$$R\ddot{\theta} = \sin\theta(\omega^2 R\cos\theta - g)$$

$$\theta = \theta_0 + \delta\theta \qquad\qquad \omega^2 R > g$$

$$\cos\theta_0 = \frac{g}{\omega^2 R}$$

$$\sin(\theta_0+\delta\theta) = \sin\theta_0 \cancel{\cos\delta\theta}^{\,1} + \cancel{\sin\delta\theta}^{\,\delta\theta}\cos\theta_0$$

$$\cos(\theta_0+\delta\theta) = \cos\theta_0 \cancel{\cos\delta\theta} - \sin\theta_0 \cancel{\sin\delta\theta}^{\,\delta\theta}$$

$$R\ddot{\delta\theta} = (\sin\theta_0 + \cos\theta_0\,\delta\theta)\cdot(\omega^2 R(\cos\theta_0 - \sin\theta_0\,\delta\theta) - g)$$

$$= \sin\theta_0\cancel{(\omega^2 R\cos\theta_0 - g)}^{\,0}$$

$$+ \delta\theta(\cancel{\omega^2 R\cos^2\theta_0 - g\cos\theta}^{\,0} - \omega^2 R\sin^2\theta)$$

$$\ddot{\delta\theta} = -\omega^2\sin^2\theta_0\,\delta\theta$$

$$\Rightarrow \text{freq} = \omega\sin\theta_0 = \sqrt{\omega^2 - \frac{g^2}{\omega^2 R^2}}$$

FORCES OF CONSTRAINT F_r, F_ϕ

$R \to R + \delta r$ FIND $F_r \ni \delta r = 0$

$$T = \frac{m}{2}\left[(\delta\dot{r})^2 + (R^2 + 2R\delta r)(\dot{\theta}^2 + \omega^2 \sin^2\theta)\right]$$

$$V = mg(R+\delta r)\cos\theta + F_r\,\delta r$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \delta\dot{r}}\right) - \frac{\partial \mathcal{L}}{\partial(\delta r)} = 0$$

$$\cancel{m\delta\ddot{r}}^{\,0} - \left[mR(\dot{\theta}^2 + \omega^2\sin^2\theta) - mg\cos\theta - F_r\right] = 0$$

$$F_r = mR(\dot{\theta}^2 + \omega^2\sin^2\theta) - mg\cos\theta$$

CENTRIPETAL GRAVITY

FORCES

$$\omega \to \omega_0 + \delta\dot{\phi}$$

$$T = \frac{m}{2}\left(R^2\dot{\theta}^2 + R^2\sin^2\theta\left(\omega_0 + \delta\dot{\phi}\right)^2\right)$$

$$V = -mgR\cos\theta + N\delta\phi$$

$$= -mgR\cos\theta + R\sin\theta\, F_\phi\, \delta\phi$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \delta\dot{\phi}} - \frac{\partial \mathcal{L}}{\partial \delta\phi} = 0$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \delta\dot{\phi}} = \frac{d}{dt}\left(mR^2\sin^2\theta\left(\omega_0 + \delta\dot{\phi}\right)\right)$$

$$= 2mR^2\cos\theta\sin\theta\left(\omega_0 + \cancelto{0}{\delta\dot{\phi}}\right)\dot{\theta}$$

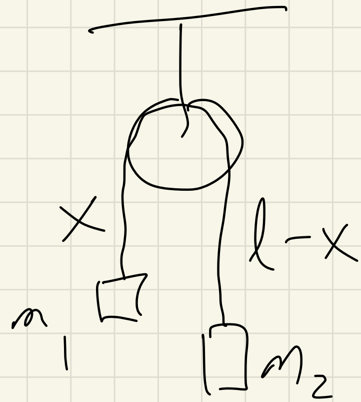
$$+ \cancelto{0}{mR^2\sin^2\theta\,\delta\ddot{\phi}}$$

$$-\frac{\partial \mathcal{L}}{\partial \delta\phi} = R\sin\theta\, F_\phi$$

$$2mR^{\cancel{2}}\omega_0\cos\theta\cancel{\sin\theta}\,\dot{\theta} + \cancel{R}\cancel{\sin\theta}\,F_\phi = 0$$

$$F_\phi = -2mR\omega_0\cos\theta\,\dot{\theta}$$

ATWOOD'S MACHINE



$$V = -m_1 g x - m_2 g (l - x)$$

$$T = \frac{1}{2}(m_1 + m_2)\dot{x}^2$$

$$\mathcal{L} = T = V$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} - \frac{\partial \mathcal{L}}{\partial x} = 0$$

$$(m_1 + m_2)\ddot{x} - (m_1 g - m_2 g)$$

$$\ddot{x} = \frac{m_1 - m_2}{m_1 + m_2} g$$

VELOCITY DEPENDENT POTENTIALS

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} = Q_j$$

if $\exists$ A FUNCTION $U \ni$

THERE EXISTS

SUCH THAT

$$Q_j = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\frac{\partial U}{\partial \dot{q}_j}$$

$$Q_j = \sum_i \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j}$$

EXAMPLE:

$$\vec{F} = q\left[\vec{E} + \vec{v} \times \vec{B}\right] \qquad \vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$

$$U = q\phi - q\vec{A}\cdot\vec{v} \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

$$\mathcal{L} = \frac{1}{2}mv^2 - q\phi + q\vec{A}\cdot\vec{v}$$

LOOK @ x-component

$$L = \frac{1}{2} m v^2 - q\phi + q\vec{A}\cdot\vec{v}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} = m\ddot{x} + q\frac{dA_x}{dt}$$

