ing values that communities hold are what determine public policy.

References and Notes

1. The volume was included in the Broadview Press publishing lists purchased in May by the University of Toronto Press.
2. D. M. Thomas, Ed., *Canada and the United States: Differences That Count* (Broadview, Peterborough, Ontario, 1993).
3. I contributed a chapter to the 1993 volume but have not been involved in either of the subsequent editions.
4. See, for example, R. Inglehart, N. Nevitte, M. Basañez, *The North American Trajectory: Cultural, Economic, and Political Ties Among the United States, Canada, and Mexico* (Aldine de Gruyter, New York, 1996).
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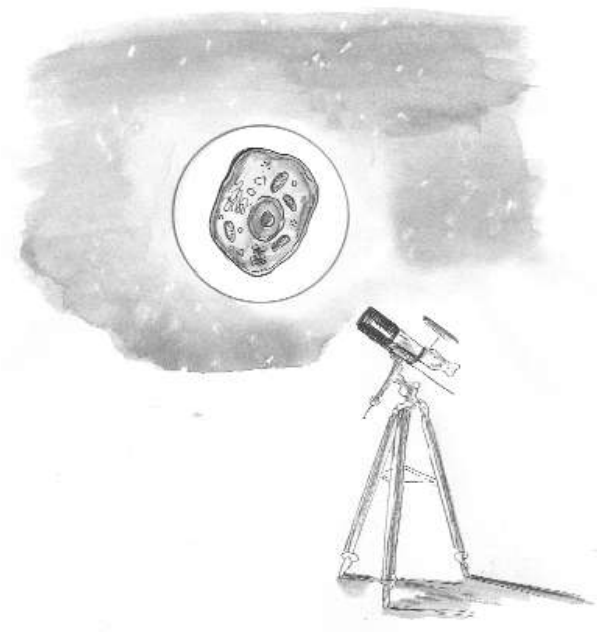
GENERAL SCIENCE

On the Back of an Envelope

Stephan Mertens

An important skill of great use in science is the ability to derive an approximate result from insufficient data. That ability allows one to determine whether or not an answer is reasonable by a quick calculation. The physicist Enrico Fermi (1901–1954) was a master of this art, and he used to practice his skills by working out "impossible" problems such as "How many piano tuners are there in the city of Chicago?" or "How many alien civilizations are there in our Galaxy?" Questions of this type are now known as Fermi problems (*1*), and they are frequently used in job interviews to test applicants' ability to think on their feet and to make inspired guesses from very little data. What matters there is not the accuracy of the answer but the way of reasoning.

Guesstimation is a collection of 73 Fermi problems that physicist Lawrence Weinstein and mathematician John A. Adam (professors at Old Dominion University, Virginia) gathered from everyday life and various fields of science. Even the science questions require little more than common sense to be answered Fermi style. Take, as an example, the question of how many years of life the average smoker loses. Well, smoking kills primarily through lung cancer and heart disease. These are late-onset diseases whose victims typically die after reaching the age of 50. On average, smoking cannot cost one more than 30 years because life expectancy is less than 80. It will cost more than one year, as otherwise it wouldn't be a major health issue, so the number of lost years is between 1 and 30. Here we apply one of the few rules of guesstimation: if you have reasonable upper and lower bounds, take the geometric mean of the bound, i.e., the square root of the product of the bounds. In this case we take the square root of 30, which is a little more than 5. On average, smokers die 5 years earlier than nonsmokers. This guess is close to the actual number of 6.5 years (*2*). It also tells us that the main factor that determines the number of lost years is not the nicotine but the fact that smoking generally starts killing at age 50.



How many cells are there in the human body?

The rule of taking the geometric mean of the bounds instead of their average reflects the fact that guessed bounds are usually orders of magnitude apart, so taking the average would give too much weight to the upper bound. How many people could we cram into a car? Certainly more than one and less than 100. The average (50) seems to be too high, but the geometric mean (10) is reasonable.

"Use the geometric mean" is the most important precept of guesstimation. Another rule says: dare to be inaccurate. For quick calculations on the back of an envelope or in your head, simplify the numbers and focus on the order of magnitude. Days have 25 hours, pi is 3, and every adult weighs 100 kg. It also helps if you know the order of magnitude of a few quantities like the population of the United States or the density of water. One needs to know only very few numbers to estimate a lot of other numbers. Suppose you want to know the number of cells in your body. An individual cell cannot be seen by the unaided eye. Among the smallest things we can see are the lines of a ruler, which are a small fraction of 1 mm wide. Hence we can assume that cells are smaller than 0.1 mm or 10^{-4} m. On the other hand we remember that cells were discovered by viewing through the early microscopes, which had a magnification less than 100. This means that cells can't be smaller than 10^{-6} m. The geometric mean of both bounds yields 10^{-5} m for the size of a cell and 10^{-15} m^3 for its volume. The volume of your body is easily derived from the density of water and the observation that we float. You do the math.

Guesstimation presents its collection of Fermi problems with an invitation to work them out yourself. After the authors work through a trio of examples, each problem is posed at the top of a right-hand page. Printed upside down below the question (and a sketch, usually humorous, by Patty Edwards) are hints to get you started if you are completely clueless. If you give up, you can turn the page for a full Fermi-style reasoning to the answer. Working out questions like "How many people are airborne over the US at any given moment?" or "How long a hot dog can be made from a typical cow?" is both entertaining and enlightening—especially if you do so along with friends or your children. It may also foster your career. Not just because you may encounter Fermi questions in job interviews, but because making correct guesses quickly establishes your reputation as an expert.

Guesstimation
Solving the World's Problems on the Back of a Cocktail Napkin

by Lawrence Weinstein and John A. Adam

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CREDIT: PATTY EDWARDS/FROM *GUESSTIMATION*

The reviewer is at the Santa Fe Institute and the Institut für Theoretische Physik, Otto von Guericke University, 39106 Magdeburg, Germany. E-mail: mertens@ovgu.de

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