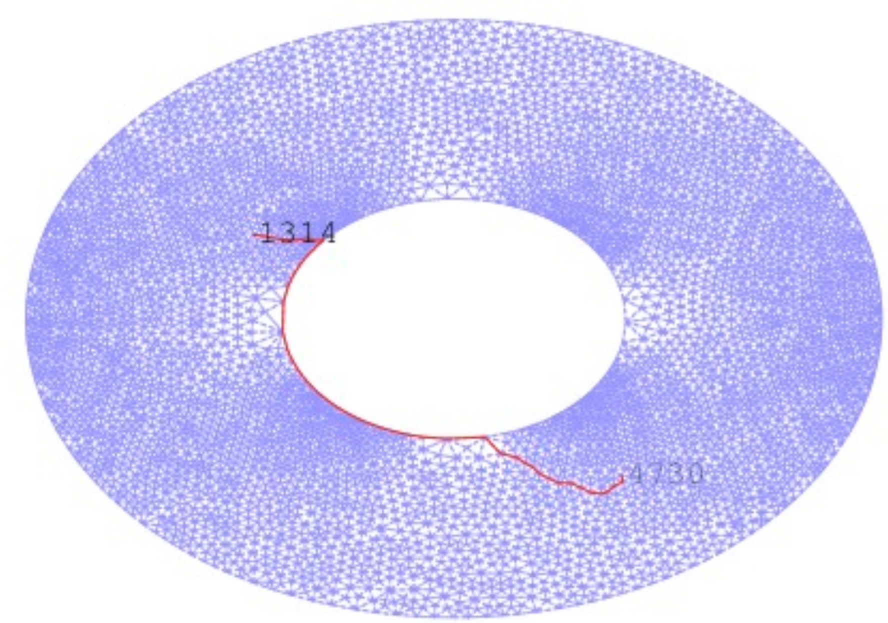
(a) Finest triangulation by [2]



