PHYSICS 603 - Winter/Spring Semester 2021 - ODU

Classical Mechanics - Problem Set 5 (Due Tuesday, Mar 9)

Problem 1)

Assume you are given a rotational matrix $\mathbf{R}$ that transforms the components of the vector $\mathbf{r}$ in the unprimed coordinate system, $\mathbf{r}^T = (x,y,z)$, into its components in the primed (rotated) coordinate system, $\mathbf{r}'^T = (x',y',z')$ where $\mathbf{r}' = \mathbf{R}\,\mathbf{r}$ (passive rotation).
Show explicitly (using known properties of derivatives and rotational matrices) that the gradient

$$\vec{\nabla}^T = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$$

transforms like a vector under this rotation,

$$\vec{\nabla}' = \mathbf{R}\vec{\nabla} \quad \text{where } (\vec{\nabla}')^T = (\frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{\partial}{\partial z'}),$$

(i.e., the components of $\vec{\nabla}$ in the primed coordinate system are simply the derivatives with respect to the primed coordinates x', y', z').
(If you find the notion of a "del operator" without anything to take the derivative of unfamiliar, just "multiply" it with any scalar function f($\mathbf{r}$), i.e., take the gradient of such a function).

Problem 2)

Check Goldstein's Eq. (4.46), pg. 153, by explicitly multiplying the three "Euler matrices" BCD. Show your intermediate steps.

Problem 3)

An infinitesimal rotation around the z-axis by an angle $d\phi$ can be described by the rotational matrix $\mathbf{R}(d\varphi) = \mathbf{1} + \mathbf{M}_3 d\varphi$ (see Goldstein pg. 171). Show that for a finite rotation around the same axis by an angle ϕ, one can write the rotational matrix as $\mathbf{R}(\varphi) = \exp(\mathbf{M}_3\varphi)$ where the "exponent of a matrix" is simply given by the usual Taylor expansion of the exp function. Note: your proof doesn't have to be rigorous – I'll give you full credit if you calculate the first few terms in the Taylor expansion and then explain how the full series will lead to the correct result.