Classical Mechanics - Problem Set 1 – Due January 28

Problem 1) Consider a vinyl record (now once more fashionable!) with a single groove that spirals inward from the edge at radius R_2 to the inner radius R_1. The spiral makes N turns, evenly spaced between R_1 and R_2. Put a tiny object of mass m into the groove, which moves without friction along the groove without rolling. The object has only one degree of freedom (it can move along the groove, but not perpendicular to it). Use as generalized coordinate the angle ϕ between the starting point (at the outer edge, radius R_2) and the instantaneous position of the marble (increasing by 2π for every full turn).

a) Express the cartesian coordinates x and y describing the marble position relative to a fixed coordinate system centered on the record axis in terms of that generalized coordinate. (Assume the record does not rotate).
b) Using the same generalized coordinate, write down the (cartesian) velocity of the marble, and the kinetic energy. Find the generalized force Q_F in terms of the force components F_x and F_y in x- and y-direction.
c) What are the equations of motion for the marble? (Don't attempt to solve them).

Problem 2) The position of a particle of mass m moving freely in two dimensions in an inertial Cartesian coordinate system is given by (x, y).

a) Write down the particle's kinetic energy $T(\dot{x},\dot{y})$ and the equations of motion.
b) Generalized coordinates $q_1(x, y, t)$ and $q_2(x, y, t)$ are defined by

$$q_1 = x\cos\omega t + y\sin\omega t$$
$$q_2 = -x\sin\omega t + y\cos\omega t$$

where ω is a constant. Write down the inverse transformations, $x(q_1,q_2,t)$ and $y(q_1,q_2,t)$ and hence find an expression for $T(q_1,q_2,\dot{q}_1,\dot{q}_2,t)$ in the new coordinates.

c) Obtain the equations of motion in the new coordinates (including the generalized forces Q_1 and Q_2). Identify the extra terms that appear, in terms of so-called "fictitious forces".

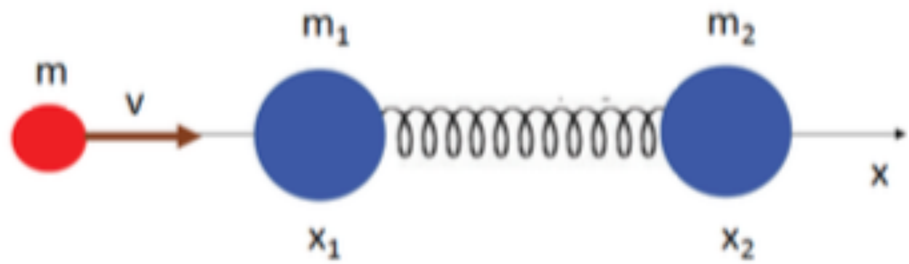
Problem 3)

Goldstein page 10 shows how one can split up the total kinetic energy of a system of particles into a part involving the motion of the center of mass only ($\frac{1}{2}MV^2, \mathbf{V} = d\mathbf{R}/dt$) and a term involving the "internal" or relative velocities, $\mathbf{v}_i' = \mathbf{v}_i - \mathbf{V}$ (eq. 1.31). The set of internal velocities $\{\mathbf{v}_i'\}$ are not all independent, since $\sum_i m_i \mathbf{v}_i' = 0$.

For a system of only two particles, we can replace $\mathbf{v}_1'$ and $\mathbf{v}_2'$ by a single variable, $\mathbf{v} = d\mathbf{r}/dt$. Here, $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = \mathbf{r}_2' - \mathbf{r}_1'$ is the relative coordinate (the distance vector) between the two mass points. Show that in this case, you can write the kinetic energy with just two terms involving only **V** and **v**: $T = \frac{1}{2}MV^2 + \frac{1}{2}\mu v^2$. Express the "reduced mass" μ in terms of the two masses m_1 and m_2.

Problem 4) (Application of Problem 3)

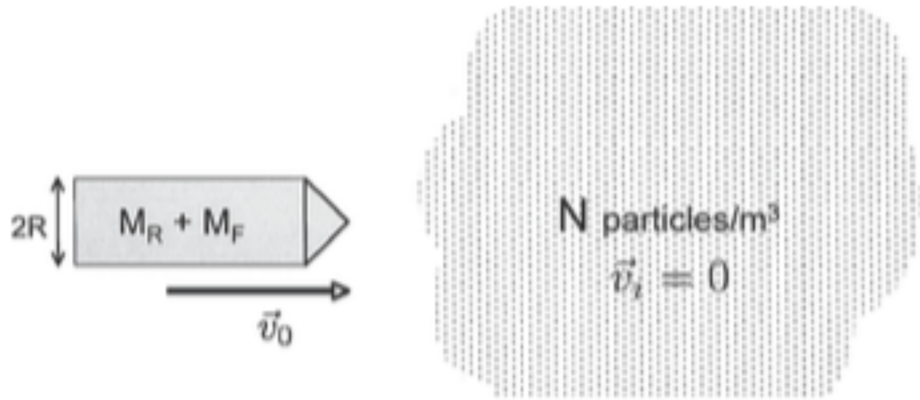
Two balls of mass m_1 and m_2 are connected by a spring with an elastic constant k. Initially, m_1 is located at $x = 0$ and m_2 at $x = L$ which corresponds to the relaxed length of the spring. Both balls are at rest. A third ball of mass m moving with a velocity v from the left hits ball 1 and sticks to it, as shown in the figure below. Ignore friction and consider only motion along the x-axis. Calculate the amplitude and frequency of oscillations after the impact. Use the results and nomenclature of Problem 3 (2^{nd} half) to solve this problem.



Problem 5)

A cylindrical rocket of diameter $2R$, mass M_R and containing fuel of mass M_F is moving through empty space at velocity v_0. At some point the rocket enters a uniform cloud of interstellar particles with volume density N (e.g, particles/m^3), each particle having mass $m \ll M_R$ and initially at rest. To compensate for the dissipative force of the particles colliding with the rocket, the rocket engines emit fuel at the rate γ ($dM_F/dt = -\gamma$) at a constant velocity u with respect to the rocket. Ignore the gravitational interaction between the rocket and the cloud particles.

a) Assuming that the dissipative drag force from the cloud particles is $F = -Av^2$, where A is a constant, derive the differential equation for the velocity $v(t)$ of the rocket through the cloud as it is firing its engines.
b) What must the rocket thrust γu be to maintain constant velocity v_0?
c) If the rocket suddenly runs out of fuel, how would its velocity $v(t)$ decrease with time as the rocket moves through the cloud?
d) BONUS: Assume that the front nose-cone of the rocket has opening angle of 90°, each cloud particle bounces off the rocket elastically, and collisions occur very frequently (see sketch below). Prove that the drag force is indeed proportional to v^2 and determine the constant A in terms of the other variables.



2R
$M_R + M_F$
$\vec{v}_0$
N particles/m³
$\vec{v}_i = 0$