(b) Ricci embedding



(c) Proposed triangulation



(d) Ricci embedding

APPLICATION AND SIMULATION RESULTS

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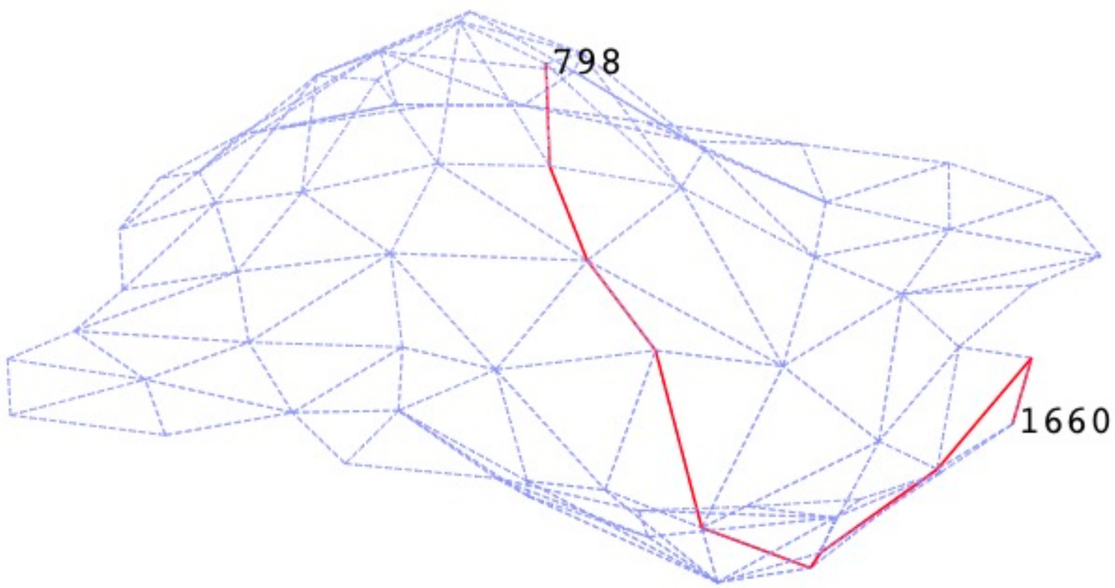
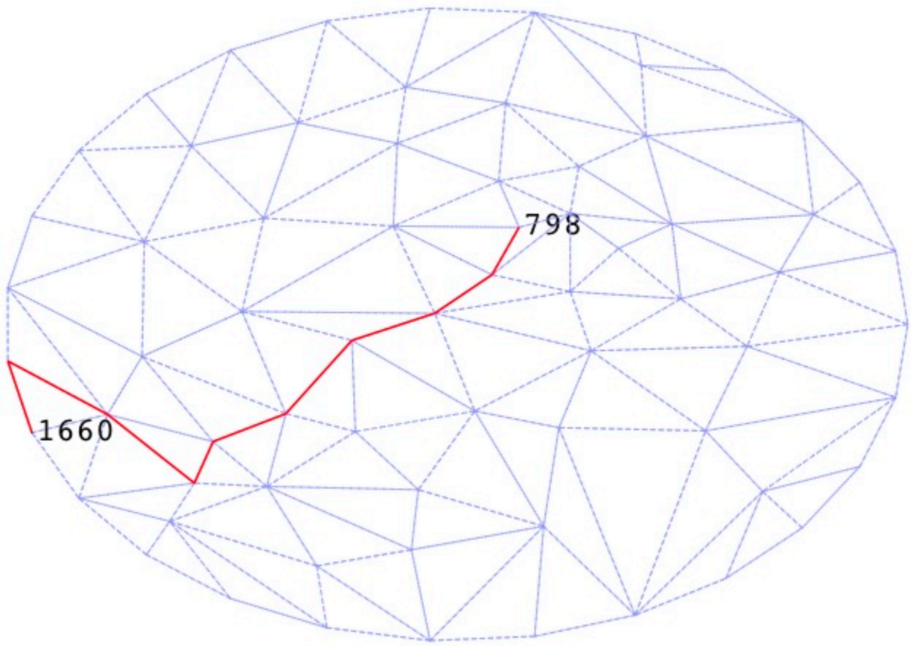
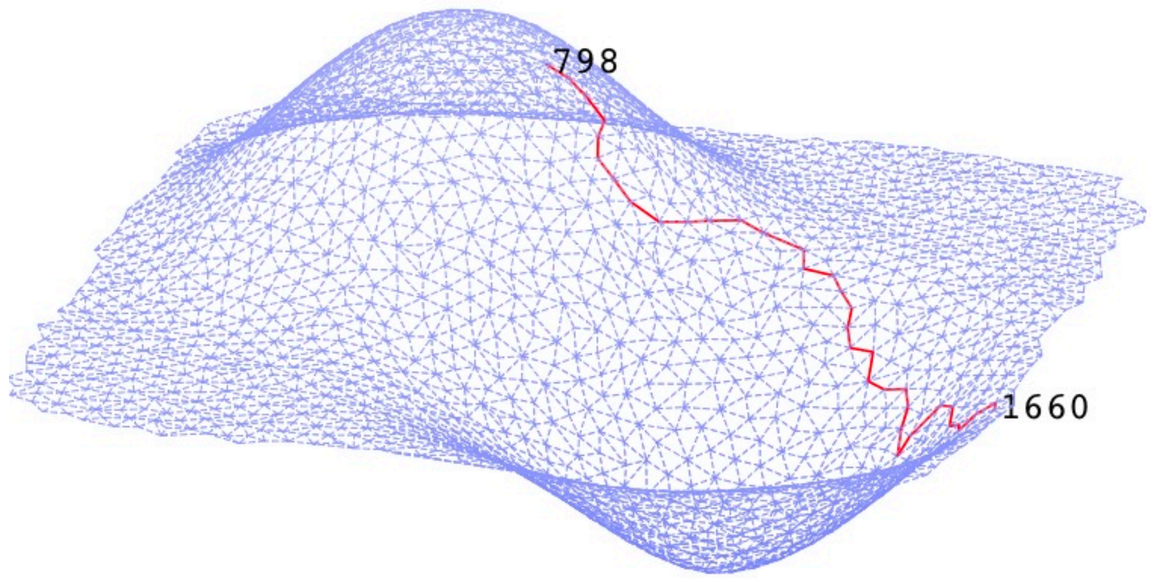


