GALILEAN RELATIVITY

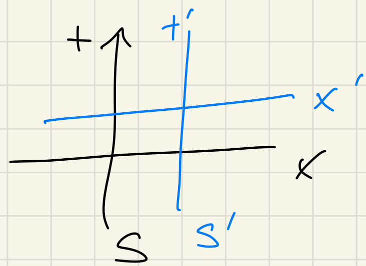
1) SIMULTANEITY INDEPENDENT OF INERTIAL REFERENCE FRAME (IRF)

2) SAME LENGTH IN EACH IRF

$$\Delta \vec{r} \cdot \Delta \vec{r} = \Delta \vec{r}' \cdot \Delta \vec{r}'$$

3) SAME TIME SCALE $\quad c \Delta t = c \Delta t'$

4) LAWS OF NATURE INDEPENDENT OF IRF



S' MOVING W/ $\vec{v}$ RELATIVE TO S

@ $t = t' = 0$, $\quad \vec{r}_i = \vec{r}_i$

$$\vec{r}_i'(t) = \vec{r}_i(t) - \vec{v} t$$

$$\dot{\vec{r}}_i'(t) = \dot{\vec{r}}_i(t) - \vec{v} \qquad \vec{p}_i' = \vec{p}_i - m_i \vec{v}$$

$$H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i<j} V(\vec{r}_j - \vec{r}_i)$$

NO EXTERNAL FORCES

$$[\vec{r}_i, \vec{p}_i] = [\vec{r}_i', \vec{p}_i'] \qquad \text{CANONICAL}$$

CANONICAL $\Rightarrow \exists$ GENERATING FN

$$F_2 = \sum \vec{r}_i \cdot \vec{p}_i' + \vec{v} \cdot \left(\sum m_i \vec{r}_i - \sum \vec{p}_i' t \right)$$

$$\frac{\partial F_2}{\partial \vec{r}_i} = \vec{p}_i = \vec{p}_i' + m_i \vec{V}$$

$$\frac{\partial F_2}{\partial \vec{p}_i'} = \vec{r}_i' = \vec{r}_i - \vec{v} t$$

FIND $K(\vec{r}', \vec{p}')$ NEW HAMILTONIAN

$$K(\vec{r}_i', \vec{p}_i') = H + \frac{\partial F}{\partial t}$$

$$= \sum \frac{(\vec{p}_i' + m_i \vec{v})^2}{2m_i} + \sum_{i<j} V(\vec{r}_j - \vec{r}_i)$$

$$+ \sum_i \left(-\vec{v} \cdot \vec{p}_i' \right)$$

$$K = \sum_i \frac{\vec{p}_i'^{\,2}}{2m} + \underbrace{\frac{1}{2} \sum_i m_i \vec{V}^2}_{\frac{1}{2} M V^2} + \sum_{i<j} V(\vec{r}_j' - \vec{r}_i')$$

HEM: $\dot{\vec{r}}_i' = \dfrac{\partial K}{\partial \vec{p}_i'} = \dfrac{\vec{p}_i'}{m_i}$

$$\dot{\vec{p}}_i' = -\frac{\partial K}{\partial \vec{r}_i'} = \vec{F}_i' \quad \text{(SAME FORCE)}$$

$S \to S'$ $\qquad x' = x - \frac{v}{c} ct \qquad ct' = ct$

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{v}{c} & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

THINKING PHYSICS
RELATIVITY VISUALIZED
} EPSTEIN

DERIVE H FROM $E = mc^2$

$$m = \frac{E}{c^2} \qquad F = \dot{p} = ma \qquad H = E$$

$$F = \dot{p} = -\frac{\partial H}{\partial x}$$

$$\dot{x} = \frac{\partial H}{\partial p}$$

$$-\frac{\partial H}{\partial x} = \dot{p} = \underbrace{\frac{H}{c^2}}_{m} \underbrace{\frac{d}{dt}\left(\frac{\partial H}{\partial p}\right)}_{a}$$

$$\frac{dp}{dt} = \frac{d}{dt}\left(\frac{H}{c^2}\frac{\partial H}{\partial p}\right).$$

$$p = \frac{H}{c^2}\frac{\partial H}{\partial p} - A$$

$$\frac{1}{c^2}\int H\, dH = \int (p + A)\, dp$$

$$\frac{H^2}{2c^2} = \frac{p^2}{2} + Ap + B$$

$$H^2 = c^2 p^2 + 2Apc^2 + 2Bc^2$$

a) $p = 0 \quad H = m_0 c^2 \Rightarrow B = \dfrac{m_0^2 c^2}{2}$

b) $p \ll mc \Rightarrow H = m_0 c^2 + \dfrac{p^2}{2m} + \dots$

$$H^2 = m_0^2 c^4 + p^2 c^2 + \dots \Rightarrow A = 0$$

$$H^2 = E^2 = p^2 c^2 + m_0^2 c^4$$

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{pc^2}{\sqrt{p^2c^2 + m_0^2c^4}} = \frac{pc^2}{E} = \beta c$$

$$\beta = \frac{pc}{E}$$

HOW DO E, p TRANSFORM?

m_0 @ REST IN S'

$$E_{s'} = E_0 = m_0 c^2 \qquad p_{s'} = 0$$

S' (AND m_0) MOVING WITH V WRT S

$$p_s = mv = \frac{v}{c}\frac{E_s}{c} = \beta \frac{E_s}{c}$$

$$E_s^2 - p_s^2 c^2 = m_0^2 c^4$$

$$E_s^2 (1 - \beta^2) = m_0^2 c^4$$

$$E_s = \gamma m_0 c^2 \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$= \gamma E_{s'}$$

$$p_s = \beta\gamma \frac{E_{s'}}{c}$$

INVERT TO GET

$$\cancel{p_s' = -\beta\gamma \frac{E_s}{c}}$$

TRY $\boxed{p_s' = -\beta\gamma \frac{E_s}{c} + \gamma p_s}$

$p_s = \beta\gamma \frac{E_{s'}}{c}$

$$= -\beta\gamma \frac{\gamma E_s'}{c} + \beta\gamma^2 \frac{E_s'}{c}$$

$$= 0 \checkmark$$

$\boxed{E_{s'} = \gamma E_s - \beta\gamma c\, p_s}$

$$= \gamma\gamma E_{s'} - \beta^2\gamma^2 E_s'$$ $\gamma^2(1-\beta^2) = \underline{1}$

$$= E_s' \checkmark$$

$$V_{s'} = \frac{p_s'}{E_{s'}} c^2 = \frac{(\cancel{\gamma} p_s - \beta\cancel{\gamma}\frac{E_s}{c})}{\cancel{\gamma} E_s - \beta\cancel{\gamma} c\, p_s} c^2$$

IF $p_s' \neq 0$

$$= \frac{\frac{p_s c^2}{E_s} - \beta c}{1 - \beta c \frac{p_s}{E_s}} = \frac{V_s - V}{1 - \frac{V_s V}{c^2}}$$

β, γ REFER TO SPEED OF S' WRT S

$$\frac{dx'}{dt'} = \frac{dx - v\,dt}{dt - dx\left(\frac{v}{c^2}\right)}$$

$\Rightarrow x, t$ TRANSFORM

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} = \overleftrightarrow{L} \begin{pmatrix} ct \\ x \end{pmatrix}$$

GALILEO: $\overleftrightarrow{L} = \begin{pmatrix} 1 & 0 \\ -\beta & 1 \end{pmatrix}$

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct' \\ x' \end{pmatrix}$$

MUST HAVE

$$\begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} = \overleftrightarrow{\mathbb{1}}$$