PHYSICS 603 - Spring 2021 - ODU

Classical Mechanics - Problem Set 9 – Due Tuesday, April 13

Problem 1)

Solve Goldstein's Problem 30, page 427.

Problem 2

1) Consider the example of a string with finite length L_0 under tension exactly as we discussed in lecture, namely in the form of a Fourier decomposition of the displacement $y(x,t)$. In addition to the potential energy due to the elongation of the string, include now the potential energy of the string in the gravitational field on Earth's surface (remember: the mass density of the string per unit length is λ), using that same Fourier expansion. Outline the solution to the stationary situation only (i.e., the string is at rest) in this Fourier decomposition framework. Calculate explicitly the "Fourier amplitude" belonging to the lowest "Fourier mode" of the string displacement for the equilibrium. Find a formula for the sagging of the string in the middle as function of its mass density and the tension, using just that lowest "Fourier mode".
2) XC: Now repeat this analysis using the Lagrangian density for continuous degrees of freedom instead of the Fourier decomposition, and write down the extended Euler-Lagrange equations of motion. Again, find (only) the static (equilibrium) solution and the maximum deflection from y=0 in the middle of the string. Compare with the result you found under 1).