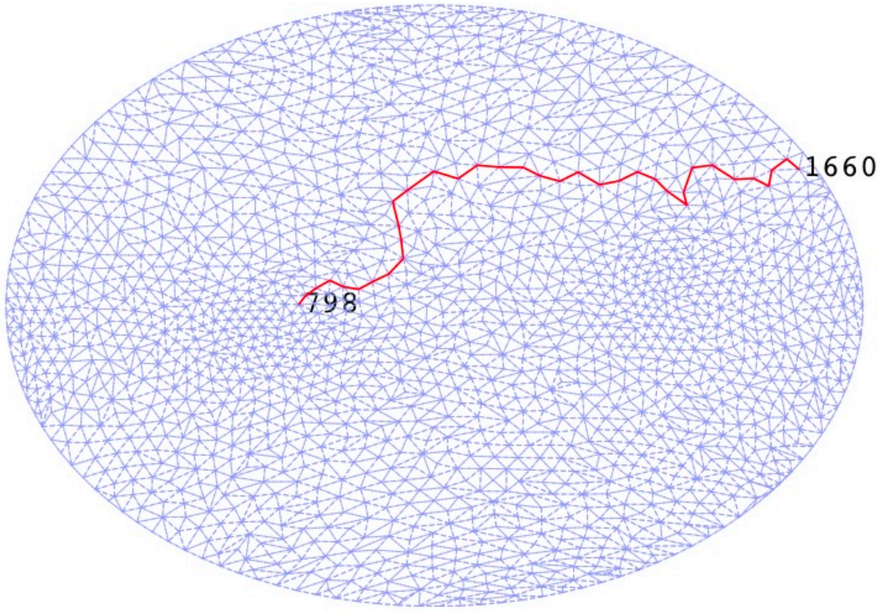
(a) Finest triangulation by [3]



