$\pi^+ p \rightarrow \pi^+ X$ or $p X$

$$P_X^\mu = P_{\pi^+}^\mu + P_p^\mu - P_{\pi^{+\prime}}^\mu$$

INCOMING OUTGOING

$$P_X^\mu P_{X\mu} = m_X^2 c^2$$

PROBLEM SOLVING (IF INCOMING P_i^μ KNOWN)

CALCULATE M_f

" THE LORENTZ TRANSFORM FROM INITIAL FRAME $\rightarrow$ COM FRAME

CALCULATE POSSIBLE "DECAYS"

BOOST BACK TO INITIAL FRAME

$p + p \rightarrow$ antiproton + STUFF

$$p + p \rightarrow p \ p \ p \ \bar{p}$$

$$M_X^{MIN} = 4 m_p$$

COM FRAME $P^\mu_{tot} = (M^{MIN}, \vec{0})$

$$M_f = \left[(m_0 + M_0)^2 + 2 \frac{T_{kin}}{c^2} M_0 \right]^{1/2}$$ FROM LAST TIME

$$m_0 = M_0 = m_p$$

$$(4m_p)^2 = (m_p + m_p)^2 + 2T_{kin} m_p / c^2$$

$$T_{kin} = 6 m_p c^2$$

Collider $\quad \bullet \xrightarrow{\vec{p}} \quad \xleftarrow{-\vec{p}} \bullet \qquad p^\nu = \left(\sqrt{m^2c^2 + p^2},\ \vec{p}\right)$

$$p^\nu_{tot} = \left(2\sqrt{m^2c^2 + \vec{p}^2},\ \vec{0}\right)$$

$$M_f c = 2\sqrt{m^2c^2 + \vec{p}^2}$$

$$(4m_pc)^2 = 4\left(m_p^2c^2 + \vec{p}^2\right)$$

$$|\vec{p}| = \sqrt{3}\, m_p c$$

$$\sqrt{E^2} = (M_pc^2 + T_{kin}) = \sqrt{m_p^2c^4 + \vec{p}^2c^2} = 2M_pc^2$$

$$T_{kin} = M_p c^2$$

BOOST FROM COLLIDER FRAME TO LAB FRAME

$$\gamma = \frac{p^0}{m_pc} = 2 \qquad \beta\gamma = \frac{\vec{p}}{M_pc} = \sqrt{3}$$

$$p^{\nu'} = L_\nu^{\nu'} p^\nu = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} E/c \\ p \end{pmatrix}$$

$$p'_{TGT} = \begin{pmatrix} 2 & -\sqrt{3} \\ -\sqrt{3} & 2 \end{pmatrix} \begin{pmatrix} 2 \\ \sqrt{3} \end{pmatrix} m_pc = m_pc \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$p'_{INC} = \begin{pmatrix} 2 & -\sqrt{3} \\ -\sqrt{3} & 2 \end{pmatrix} m_pc \begin{pmatrix} 2 \\ -\sqrt{3} \end{pmatrix} = m_pc \begin{pmatrix} 7 \\ -4\sqrt{3} \end{pmatrix}$$

$$(p'_{INC})^2 = \frac{E^2}{c^2} - p^2 = m_p^2c^2(49 - 48) = m_p^2c^2$$

$$(p_{TOT})^2 = (p'_{TGT} + p'_{INC})^2 = m_p^2c^2(64 - 48) = 16m_p^2c^2$$

RELATIVISTIC LAGRANGIAN

$$\delta I = \delta \int_{t_1}^{t_2} \mathcal{L}\,dt = 0$$

$$\mathcal{L} = -mc^2\sqrt{1-\beta^2} - V$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v_i} - \frac{\partial \mathcal{L}}{\partial x_i} = 0$$

$$\frac{\partial \mathcal{L}}{\partial v^i} = \frac{mv^i}{\sqrt{1-\beta^2}} = \gamma m v^i = P^i$$

ASSUMES $V = V(x^i)$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v^i} = \frac{d}{dt}\frac{mv^i}{\sqrt{1-\beta^2}} = -\frac{\partial V}{\partial x^i} = F^i$$

$\mathcal{L} \neq T - V$ BUT $\dfrac{\partial \mathcal{L}}{\partial \vec{v}} = \vec{P}$

