$$x^\nu = (ct, \vec{r}) \qquad g_{\mu\nu} = g^{\mu\nu}$$

$$x_\mu = g_{\mu\nu} x^\nu = (ct, -\vec{r})$$

1-form vector

$$u^\alpha = (a, \vec{b})$$

$$x^\mu = g^{\mu\nu} x_\nu \qquad u_\alpha = (a, -\vec{b})$$

Tensors $T^{\mu\nu\ldots}_{\alpha\beta\ldots}$ Boost $\Lambda^\mu_{\ \nu}$ $x^{\mu'} = \Lambda^\mu_{\ \nu} x^\nu$

BOOST ALONG Z

$$\Lambda = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

$$x^\mu = (ct, \vec{r}) \qquad \vec{v} = \frac{d\vec{r}}{dt} \Rightarrow u^\mu = \frac{dx^\mu}{d\tau} = \gamma \frac{dx^\mu}{dt}$$

$$= (\gamma c, \gamma \vec{v})$$

$$d\tau = \frac{1}{c}\sqrt{(dct)^2 - (d\vec{r})^2}$$

$$= \sqrt{(1 - v^2/c^2)}\, dt = \frac{1}{\gamma} dt$$

$$p^\mu = m_0 u^\mu = (\gamma m_0 c, \gamma m_0 \vec{v}) = \left(\frac{E}{c}, \vec{p}\right)$$

$$\vec{\nabla} \to \partial_\mu = \left(\partial/\partial x^0, \partial/\partial x^1, \partial/\partial x^2, \partial/\partial x^3\right)$$

$$\frac{\partial j^\mu}{\partial x^\mu} = \Box \cdot J = \frac{\partial \rho c}{\partial ct} + \vec{\nabla} \cdot \vec{j} = \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j}$$

$$\Box^2 = g^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^\nu} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$

SCALAR PRODUCT

$$x^\mu x_\mu = (ct, \vec{x}) \cdot (ct, -\vec{x}) = c^2t^2 - \vec{x}^{\,2}$$

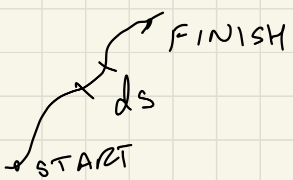
$$g_{\mu\nu} x^\mu x^\nu = (ct, \vec{x}) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} ct \\ \vec{x} \end{pmatrix}$$

$$= g^{\mu\nu} x_\mu x_\nu = (ct, -\vec{x}) \begin{pmatrix} & & \\ & g & \\ & & \end{pmatrix} \begin{pmatrix} ct \\ -\vec{x} \end{pmatrix}$$

EXPRESS ALL PHYSICAL QUANTITIES AS

1) LORENTZ SCALARS $\quad S \xrightarrow{L} S$
2) " VECTORS (OR 1-FORMS) $\quad V^\mu \xrightarrow{L} V'^\mu = L^\mu{}_\nu V^\nu$
3) " TENSORS

"DISTANCE" $\quad ds^2 = (dct)^2 - (d\vec{x})^2 \quad$ SCALAR



$$ds^2 > 0$$
$$d\tau = \sqrt{ds^2}\,/\,c$$

$$\tau = \int_{\text{START}}^{\text{FINISH}} d\tau$$

ELAPSED REST TIME (ACCORDING TO A CLOCK MOVING WITH THE PARTICLE)

PATH $x^\mu(\tau)$

$$U^\mu = \frac{dx^\mu}{d\tau} = \gamma(c, \vec{v})$$

$$U^\nu U_\nu = \gamma^2 c^2 - \gamma^2 \vec{v}^{\,2} = \gamma^2 c^2 (1 - v^2/c^2) = c^2$$

$$p^\nu = m_0 U^\nu \qquad p^0 = \gamma m_0 c = \frac{E}{c}$$

$$\vec{p} = \gamma m_0 \vec{v} = (p^1, p^2, p^3)$$
$$= -(p_1, p_2, p_3)$$

SMALL β LIMIT

$$E = p^0 c = \gamma m_0 c^2 = m_0 c^2 \frac{1}{\sqrt{1-\beta^2}}$$

$$\cancel{(1-\beta^2)^{-1/2}} \simeq f(0) + x f'(0) + \frac{1}{2} x^2 f''(0) + \dots$$

$$(1-x)^{-1/2} \simeq 1 - \frac{1}{2} x \left. \frac{-1}{\sqrt{1-x}} \right|_0 + \dots$$

$$\frac{1}{\sqrt{1-\beta^2}} \simeq \left(1 + \frac{1}{2}\beta^2\right)$$

$$E = m_0 c^2 \left(1 + \frac{1}{2}\beta^2 + \dots\right) = m_0 c^2 + \frac{1}{2} m_0 v^2 + \dots$$

γm_0 = relativistic mass

$$p_\nu p^\nu = m_0^2 u_\nu u^\nu = m_0^2 c^2$$

$$= p^{0\,2} - \vec{p}^{\,2} = \frac{E^2}{c^2} - \vec{p}^{\,2}$$

$$\Rightarrow \quad E^2 = \vec{p}^{\,2} c^2 + m_0^2 c^4$$

FORCE $\quad K^\nu = \dfrac{dp^\nu}{d\tau} = \gamma \dfrac{dp^\nu}{dt}$

PSEUDOFORCE $\quad F^\nu = \dfrac{1}{\gamma} K^\nu = \dfrac{dp^\nu}{dt} = \left(\dfrac{dp^0}{dt}, \vec{F}\right)$