(b) Ricci embedding



(c) Proposed triangulation



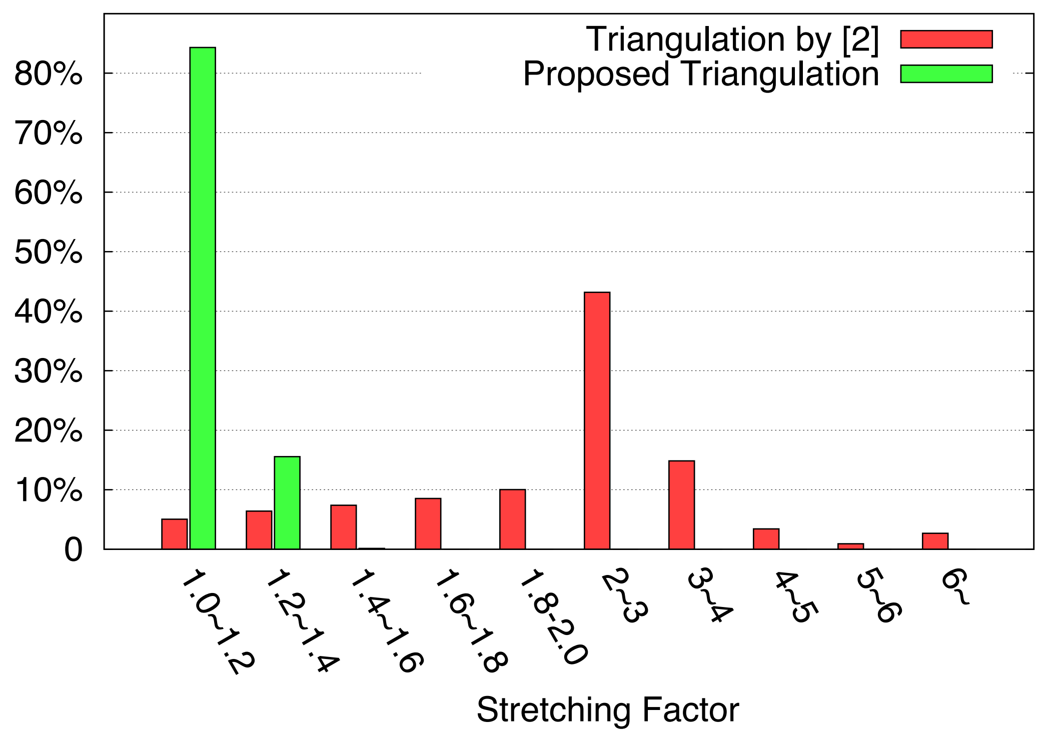
(d) Ricci embedding

Application and Simulation Results

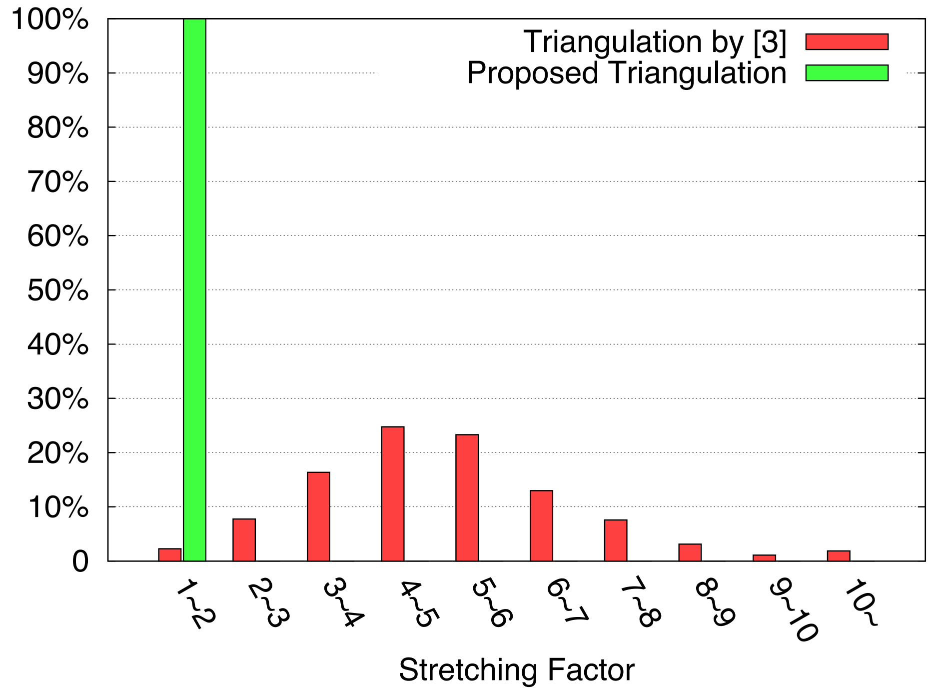
Comparison of stretch factor in greedy routing.

	2D networks	3D networks
Triangulation by [2], [3]	2.065	5.417
Proposed Triangulation	1.214	1.091

Distribution of stretch factor in greedy routing



(a) 2D networks



(b) 3D networks

CONCLUSION

- In summary, we have proposed a distributed algorithm that can triangulate an arbitrary sensor network without position information.
- The algorithm can achieve the finest triangulation and tolerate distance measurement errors.
- We have proven its correctness in 2D and extended it to 3D surface.
- A range of geometry-based network algorithms can benefit from the proposed triangulation.