Classical Mechanics - Problem Set 4

Problem 1)

Solve Goldstein's problem 6.4 on p.272. Make use of the small-angle approximation $\sin(\theta) = \theta$ and $\cos(\theta) = 1 - \frac{1}{2}\theta^2$ for all angles. Discard all products of 3 or more small quantities.

Problem 2)

Three massless rods are free to rotate independently around a common axis at the end of each rod (where they meet). The other end of each rod is attached to a mass m (all 3 masses are identical). Rods 1 and 2, 2 and 3, and 3 and 1 are each connected by identical torsional springs with torque $\tau = -\kappa(\theta - \theta_0)$, where $\theta(t)$ is the instantaneous angle between two adjacent rods, $\theta_0 = 2\pi/3$ is the equilibrium angle for the spring, and κ is a constant. Write the Lagrangian for this system, find the equation of motion for each mass and determine the normal mode frequencies and eigenvectors. What motion does each eigenvector describe?