Classical Mechanics - Problem Set 3 – DUE: Thursday, February 18

Problem 1)

A particle of mass m moving along the x-axis is subject to the force $F = -kx + a/x^3$ where both k and a are positive.

1) What are the equilibrium positions of the particle and are they stable?
2) Assume that the particle undergoes small oscillations around an equilibrium point. Calculate the period of these oscillations.

Problem 2)

Solve Goldstein's Exercise 11, p.128: Two particles move about each other in circular orbits under the influence of gravitational forces, with a period τ. Their motion is suddenly stopped at a given instant of time, and they are then released and allowed to fall into each other. Prove that they collide after a time $T = \tau/(4\sqrt{2})$.
(Note: this problem can be solved either by brute force integration – not recommended – or by using a trick that mostly requires some creative thinking and nearly NO math).

Problem 3)

A planet is circling Sun on an elliptic orbit with eccentricity e. Calculate the ratio between its maximum velocity (where does this occur? Why?) and its minimum velocity (ditto) as a function of e. By how much does the velocity of Earth ($e = 0.0167504$) vary (in %) over one orbit?