QUANTIZED OR NOT

FINITE DoF OR INFINITE

RELATIVISTIC OR NOT

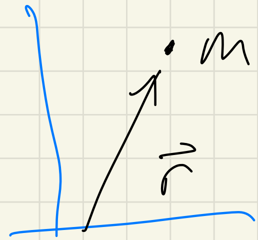
LORENTZ GALILEAN

	REL		NON REL	
	Q	NOT	Q	NOT
FINITE DoF	REL QUANT MECH	SPECIAL REL	QUANTUM MECH	CLASSIC
∞ DoF	REL QUANT FIELD THEORY	REL FIELD THEORY E+M	QUANTUM FIELD TH	FLUID DYNAM

INCOMPLETE INFORMATION

THERMO / STAT MECH

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{p} = m\vec{v}$$



$$\vec{F} = \frac{d\vec{p}}{dt} = \dot{\vec{p}} = m\dot{\vec{v}}$$

IF m = constant

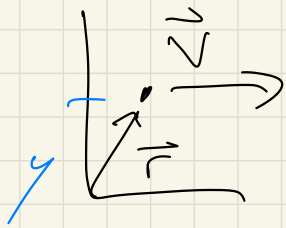
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$\vec{F} = 0 \iff \dot{\vec{p}} = 0 \iff \vec{p}$$

CONSERVED

$$\vec{L} = \vec{r} \times \vec{p}$$ ABOUT THE ORIGIN

$$= ymv\,(-\hat{z})$$



$$\vec{L} = \vec{r} \times \vec{p}$$

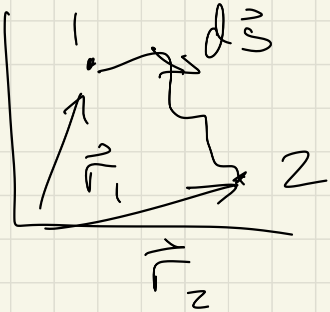
$$\vec{N} = \vec{r} \times \vec{F} = \vec{r} \times \frac{d}{dt}(m\vec{v})$$

$$\frac{d}{dt}(\vec{r} \times m\vec{v}) = \underbrace{\dot{\vec{r}} \times m\vec{v}}_{\text{zero}} + \vec{r} \times$$

$$\vec{N} = \frac{d}{dt}(\vec{r} \times m\vec{v}) = \frac{d\vec{L}}{dt} = \dot{\vec{L}}$$

$$\vec{N} = 0 \iff \vec{L} \text{ CONSERVED}$$

WORK $W_{12} = \int_1^2 \vec{F} \cdot \underbrace{d\vec{s}}_{\text{PATH ELEMENT}}$



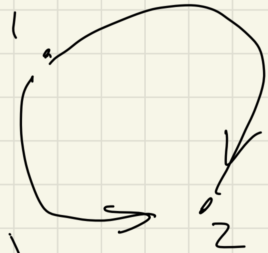
$$d\vec{s} = \vec{v}\,dt$$

$$W_{12} = \underbrace{m \int_1^2 \frac{d\vec{v}}{dt}}_{F} \cdot \underbrace{\vec{v}\,dt}_{d\vec{s}} = \frac{m}{2} \int_1^2 \frac{d}{dt}(v^2)\,dt$$

$$= \frac{m}{2}\left(v_2^2 - v_1^2\right) = \underbrace{T_2 - T_1}_{\text{KINETIC ENERGY}}$$

IF W_{12} IS PATH INDEPENDENT

THEN F IS CONSERVATIVE



$$\oint \vec{F}\cdot d\vec{s} = 0$$

$$\Longleftrightarrow \quad \vec{F} = -\vec{\nabla} V(\vec{r})$$

$$V'(\vec{r}) = V(\vec{r}) + V_0$$

POTENTIAL E

ENERGY CONSERVED

$$T_1 + V_1 = T_2 + V_2$$

IF V IS TIME DEPENDENT

$$V = V(\vec{r}, t) \quad \text{then}$$

$$\underline{dW} = \vec{F}\cdot d\vec{s} = -\frac{\partial V}{\partial s}\,ds \neq \underline{-dV}$$

$$-\vec{\nabla} V\cdot d\vec{s} = -\frac{\partial V}{\partial s}\,ds$$

$$+dV = \frac{\partial V}{\partial s}\,ds + \frac{\partial V}{\partial t}\,dt$$

E NOT CONSERVED

LOTS OF PARTICLES

$$\Rightarrow \vec{F} = \dot{\vec{p}} \Rightarrow \sum_j \vec{F}_{ji} + \vec{F}_i^{(e)} = \dot{\vec{p}}_i$$

