PHYSICS 603 - Spring 2021 - ODU

Classical Mechanics - Problem Set 8 – Due Tuesday, April 6

Problem 1)

Show directly that the transformation

$$Q = \log\left(\frac{1}{q}\sin p\right), \qquad P = q\cot p$$

is canonical (Problem 4 on page 422 in Goldstein).

Problem 2)

a) Show that the following transformation is canonical:

$$x = \frac{1}{\alpha}\left(\sqrt{2P_1}\sin Q_1 + P_2\right), \qquad p_x = \frac{\alpha}{2}\left(\sqrt{2P_1}\cos Q_1 - Q_2\right)$$

$$y = \frac{1}{\alpha}\left(\sqrt{2P_1}\cos Q_1 + Q_2\right), \qquad p_y = -\frac{\alpha}{2}\left(\sqrt{2P_1}\sin Q_1 - P_2\right)$$

where α is some fixed parameter.

b) Apply this transformation to the problem of a particle of charge q moving in the x-y plane perpendicular to a constant magnetic field $\mathbf{B}^*$. Express the Hamiltonian for this problem in the (Q_i, P_i) coordinates letting the parameter α take the form

$$\alpha^2 = \frac{qB}{c}\ .$$

From this Hamiltonian, obtain the motion of the particle as a function of time.

* Use the vector potential $\vec{A} = -\frac{1}{2}\vec{r}\times\vec{B} = \frac{1}{2}\left(-yB_0\hat{x} + xB_0\hat{y}\right)$. Note that you can drop the factor "*c*" in the denominator in the definition for α since it doesn't occur if all quantities are calculated in the SI system.