$$\frac{\partial T}{\partial x} = q\left(v_x \frac{\partial A_x}{\partial x} + v_y \frac{\partial A_y}{\partial x} + v_z \frac{\partial A_z}{\partial x}\right) - q\frac{\partial \phi}{\partial x}$$

$$\frac{dA_x}{dt} = \frac{\partial A_x}{\partial t} + \frac{\partial A_x}{\partial x}\frac{\partial x}{\partial t}$$

$$v_x \frac{\partial A_x}{\partial x} + v_y \frac{\partial A_x}{\partial y} + v_z \frac{\partial A_x}{\partial z}$$

$$(\vec{v}\times\vec{B})_x = \vec{v}\times(\vec{\nabla}\times\vec{A})\Big|_x$$

$$= v_y\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) + v_z\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)$$

$$m\ddot{x} = q\left(\cancel{v_x \frac{\partial A_x}{\partial x}} + v_y \frac{\partial A_y}{\partial x} + v_z \frac{\partial A_z}{\partial x}\right)$$

$$- q\frac{\partial \phi}{\partial x} - q\left(\frac{\partial A_x}{\partial t} + \cancel{v_x \frac{\partial A_x}{\partial x}} + v_y \frac{\partial A_x}{\partial y} + v_z \frac{\partial A_x}{\partial z}\right)$$

$$m\ddot{x} = q\left[-\frac{\partial\phi}{\partial x} + v_y\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) + v_z\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)\right]$$

$$= q\left[E_x + (\vec{v}\times\vec{B})_x\right]$$

VELOCITY DEPENDENT DRAG

$$F_x = -k_x v_x$$

DERIVE IT FROM

$$\mathcal{F} = \frac{1}{2}\sum_i \left(k_x v_{i_x}^2 + k_y v_{i_y}^2 + k_z v_{i_z}^2\right)$$

DISSIPATION FUNCTION

$$F_{x_i} = -\frac{\partial \mathcal{F}}{\partial v_{i_x}} \qquad \vec{F}_i = -\vec{\nabla}_{v_i}\mathcal{F}$$

WORK DONE OPPOSING DRAG

$$dW = -\vec{F}\cdot d\vec{r} = -\vec{F}\cdot\vec{v}\,dt$$
$$= \left(k_x v_x^2 + k_y v_y^2 + k_z v_z^2\right)dt$$
$$= 2\mathcal{F}\,dt$$

$$Q_j = \sum_i \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j} = -\sum_i \vec{\nabla}_v \mathcal{F} \cdot \frac{\partial \vec{r}_i}{\partial q_j}$$
$$= -\sum_i \vec{\nabla}_v \mathcal{F} \cdot \frac{\partial \vec{v}_i}{\partial \dot{q}_j} = \frac{\partial \mathcal{F}}{\partial \dot{q}_j}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_j} - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j$$

$$\Longrightarrow \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_j} - \frac{\partial \mathcal{L}}{\partial q_j} + \frac{\partial \mathcal{F}}{\partial \dot{q}_j} = 0$$

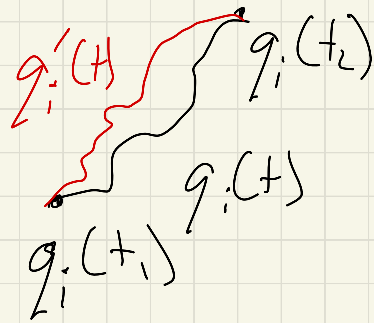
VISCOUS FLUID $\quad \vec{F} = -6\pi\eta a\vec{v}$

HAMILTON'S PRINCIPLE

ASSUME FORCES DERIVABLE FROM A SCALAR POTENTIAL

$$V(\{q_i\}, \{\dot{q}_i\}, t)$$

THEN ACTION $I = \int_{t_1}^{t_2} L\,dt$ IS STATIONARY WITH RESPECT TO THE PATH



SMALL VARIATIONS

$\Rightarrow$ ACTION UNCHANGED

$$q_i'(t) = q_i(t) + \alpha \eta_i(t)$$

$$\frac{\partial}{\partial\alpha}\int_{t_1}^{t_2}\mathcal{L}\left(\{q_i(t)+\alpha\eta_i(t)\},\{\dot q_i+\alpha\dot\eta_i\},t\right)dt=0$$

$$\int_{t_1}^{t_2}\sum_i\left[\frac{\partial\mathcal{L}}{\partial q_i}\left(\frac{\partial(q_i+\alpha\eta_i)}{\partial\alpha}\right)+\frac{\partial\mathcal{L}}{\partial\dot q_i}\left(\frac{\partial(\dot q_i+\alpha\dot\eta_i)}{\partial\alpha}\right)\right]dt=0$$

$$\int_{t_1}^{t_2}\left[\sum_i\left(\frac{\partial\mathcal{L}}{\partial q_i}\eta_i+\underbrace{\frac{\partial\mathcal{L}}{\partial\dot q_i}\dot\eta_i}\right)\right]dt=0$$

$$\frac{d}{dt}ab=\dot a b+a\dot b$$

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot q_i}\eta_i\right)=\boxed{\frac{\partial\mathcal{L}}{\partial\dot q_i}\dot\eta_i}+\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot q_i}\right)\eta_i$$

$$\sum_i\int_{t_1}^{t_2}\cancelto{0}{\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot q_i}\eta_i\right)dt}\qquad \eta_i(t_1)=\eta_i(t_2)=0$$

$$+\int\sum_i\boxed{\left[\frac{\partial\mathcal{L}}{\partial q_i}-\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot q_i}\right)\right]}^{=0}\eta_i\,dt=0$$