GENERALIZE TO MORE PARTICLES AND NON CARTESIAN COORDS

$$P_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$

IF $\mathcal{L} \neq \mathcal{L}(t)$ THEN

$$h = \dot{q}^i P_i - \mathcal{L}$$

h IS A CONSTANT OF THE MOTION

$$h = \frac{mv_i v^i}{\sqrt{1-\beta^2}} + mc^2\sqrt{1-\beta^2} + V$$

$$= \frac{mc^2\beta^2}{\sqrt{1-\beta^2}} + \frac{mc^2(1-\beta^2)}{\sqrt{1-\beta^2}} + V$$

$$h = mc^2\gamma + V = (T + mc^2) + V = E$$

ADD POTENTIALS (VELOCITY-DEPENDENT)

$$\mathcal{L} = -mc^2\sqrt{1-\beta^2} - q\phi + q\vec{A}\cdot\vec{v}$$

$$p^i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i} = mu^i + qA^i$$

$$\gamma v_i = u_i \qquad u^\mu = \frac{\partial x^\mu}{\partial \tau}$$

EXAMPLES

1) CONSTANT FORCE

$$\mathcal{L} = -mc^2\sqrt{1-\beta^2} - max$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v} - \frac{\partial \mathcal{L}}{\partial x} = 0$$

$$\frac{d}{dt}\frac{mv}{\sqrt{1-\beta^2}} = ma$$

$$\frac{d}{dt}\frac{\beta}{\sqrt{1-\beta^2}} = \frac{a}{c}$$

$$\frac{\beta}{\sqrt{1-\beta^2}} = \frac{at+\alpha}{c} \qquad \alpha = v_0$$

$$\beta = \frac{at+\alpha}{\left[c^2+(at+\alpha)^2\right]^{1/2}}$$

small t $\quad \beta = \frac{at+\alpha}{c}$

large t $\quad \beta \to 1$

$$x - x_0 = c\int_0^t \beta\, dt$$

if $x_0 = 0$, $v_0 = \alpha = 0$

$$\left(x + \frac{c^2}{a}\right)^2 - c^2 t^2 = \frac{c^4}{a^2}$$

HYPERBOLA IN $x-t$ PLANE

NR: CONSTANT $a \rightarrow x = \frac{1}{2} a t^2$ PARABOLA

ACCELERATION OF e^- USING $\vec{E}$ FIELD

THE ULTIMATE SPEED W. BERTOZZI

2) HARMONIC OSCILLATOR

$$E = \frac{mc^2}{\sqrt{1-\beta^2}} + \frac{1}{2} k x^2$$

$\mathcal{L} \neq \mathcal{L}(t)$ AND $V \neq V(\dot{x})$ $\therefore$ E CONSTANT

$$(E - V)^2 = \frac{m^2 c^4}{1 - \beta^2} \qquad 1 - \beta^2 = \frac{m^2 c^4}{(E-V)^2}$$

$$\beta^2 = \frac{1}{c^2}\left(\frac{dx}{dt}\right)^2 = 1 - \frac{m^2 c^4}{(E-V)^2}$$

$V(x) = V(-x)$ $V(0)$ IS MINIMUM

IF $V_{MIN} < E < V_{MAX} \Rightarrow$ BOUND

OSCILLATORY BEHAVIOR BETWEEN

$x = \pm b$ WHERE $V(\pm b) = E$

LOOK @ RELATIVISTIC CORRECTIONS WHEN $V \ll mc^2$

$$E = mc^2(1+\epsilon) \qquad \beta = \left[1 - \frac{m^2c^4}{(E-V)^2}\right]^{1/2}$$

$$\frac{E - V}{mc^2} = 1 + \epsilon - \alpha x^2 = 1 + \alpha(b^2 - x^2)$$

$$\alpha = \frac{k}{2mc^2}$$

$$\tau = \frac{4}{c}\int_0^b \frac{dx}{\beta} \simeq \frac{4}{c}\int_0^b \frac{dx}{\left[2\alpha(b^2-x^2)\right]} \left[1 - \frac{3\alpha}{4}(b^2-x^2)\right]$$

PERIOD

TO 1ST ORDER

$$= 2\pi\sqrt{\frac{m}{k}}\left(1 - \frac{3}{16}\frac{kb^2}{mc^2}\right)$$

PERIOD IS NOW AMPLITUDE-DEPENDENT

3) CONSTANT B FIELD

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\frac{d\vec{p}}{dt} = q(\vec{v} \times \vec{B}) = \frac{q}{m\gamma}(\vec{p} \times \vec{B})$$

SINCE $\vec{F} \cdot \vec{v} = 0$, E, $P_\perp$, γ CONSTANT

$P_\parallel$ IS ALSO CONSTANT

CONSIDER $P_\parallel = 0$

$$\Omega = \frac{qB}{m\gamma} \quad \text{CYCLOTRON FREQUENCY}$$

$$F = mu^2/r \quad \text{CENTRIPETAL FORCE}$$

$$\vec{p} = m\vec{u} = m\vec{v}\gamma$$

$$\vec{p} = m\gamma r\Omega$$

$$r = \frac{p}{qB}$$

NON RELATIVISTICALLY $r = \dfrac{mv}{qB}$