EXTERNAL FORCE

FORCE OF j ON i

ASSUME
WEAK LAW OF ACTION/REACTION

$$\vec{F}_{ij} = -\vec{F}_{ji}$$

$$\frac{d^2}{dt^2}\sum_i m_i \vec{r}_i = \sum_i \vec{F}_i^{e} + \underbrace{\sum_{\substack{ij \\ i \neq j}} \vec{F}_{ji}}_{=0}$$

$$\bar{R} = \frac{\sum m_i \bar{r}_i}{\sum m_i}$$

CENTER OF MASS POSITION

$$M\vec{R} = \sum m_i \left(\frac{\sum m_i \vec{r}_i}{\sum m_i}\right) = \sum m_i \vec{r}_i$$

$$M\ddot{\vec{R}} = \sum \vec{F}_i^e = \vec{F}^e$$

$$\dot{\vec{P}} = M\dot{\vec{R}}$$

$\vec{F}_e = 0 \iff \vec{P}$ IS CONSERVED

ANGULAR MOMENTUM

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$$

$$\dot{\vec{L}} = \sum_i \frac{d}{dt}(\vec{r}_i \times \vec{p}_i)$$

$\dot{\vec{r}}_i \times \vec{p}_i = 0$

$$\underline{\dot{\vec{L}}} = \underbrace{\sum_i \vec{r}_i \times \vec{F}_i^e}_{\vec{N}^e} + \underline{\sum_{\substack{ij \\ i \neq j}} \vec{r}_i \times \vec{F}_{ji}}$$

$$\sum \vec{r}_i \times \vec{F}_{ji} = \frac{1}{2}\left(\sum \vec{r}_i \times \vec{F}_{ji} + \vec{r}_j \times \vec{F}_{ij}\right)$$

$$= \sum (\vec{r}_i - \vec{r}_j) \times \vec{F}_{ji}$$

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$$

$$= \vec{r}_{ij} \times \vec{F}_{ji}$$

STRONG ACTION REACTION

$$\vec{r}_{ij} \times \vec{F}_{ji} = 0$$

CENTRAL FORCE

$$\dot{\vec{L}} = \vec{N}^e \qquad \vec{N}^e = 0$$

$$\Longleftrightarrow \vec{L}$$

CONSERVED

$$\vec{r}_i = \vec{R} + \vec{r}_i{}'$$

RELATIVE COORDS

$$\vec{v}_i = \vec{V}_{TOT} + \vec{v}_i{}'$$

$$\vec{L} = \sum (\vec{R} + \vec{r}_i{}') \times m_i (\vec{V}_{TOT} + \vec{v}_i{}')$$

$$= \sum_i (\vec{R} \times m_i \vec{V}_{TOT})$$

$$+ \sum_i \vec{r}_i{}' \times m_i \vec{v}_i{}'$$

$$+ \sum_i \cancel{(m_i \vec{r}_i{}')}^{\,0} \times \vec{V}_{TOT} + \vec{R} \times \frac{d}{dt} \cancel{\sum_i m_i \vec{r}_i{}'}^{\,0}$$

$$\vec{L} = \sum_i \vec{R} \times M \vec{V}_{TOT} + \sum_i \vec{r}_i{}' \times \vec{p}_i{}'$$

CoM

RELATIVE

ENERGY $$W_{12} = \sum_i \int_1^2 \vec{F}_i \cdot d\vec{s}_i$$

$$= \sum_i \int_1^2 \underbrace{m\dot{\vec{v}}_i}_{\vec{F}_i} \cdot \underbrace{\vec{v}_i \, dt}_{d\vec{s}_i} = \sum_i \frac{1}{2} m v_i^2 \Big|_1^2$$

$$= \sum_i T_2^i - T_1^i$$

$$T = \frac{1}{2} \sum_i m_i \left(\vec{V}_{TOT} + \vec{v}_i' \right) \cdot \left(\vec{V}_{TOT} + \vec{v}_i' \right)$$

$$= \frac{1}{2} \sum_i m_i V_{TOT}^2 + \frac{1}{2} \sum_i m_i \vec{v}_i'^2$$

$$\vec{V}_{TOT} \cdot \sum \vec{v}_i' = 0$$

IF $\vec{F}^e_i$ ARE CONSERVATIVE

THEN $\vec{F}^e_i = -\vec{\nabla}_i V_i(\vec{r}_i)$

IF $\vec{F}_{ij}$ CONSERVATIVE

THEN ALSO HAVE A POTENTIAL

STRONG ACTION REACTION

$$\Rightarrow V_{ij} = V_{ij}(|\vec{r}_i - \vec{r}_j|)$$

$$\vec{F}_{ji} = -\vec{\nabla}_i V_{ij} = +\vec{\nabla}_j V_{ij}$$

$$= -\vec{F}_{ij}$$

EXTERNAL INTERNAL

$$V = \sum_i V(\vec{r}_i) + \sum_{i \neq j} V_{ij}(|\vec{r}_{ij}|)$$