$$= \left(\frac{1}{c}\frac{dE}{dt}, \vec{F}\right)$$

$$\frac{dp^\nu}{dt} u_\nu = \underbrace{\gamma \frac{dE}{dt} - \gamma \vec{F} \cdot \vec{v}}_{} = \frac{m_0}{2} \frac{d}{dt} u^\nu u_\nu$$

$$= \frac{m_0}{2} \frac{d}{dt} c^2 = 0$$

$$\frac{dE}{dt} = \underbrace{\vec{F} \cdot \vec{v}}_{} \qquad \text{POWER}$$

E + M $\qquad A^\mu = (\phi/c, \vec{A})$

LORENZ CONDITION $\quad \Box \cdot A = \dfrac{\partial A^\mu}{\partial x^\mu} = \vec{\nabla}\cdot\vec{A} + \dfrac{1}{c^2}\dfrac{d\phi}{dt}$

then $\quad \Box^2 \vec{A} = \dfrac{1}{c^2}\dfrac{\partial^2 \vec{A}}{dt^2} - \nabla^2 \vec{A} = \mu_0 \vec{J} \qquad \dfrac{1}{c^2} = \mu_0 \epsilon_0$

current

$$\Box^2 \phi = \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = \rho/\epsilon_0$$

density

USING $\quad \vec{F} = q\left(-\vec{\nabla}\phi + \frac{1}{c}\frac{\partial \vec{A}}{\partial t} + \vec{v} \times (\vec{\nabla} \times \vec{A})\right)$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

GENERALIZE TO

$$\frac{dp_\mu}{d\tau} = q\left(\frac{\partial (U^\nu A_\nu)}{\partial x^\mu} - \frac{dA_\mu}{d\tau}\right)$$

$$\frac{1}{q}\frac{dp_\mu}{d\tau} = \Box_\mu \left[U^0 A_0 - \vec{U}\cdot\vec{A}\right] - \frac{dA_\mu}{d\tau}$$

LOOK AT $\vec{P}$

$$\frac{1}{q}\cancel{\gamma}\frac{d\vec{p}}{dt} = -\vec{\nabla}\left[\cancel{\gamma} c \frac{\phi}{c} - \vec{U}\cdot\vec{A}\right) + \frac{\cancel{\gamma}\, d\vec{A}}{dt}$$

ONCE AGAIN! WITH TENSORS!

$$\frac{dp^\mu}{d\tau} = q F^\mu{}_\nu U^\nu = q F^{\mu\nu} U_\nu$$

$$c F^\alpha{}_\nu = \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & cB_z & -cB_y \\ E_y & -cB_z & 0 & cB_x \\ E_z & cB_y & -cB_x & 0 \end{pmatrix}$$

$$cF^{\mu\nu} = cF^{\mu}{}_{\sigma}\, g^{\sigma\nu} = \begin{pmatrix} & & \\ & F & \\ & & \end{pmatrix}\begin{pmatrix} & & \\ & g & \\ & & \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}.$$

$$= \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{pmatrix}$$

SIMILARLY $\quad F_{\mu\nu} = g_{\mu\sigma} F^{\sigma}{}_{\nu}$

COLLISIONS $\quad$ ENERGY-MOMENTUM CONSERVATION

$$P^{\mu}_{INIT} = \sum_i P^{\mu}_{i\,INIT} = P^{\mu}_{FIN} = \sum_j P^{\mu}_{j\,FIN}$$

E AND P ALWAYS CONSERVED IN EVERY FRAME

i) INELASTIC COLLISION

$$m_0 \xrightarrow{\vec{V}} M_0 \Rightarrow M_f, \vec{V}_f$$

$$P^{\mu}_{IN} = (\gamma m_0 c + M_0 c, \gamma m_0 \vec{V})$$

$$P^{\mu}_{FIN} = (\gamma_f M_f c, \gamma_f M_f \vec{V}_f)$$

$$(P_{fin})^2 = M_f^2 c^2 = (\gamma_f^2 M_f^2 c^2 - \gamma_f^2 M_f^2 \vec{V}^2)$$

$$= (P_{in})^2 = \underline{\gamma^2 m_0^2 c^2} + 2\gamma m_0 M_0 c^2 + M_0^2 c^2 \quad \underline{-\gamma^2 m_0^2 \vec{V}^2}$$

$$= \gamma^2 m_0^2 c^2 (1-\beta^2) = m_0^2 c^2$$

$$(P_{in})^2 = m_0^2c^2 + M_0^2c^2 + 2\gamma m_0 M_0 c^2$$

$$= (m_0 + M_0)^2 c^2 + \underbrace{2(\gamma-1)m_0 M_0 c^2}_{2T_{kin}M_0}$$

$$M_f = \left[(m_0 + M_0)^2 + \frac{2T_{kin}M_0}{c^2}\right]^{1/2} \qquad T_{kin} = (\gamma-1)m_0c^2$$

MAKE HIGHER MASS PARTICLES

E CONS: $E_f = \gamma_f M_f c = (\gamma m_0 c + M_0 c)$

$$\gamma_f = \frac{\gamma m_0 + M_0}{M_f} = \frac{\gamma m_0 + M_0}{\left[(m_0+M_0)^2 + \frac{2T_{kin}M_0}{c^2}\right]^{1/2}}$$

$$\vec{P}_f = \gamma m_0 \vec{v}$$

$$\vec{v}_f = \frac{\vec{P}_f}{E_f/c} = \frac{\gamma m_0 \vec{v}}{\gamma m_0 c + M_0 c}$$

$$\pi^+ + p \to \pi^+ + p$$
$$\to X$$

$$M_f = \left[\left[(140\,\mathrm{MeV}/c^2) + (938\,\mathrm{MeV}/c^2)\right]^2 + \frac{2T_{kin}}{c^2}\,938\,\mathrm{MeV}/c^2\right]^{1/2}$$