PHYSICS 603 - Spring 2021 - ODU

Classical Mechanics - Problem Set 10 – due Tuesday, April 27

Problem 1) (Qualifier Problem)

A particle named Z^0 of mass 90 GeV/c^2 is produced by a head-on collision of an electron e^- and a positron e^+ (having the same mass as the electron –look it up!) moving with identical magnitudes of their momenta but in opposite directions (e^+e^- collider)

(1) Calculate the energy (in GeV), the gamma factor $\gamma(v)$, and the linear momentum of the electron (or positron).

(2) If the Z^0 were to be produced by colliding e^+ on an e^- at rest (fixed target), what would the energy of e^+ (in GeV) have to be?

Problem 2) (Qualifier Problem)

In Compton scattering an initial photon with the energy E_1 scatters elastically on a free electron (at rest) at an angle θ. Find the energy E_2 of the scattered photon and a relationship between wave lengths of the incoming and scattered photons.

Problem 3)

The relativistic Hamiltonian for a particle with charge q and mass m in a constant magnetic field along the z-axis is given by $H = \sqrt{m^2c^4 + \left(\frac{P_\varphi}{r} - q\frac{rB}{2}\right)^2 c^2 + P_r^2c^2 + P_z^2c^2}$

using cylindrical coordinates (r,ϕ,z). Here, the P's are the canonical momenta conjugate to the cylindrical coordinates.

1) Write down Hamilton's equations of motion for each of the three coordinates and each of the three canonical momenta

2) Which of the canonical momenta are conserved? How are they related to the "actual" spatial components of the relativistic Cartesian (linear) 4-momentum?

3) Show that a solution with fixed $r(t) = r_0$ and constant velocity $dz/dt = v_{z0}$ in z-direction exists given the correct relationship between the canonical momenta and r_0 and B. What

is this relationship? What is the corresponding relationship between the ordinary momenta and r_0 and B? How long does it take for the particle to complete one full cycle around the z-axis?

Hint: You can use the fact that the Hamiltonian is equal to the relativistic energy and conserved (why?) and simplify the expressions you get by inserting its value, E (only **after** calculating derivatives!).