Classical Mechanics - Problem Set 5 (Due Tuesday, Mar 9)

**Problem 1)**

Assume you are given a rotational matrix \( \mathbf{R} \) that transforms the components of the vector \( \mathbf{r} \) in the unprimed coordinate system, \( \mathbf{r}^T = (x,y,z) \), into its components in the primed (rotated) coordinate system, \( \mathbf{r}'^T = (x',y',z') \) where \( \mathbf{r}' = \mathbf{R} \mathbf{r} \) (passive rotation).

Show explicitly (using known properties of derivatives and rotational matrices) that the gradient
\[
\nabla^T = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)
\]
transforms like a vector under this rotation,
\[
\nabla' = \mathbf{R}\nabla \quad \text{where} \quad (\nabla')^T = \left( \frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{\partial}{\partial z'} \right),
\]
(i.e., the components of \( \nabla \) in the primed coordinate system are simply the derivatives with respect to the primed coordinates \( x', y', z' \)).

(If you find the notion of a “del operator” without anything to take the derivative of unfamiliar, just “multiply” it with any scalar function \( f(\mathbf{r}) \), i.e., take the gradient of such a function).

**Problem 2)**

Check Goldstein’s Eq. (4.46), pg. 153, by explicitly multiplying the three “Euler matrices” BCD. Show your intermediate steps.

**Problem 3)**

An infinitesimal rotation around the z-axis by an angle \( d\phi \) can be described by the rotational matrix \( \mathbf{R}(d\phi) = \mathbf{I} + \mathbf{M}_3 d\phi \) (see Goldstein pg. 171). Show that for a finite rotation around the same axis by an angle \( \phi \), one can write the rotational matrix as \( \mathbf{R}(\phi) = \exp(\mathbf{M}_3 \phi) \) where the “exponent of a matrix” is simply given by the usual Taylor expansion of the exp function. Note: your proof doesn’t have to be rigorous – I’ll give you full credit if you calculate the first few terms in the Taylor expansion and then explain how the full series will lead to the correct result.