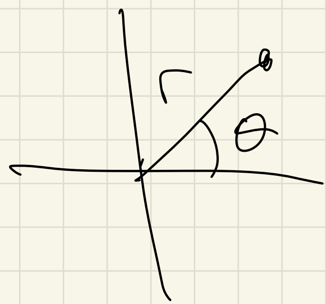
$$Q_j = \sum_i \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = 0 \qquad \mathcal{L} = T - V$$



$$x = r\cos\theta$$

$$\dot{x} = \dot{r}\cos\theta - r\dot{\theta}\sin\theta$$

$$y = r\sin\theta$$

$$\dot{y} = \dot{r}\sin\theta + r\dot{\theta}\cos\theta$$

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$$

$$Q_r = F_x \frac{\partial x}{\partial r} + F_y \frac{\partial y}{\partial r} = F_x\cos\theta + F_y\sin\theta$$

$$= \vec{F} \cdot \hat{r}$$

$$Q_\theta = F_x \frac{\partial x}{\partial \theta} + F_y \frac{\partial y}{\partial \theta}$$

$$= F_x(-r\sin\theta) + F_y(r\cos\theta)$$

$$= -yF_x + xF_y = |\vec{r} \times \vec{F}| = |\vec{N}|$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{r}} - \frac{\partial T}{\partial r} = Q_r$$

$$m\ddot{r} - mr\dot{\theta}^2 = \vec{F} \cdot \hat{r}$$

centripetal force

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\theta}} - \frac{\partial T}{\partial \theta} = Q_\theta$$

$$\frac{d}{dt}(mr^2\dot{\theta}) = 2mr\dot{r}\dot{\theta} + mr^2\ddot{\theta}$$

$$= Q_\theta = |\vec{N}|$$

$$mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} = |\vec{N}|$$

Coriolis force

$$T = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2$$

$$V = \frac{1}{2} k (r - \ell)^2$$

$$\mathcal{L} = T - V$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{r}} - \frac{\partial \mathcal{L}}{\partial r} = 0$$

$$m \ddot{r} - m r \dot{\theta}^2 + k(r - \ell) = 0$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} = m r^2 \ddot{\theta} + 2 m r \dot{r} \dot{\theta} = 0$$

$$\dot{L} = 0$$ ANGULAR MOMENTUM CONSERVED

$$\ddot{r} = \ddot{\theta} = 0$$

STATIONARY SOLUTION

$$\dot{r} = \overset{0}{\cancel{\text{CONST}}} \qquad \dot{\theta} = \omega_0$$

$$\Rightarrow m r_0 \omega_0^2 = k(r_0 - \ell)$$

$$\omega_0^2 = \frac{k(r_0 - \ell)}{m r_0} \qquad r_0 > \ell$$

SMALL OSCILLATIONS

$$r(t) = r_0 + \delta r \qquad \delta r \ll r_0$$

$$\dot\theta(t) = \omega_0 + \delta\dot\theta \qquad \delta\dot\theta \ll \omega_0$$

$$m\ddot r - m r\dot\theta^2 + k(r-l) = 0$$

$$\Rightarrow m\delta\ddot r = m(r_0+\delta r)(\omega_0+\delta\dot\theta)^2 - k(r_0+\delta r - l)$$

$$\left[\begin{array}{l} m r_0 \omega_0^2 = k(r_0 - l) \\ \dfrac{d}{dt}(m r^2\dot\theta) = 0 \qquad m r^2\dot\theta = m r_0^2 \omega_0 \end{array}\right.$$

WORK TO 1ST ORDER

$$m(r_0+\delta r)^2(\omega_0+\delta\dot\theta) = m r_0^2\omega_0$$

$$\cancel{m r_0^2\omega_0} + 2\cancel{m}\cancel{r_0}\,\delta r\,\omega_0 + \cancel{m}r_0^{\cancel{2}}\delta\dot\theta = \cancel{m r_0^2\omega_0}$$

$$\delta\dot\theta = -\frac{2\omega_0}{r_0}\delta r$$ FROM $\dot L = 0$

$$m\delta\ddot{r} = m(r_0 + \delta r)(\omega_0 + \delta\dot{\theta})^2 - k(r_0 + \delta r - \ell)$$

$$\delta\dot{\theta} = -\frac{2\omega_0}{r_0}\delta r$$

$$m\delta\ddot{r} = \cancel{m r_0 \omega_0^2 - k(r_0 - \ell)} \rightarrow 0$$

FROM STATIONARY SOL'N

$$+ 2 m r_0 \delta\dot{\theta}\,\omega_0 + m\delta r\,\omega_0^2 - k\delta r$$

$$m\delta\ddot{r} = \delta r\left[m\omega_0^2 - k + 2m\cancel{r_0}\omega_0\left(-\frac{2\omega_0}{\cancel{r_0}}\right)\right]$$

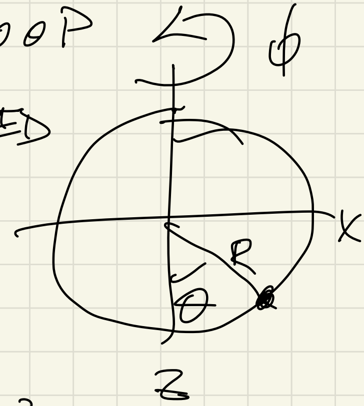
$-4m\omega_0^2$

$$\delta\ddot{r} = \delta r\left[-3\omega_0^2 - \frac{k}{m}\right]$$

$$\Rightarrow \omega_r = \sqrt{3\omega_0^2 + \frac{k}{m}}$$

SMALL OSCILLATION FREQ

BEAD ON ROTATING HOOP



$\omega = \dot{\phi}$ ANGULAR SPEED

ROTATION AROUND AXIS

$$T = \frac{1}{2} m R^2 \dot{\theta}^2$$ FROM BEFORE

$$+ \frac{1}{2} m (\omega R \sin\theta)^2$$ AROUND Z-AXIS

$$V = -mgR\cos\theta + \cancel{231.6\,J}$$ IRRELEVANT

$$\mathcal{L} = T - V$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \frac{d}{dt}\left(mR^2\dot{\theta}\right) = mR^2\ddot{\theta}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = m\omega^2 R^2 \sin\theta\cos\theta - mgR\sin\theta$$

$$\cancel{m}R^{\cancel{2}}\ddot{\theta} = \cancel{m}\omega^2 R^{\cancel{2}}\sin\theta\cos\theta + \cancel{m}g\cancel{R}\sin\theta$$

$$R\ddot{\theta} = \sin\theta\,(\omega^2 R\cos\theta - g)$$

a) NO ROTATION, SMALL ANGLES

$\omega = 0 \qquad \sin\theta \simeq \theta$

$30^\circ \qquad \sin 30^\circ = \frac{1}{2} = 0.50$

$30^\circ = \frac{\pi}{6} = \frac{3.14159}{6} \simeq 0.52$

$$R\ddot{\theta} = -g\theta/R$$

$$\theta = A\cos\left(\sqrt{\frac{g}{R}}\,t + \cancel{\phi}\,\delta\right)$$

OSCILLATIONS W/FREQ $\sqrt{g/R}$

b) $\omega \neq 0$ STATIONARY $\theta = \text{CONST}$

$\omega^2 R\cos\theta = g \qquad \sin\theta = 0$

$\theta = 0, \pi$

$$\cos\theta = g/\omega^2 R$$

IF $\omega^2 R < g \Rightarrow$ THEN $\theta = 0$