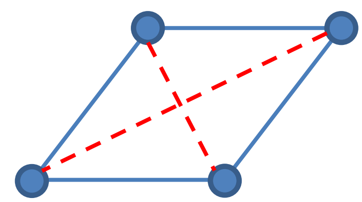
Classical Mechanics - Midterm Exam

NOTE: This midterm is take-home and due before class on Thursday, March 18. All problems are taken from Qualifying Exams. Try not to use our course book and lecture notes or the material posted on our course website (but peek if you must). You may **not** collaborate with anybody else on this midterm or use any other notes. Please email **me** if you have questions. Do not use Mathematica to solve any equations. Remember: During the actual Qualifier you will be completely on your own!

All problems are taken from the Physics Qualifying Exam

Problem 1)

Four massless rods of length L are hinged together at their ends to form a rhombus. A particle of mass m is attached at each joint. The opposite corners of the rhombus are joined by springs, each with spring constant k. In the equilibrium square configuration, the springs are unstretched. The motion is confined to a plane. Ignore the motion of the center of mass and assume that the system does not rotate.



1. The system has a single degree of freedom. Starting from the eight degrees of freedom of four unconstrained masses, explain why this system only has one degree of freedom.
2. Choose a suitable generalized coordinate and obtain the Lagrangian.
3. Deduce the equation of motion
4. Obtain the frequency of small oscillations about the equilibrium configuration.

Problem 2)

A smooth wire is bent into the shape of a spiral helix. In cylindrical polar coordinates (ρ, φ, z) it is specified by equations $\rho = R\varphi^2$ and $z = \lambda\varphi^2$, where R and λ are constants and the z axis is vertically up (and gravity is vertically down).

1. Using z as your generalized coordinate, write down the Lagrangian for a bead of mass m threaded on the wire.
2. Find the Lagrange equations of motion and find from it the expression for the bead's vertical acceleration $\ddot{z}$ as a function of z and $\dot{z}$.
3. Find acceleration $\ddot{z}$ in two limits: (i) when $R \rightarrow 0$ but λ is fixed, and (ii) when $\lambda \rightarrow \infty$ but R is fixed. Discuss if your results for $\ddot{z}$ in these limits make sense.

Problem 3)

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Two bodies move under the influence of the central-force potential $V(r) = kr^{\alpha}$ where $\vec{r}$ is the relative coordinate and k and α are constants (ignore the center-of-mass motion).

1. Assume that $\vec{r}(t)$ is a solution to the equations of motion. Show that $\vec{r}\,'(t) = \lambda\vec{r}(\lambda^{\sigma}t)$ is also a solution to the equations of motion for any constant λ, provided the exponent σ is suitably chosen. What is the value of σ?
2. Apply the result from 1. to the cases $\alpha = 2$ (harmonic oscillator) and $\alpha = -1$ (Kepler problem). Comment on the results and on the properties you can derive for them.
   (Qualifier Problem)

Hint: This does not require a lot of complicated math, just some clever argument. If you get stuck, email me – do NOT collaborate with your fellow students!