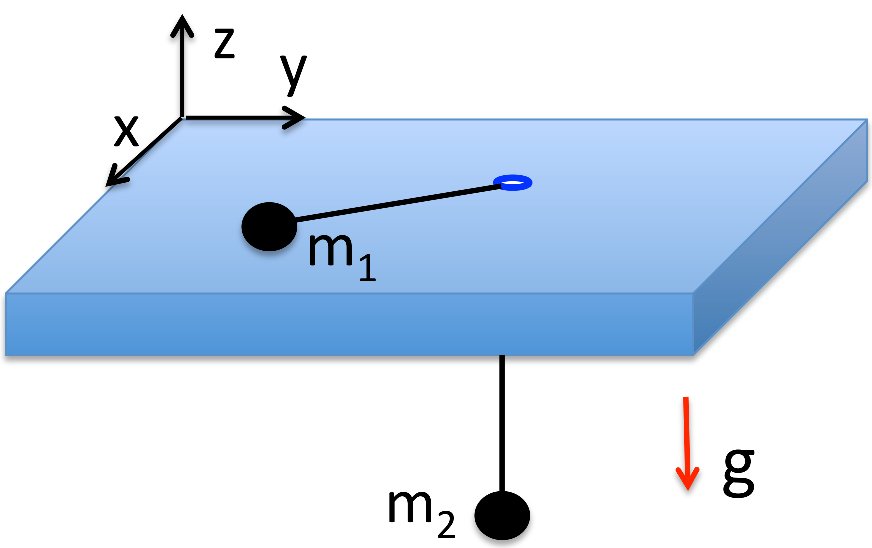
PHYSICS 603 -- Winter/Spring Semester 2021 - ODU

Classical Mechanics - Problem Set 2 – Due Tuesday, February 4

Problem 1)

Two particles of mass m_1 and m_2 are connected by a string of length L passing through a small hole in a smooth table so that m_1 is situated on the table and m_2 hangs suspended in a gravitational field with acceleration g. m_1 moves without friction in the x-y plane of the table, and m_2 moves only vertically along the z-axis parallel to **g**. Assume that neither m_1 nor m_2 passes through the hole.

Write down the Lagrange equations in terms of generalized coordinates of the system. Do not solve it.
Can this system be in equilibrium? If so, find the equilibrium condition.



Problem 2)

A mass point (mass m) is moving in the x-y plane. There are no external forces present, but the mass is tethered to the origin ($x = y = 0$) by a string of fixed length R. This constraint can be expressed by requiring that the function $g(x,y) = 0$, where $g(x,y) = x^2 + y^2 -$

R^2 . Using Lagrange multipliers, write down the Euler-Lagrange equations of motion for the variables x and y. A solution can be found by making the ansatz $x = R\cos\phi$ and $y = R\sin\phi$ for some parameter $\phi(t)$ (of course we know what that is, but the point is that we're not supposed to start out with ϕ as generalized coordinate, but only use it for now). Obviously, this solution "automatically" fulfills the constraint.

Show that you can now solve the set of two equations for x and y both for $\phi(t)$ and for the Lagrange multiplier. From your solution, extract the x- and y-components of the force of constraint exerted by the string on the mass point.

Problem 3)

Two masses m_1 and m_2 move under their mutual gravitational attraction in a uniform external gravitational field whose acceleration is g. Choose as coordinates the Cartesian coordinates X, Y, Z of the center of mass vector $\mathbf{R}$ (taking Z in the direction of g) and the spherical coordinates r, θ and ϕ that define the relative vector $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$.

Write down the Lagrangian in terms of these 6 coordinates. Calculate all 6 generalized momenta. Which of these 6 momenta is conserved? Write down the 6 Euler-Lagrange equations of motion.

Problem 4)

A body in a gravitational field (acceleration g) is released from a height h and after a time T it strikes the ground, $T = \sqrt{2h/g}$. The equation for the distance of fall s after a time t is of course $s = 1/2\, gt^2$, but assume someone proposes as an alternative the more general form $s = 1/2\, gt^2 + A\sin(\pi t/T)$, where A is an arbitrary constant of dimension length. Note that this gives the same answer for $t = 0$ and $t = T$, meaning the total distance h between release and ground contact is traveled in the same time T.

Calculate the Lagrange function for this proposed trajectory (with s as the single generalized coordinate) in order to find out which value of A yields a minimum for the integral in Hamilton's principle:

$$\int_{t=0}^{t=T} L(s,\dot{s},t)\,dt\ .$$

(Note that you have to express L as a function of t first before calculating the integral. Do **not** appeal to Hamilton's principle for your solution – you must evaluate the integral explicitly and then find its minimum as a function of A.)